A Study of Jordan bi-derivations in Prime rings with parameters (α, β)

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Abstract:
Assume that$S$ is a prime ring with α and β as automorphisms. Bi-additive mapping If *"*$D(k^2, s) = D(k, s)\alpha(k) + \beta(k)D(k, s).$*"* then D is a *(α, β)* Jordan bi-derivation. In this study, the researcher determines the circumstances in which a symmetric Jordan bi-derivation *(α, β)* transforms into a symmetric bi-derivation *(α, β).* Furthermore, we characterise the symmetric Jordan bi-derivations *(α, β).*

Keywords: Jordan bi-derivation, Jordan derivation, Prime rings, Symmetric (α, β).

1. Introduction

The paper treats S as an associative ring, with *"Z(S)"* serving as its centre, unless otherwise specified. A prime ring (semiprime ring) is defined as "$k, l \in S, if\ kSl = \{0\}\ (kSk = \{0\})$" means that *"k = 0 or l = 0 (k = 0)."* We refer to the representations "$[k, l] = kl - lk\ and\ k \lesssim l = kl + lk$" as *"the commutator (Lie product) and the skew-commutator (Jordan product), respectively,"*for every *"k, l ∈S." "na = 0"* implies *"a = 0"* for any *"a ∈S,"* and so S is an n-torsion free ring. "$[g(k), l] \in Z(S)\ (resp. [g(k), l] = 0)\ \forall\ k, l \in S$" designates a centralising (resp. commuting) map on S for a given map g"$\forall\ k, l \in S$" is a *"derivation (resp. Jordan derivation) of an additive map*$f : S \to S$" fulfilling "$f(kl) = f(k)l + k\,f(l)\ (resp. f(k\,2) = f(k)k + k\,f(k))$". It is evident that all derivations are Jordan derivations; nevertheless, this is not always the case. Herstein [1, Theorem 4.1] demonstrated in 1957 that for any "$k, l \in S,$"*"where S is a prime ring that is not a commutative integral domain,"*"$d(klk) = d(k)lk + kd(l)k + kld(k)$" is satisfied by d, making *"the Jordan derivation of ring S a derivation."*

A *σ-derivation (resp. Jordan σ derivation)* for *"an additive map d from S to S"*is defined as follows: for any "$k, l \in S\ (resp. d(k\,2\,) = d(k)k + \sigma(k)d(k)),$" there is an accompanying endomorphism "$\sigma\ if\ d(kl) = d(k)l + \sigma(k)d(l)$". Lee[2]established in 2015 that any Jordan σ-derivation for *"a non-commutative prime ring S is a σ-derivation"*if and only if it fulfils the following condition: "$d(klk) = d(k)lk + \sigma(k)d(l)k + \sigma(kl)d(k)$" for all "$k, l \in S.$"

Bi-additive maps denote maps *"from S × S to S that are additive in both arguments:* $d(k, l) = d(l, k).$"Assumed be *"a symmetric bi-additive map"* such that for any "$k, l, s \in S,$" it satisfies the following condition: "$d(kl, s) = d(k, s)l + kd(l, s)\ (resp. d(k\,2, s) = d(k, s)k + kd(k, s)).$" This indicates that d is a symmetric bi-derivation (i.e., a symmetric Jordan bi-derivation on the contrary). Maksa[3]was the first to propose the concept of *"symmetric bi-derivation"* in 1980.Vukman[4]Bell &Daif[5] and Daif M.N.[6] then used prime and semiprime rings to verify certain conclusions relating to symmetric bi-derivation. In 2017, Abdioglu and Lee [7] studied a basic functional identity on a non-commutative prime ring S with bi-additive mappings. Specifically, they have shown that: "$Let\ B: S \times S \to Qml(S)$" be a bi-additive map and let S be a non-commutative prime ring. Assume that for every "$x, y \in S,$" "$[B(x, y), [x, y]] = 0.$" After that, for any "$x, y \in S,$" there is a bi-additive map "$\beta : S \times S \rightarrow C$" $and\ \lambda \in C\ such\ that$ "$B(x,y) = \lambda[x,y] + \beta(x,y)$". Furthermore, we may look at Jordan σ-derivation by using certain related conclusions [2, Theorem 2.1] and their application.

In this study, we developed *"symmetric (α, β) Jordan bi derivation in prime rings,"* motivated by all the findings given above. For any $k,\ s \in S$, the *"map D from S×S to S"* is defined as follows: $D(k^2, s) = D(k,s)\alpha(k) + \beta(k)D(k,s)$. In most cases, the inverse is not correct, since all bi-derivatives of *"symmetric (α, β) Jordan"* forms are also symmetric *(α, β).* We find the requirement in this article for "the (α, β) Jordan bi-derivation symmetric on the map" to become a *(α, β)* bi-derivation in prime rings *S*. Additionally, we describe "the Jordan bi-derivation" on any symmetric ring *S (α, β).*

2. Preliminaries

Lemma 2.1. [2, Theorem 2.1] Let S be a prime ring that is non-commutative and let "$f : S \times S \rightarrow D_{mr}(S)$" be a bi-additive map. When every *"k, l ∈ S"* has the value "$f(k,l)[k,l] = 0$," $then$ "$f = 0.$"

3. Results

Theorem 3.1: Assume that char(S) is not equal to 2, and that D is "a Jordan bi-derivation symmetric (α, β) on a ring S." Therefore, for any k, l, and s∈S, the following claims hold.

"$D(kl + lk, s) = D(k,s)\alpha(l) + D(l,s)\alpha(k) + \beta(l)D(k,s) + \beta(k)D(l,s)$.

(1) $D(klk, s) = D(k,s)\alpha(l)\alpha(k) + \beta(k)\beta(l)D(k,s) + \beta(k)D(l,s)\alpha(k)$.

(2) $D(klr + rlk, s) = D(k,s)\alpha(l)\alpha(r) + \beta(k)D(l,s)\alpha(r) + D(r,s)\alpha(l)\alpha(k) + \beta(r)D(l,s)\alpha(k) + \beta(r)\beta(l)D(k,s) + \beta(k)\beta(l)D(r,s)$.

(3) $D(klr + lrk, s) = D(lr,s)\alpha(k) + D(k,s)\alpha(l)\alpha(r) + \beta(k)D(lr,s) + \beta(l)\beta(r)D(k,s)$.

(4) $D(l\,2, s)\alpha(k) + \beta(k)D(l\,2, s) = D(l,s)\alpha(l)\alpha(k) + \beta(k)D(l,s)\alpha(l) + \beta(l)D(l,s)\alpha(k) + \beta(k)D(l,s)\alpha(l) + \beta(k)\beta(l)D(l,s)$."

Proof: (1) With the substitution of *"k + l"* for *k* in the formulation of the symmetric Jordan bi-derivation *(α, β),* we get the following:

$$"D((k+l)^2, s) = D(k + l, s)\alpha(k + l) + \beta(k + l)D(k + l, s)$$
$$= D(k,s)\alpha(k) + D(k,s)\alpha(l) + D(l,s)\alpha(k) + D(l,s)\alpha(l) + \beta(k)D(k,s) + \beta(l)D(k,s) + \beta(k)D(l,s) + \beta(l)D(l,s) \ldots\ldots\ldots(3.1.)$$

$$D((k+l)^2, s) = D(k^2 + kl + lk + l^2, s)$$
$$= D(k^2, s) + D(kl + lk, s) + D(l^2, s)$$
$$= D(k,s)\alpha(k) + \beta(k)D(k,s) + D(kl + lk, s) + D(l,s)\alpha(l) + \beta(l)D(l,s)." \ldots\ldots\ldots\ldots\ldots(3.2.)$$

Based on equations (3.1) & (3.2), we may deduce that

$$D(kl + lk, s) = D(k,s)\alpha(l) + D(l,s)\alpha(k) + \beta(l)D(k,s) + \beta(k)D(l,s) \ldots\ldots..(3.3.)$$

(2) By substituting l with $kl + lk$ *in* equation (3.3), we get

$$D(k(kl + lk) + (kl + lk)k, s) = D(k,s)\alpha(kl + lk) + D(kl + lk, s)\alpha(k) + \beta(kl + lk)D(k,s) + \beta(k)D(kl + lk, s) \ldots\ldots\ldots..(3.4)$$

Applying Theorem 3.1 (1) to the expression on the right side of equation (3.4), and simplifying, we obtain;

$$"D(k,s)\alpha(k)\alpha(l) + D(k,s)\alpha(l)\alpha(k) + D(k,s)\alpha(l)\alpha(k) + \beta(k)D(l,s)\alpha(k) + D(l,s)\alpha(k)\alpha(k) + \beta(l)D(k,s)\alpha(k) + \beta(k)\beta(l)D(k,s) + \beta(l)\beta(k)D(k,s) + \beta(k)D(k,s)\alpha(l) + \beta(k)\beta(k)D(l,s) + \beta(k)D(l,s)\alpha(k) + \beta(k)\beta(l)D(k,s) \ldots\ldots (3.5)"$$

Now, using equation (3.3) and calculating the left side of formula (3.4), we get

$$D(k(kl+lk)+(kl+lk)k,s) = D(k^{2}l + klk + klk + lk^{2},s)$$
$$= D(k^2l + lk^2,s) + 2D(klk,s)$$
$$= D(k^2,s)\alpha(l) + \beta(k^2)D(l,s) + D(l,s)\alpha(k^2) + \beta(l)D(k^2,s) + 2D(klk,s)$$
$$= "D(k,s)\alpha(k)\alpha(l) + \beta(k)D(k,s)\alpha(l) + \beta(k)\beta(k)D(l,s) + D(l,s)\alpha(k)\alpha(k) + \beta(l)D(k,s)\alpha(k) + \beta(l)\beta(k)D(k,s) + 2D(klk,s)" \ldots\ldots\ldots (3.6)$$

By examining equations (3.5) and (3.6), it is evident that

$$2D(klk,s) = 2D(k,s)\alpha(l)\alpha(k) + 2\beta(k)\beta(l)D(k,s) + 2\beta(k)D(l,s)\alpha(k) \ldots\ldots (3.7)$$

Given that "$char(S) \neq 2$," we may deduce the desired outcome from the last relation:

$$D(klk,s) = D(k,s)\alpha(l)\alpha(k) + \beta(k)\beta(l)D(k,s) + \beta(k)D(l,s)\alpha(k).$$

(3) By replacing k with k + r in equation (2) for any r belonging to the set S, we get;

$$"D((k + r)l(k + r),s = D(k + r,s)\alpha(l)\alpha(k + r) + \beta(k + r)\beta(l)D(k + r,s) + \beta(k + r)D(l,s)\alpha(k + r)" \ldots\ldots\ldots (3.8)$$

Now, by calculating the left-hand side of equation (3.8), we get

$$"D((k + r)l(k + r),s == D(klk + klr + rlk + rlr,s)$$
$$= D(klk,s) + D(klr + rlk,s) + D(rlr,s)$$
$$= D(k,s)\alpha(l)\alpha(k) + \beta(k)\beta(l)D(k,s) + \beta(k)D(l,s)\alpha(k) + D(klr + rlk,s) + D(r,s)\alpha(l)\alpha(r) + \beta(r)\beta(l)D(r,s) + \beta(r)D(l,s)\alpha(r) \ldots\ldots (3.9)"$$

Now, by calculating the right-hand side of equation (3.8), we get

$$"D(k,s)\alpha(l)\alpha(k) + D(r,s)\alpha(l)\alpha(k) + D(k,s)\alpha(l)\alpha(r) + D(r,s)\alpha(l)\alpha(r) + \beta(k)\beta(l)D(k,s) + \beta(r)\beta(l)D(k,s) + \beta(k)\beta(l)D(r,s) + \beta(r)\beta(l)D(r,s) + \beta(k)D(l,s)\alpha(k) + \beta(r)D(l,s)\alpha(k) + \beta(k)D(l,s)\alpha(r) + \beta(r)D(l,s)\alpha(r) \ldots\ldots (3.10.)"$$

By referring to equations (3.9) & (3.10), we determine that

$$"D(klr + rlk,s) = D(r,s)\alpha(l)\alpha(k) + D(k,s)\alpha(l)\alpha(r) + \beta(r)\beta(l)D(k,s) + \beta(k)\beta(l)D(r,s) + \beta(r)D(l,s)\alpha(k) + \beta(k)D(l,s)\alpha(r)" \ldots\ldots (3.11)$$

This is the necessary outcome.

(4) When we replace k in the formulation of the *(α, β)* Jordan bi-derivation with *k* + *lr*, we get

$$"D((k + lr)^2,s = D(k + lr,s)\alpha(k + lr) + \beta(k + lr)D(k + lr,s") \ldots\ldots (3.12.)$$

It is clear that it is possible to solve the left side of equation (3.12) using the concept of a "(α, β) Jordan bi-derivation."

$$D((k + lr)^2,s = \mathrm{D}(k^2 + \mathrm{klr} + \mathrm{lrk} + (lr)^2,\mathrm{s})$$
$$= D(k^2,s) + D(klr + lrk,s) + D((lr)^2,s)$$
$$= "D(k,s)\alpha(k) + \beta(k)D(k,s) + D(klr + lrk,s) + D(lr,s)\alpha(lr) + \beta(lr)D(lr,s)" \ldots\ldots\ldots (3.13)$$

By calculating the right side of equation ("3.12) and using the notion of a (α, β) Jordan bi-derivation," we determine that
"$D(k + lr, s)\alpha(k + lr) + \beta(k + lr)D(k + lr, s) = D(k, s)\alpha(k) + D(k, s)\alpha(lr) + D(lr, s)\alpha(k) + D(lr, s)\alpha(lr) + \beta(k)D(k, s) + \beta(lr)D(k, s) + \beta(k)D(lr, s) + \beta(lr)D(lr, s$") ...... (3.14)
By comparing equation (3.13) with equation (3.14), we may see the following relationship:

$$"D(klr + lrk, s) = D(k, s)\alpha(lr) + D(lr, s)\alpha(k) + \beta(lr)D(k, s) + \beta(k)D(lr, s)" \ldots\ldots (3.15)$$

Our task is complete.

(5) After deducting (3.11) from (3.15), we have

$$"D([r, l]k, s) = \beta(k)D(l, s)\alpha(r) + D(r, s)\alpha(l)\alpha(k) + \beta(r)D(l, s)\alpha(k) + [\beta(r), \beta(l)]D(k, s) + \beta(k)\beta(l)D(r, s) - D(lr, s)\alpha(k) - \beta(k)D(lr, s)" \ldots\ldots (3.16)$$

Specifically, when r is equal to l, equation (3.16) simplifies to

$$0 = "\beta(k)D(l, s)\alpha(l) + D(l, s)\alpha(l)\alpha(k) + \beta(l)D(l, s)\alpha(k) + \beta(k)\beta(l)D(l, s) - D(l^2, s)\alpha(k) - \beta(k)D(l^2, s)"$$

Which indicates that

$$D(l^2, s)\alpha(k) + \beta(k)D(l^2, s) = "\beta(k)D(l, s)\alpha(l) + D(l, s)\alpha(l)\alpha(k) + \beta(l)D(l, s)\alpha(k) + \beta(k)\beta(l)D(l, s)" \ldots\ldots (3.18)$$

That concludes the last equality. This proves the completion of our theorem.

Definition 3.2. Consider the ring *S* to have "$char(S) \neq 2.$" We explain what the sign means.

$$"k^{\ \ l} = D(l, s)\alpha(k) + \beta(l)D(k, s) - D(kl, s)"$$

considering each *"k, l, s ∈S."*

Presenting now is a theorem about k^l under "symmetric *(α, β)* Jordan bi-derivation" action.

Theorem 3.3. Consider a ring *S* and assume*"D* be a symmetric *(α, β)* Jordan bi-derivation" on S. Hence, the following assertions hold true for all elements *"k, ki, l, li, s"* belonging to the set *S*, where *"i"* belongs to the set *{1, 2}*.

$$(1)\ k^{\ l} + l^{\ \ k} = 0.$$
$$(2)\ k^{l_1 + l_2} = k^{l_1} + k^{l_2}$$
$$(3)\ (k_1 + k_2)^l = k_1^l + k_2^l$$

Proof: (1) Assume that *k, l, and s* are elements of the set *S.* According to Definition 3.2, we may derive that

$$"k^{\ l} + l^{\ \ k} = D(l, s)\alpha(k) + \beta(l)D(k, s) - D(kl, s) + D(k, s)\alpha(l) + \beta(k)D - D(lk, s)" \ldots\ldots (3.19)$$

The aforementioned relationship may be reformulated as

$$"k^{\ l} + l^{\ \ k} = D(l, s)\alpha(k) + \beta(l)D(k, s) + D(k, s)\alpha(l) + \beta(k)D(l, s) - D(kl + lk, s)" \ldots\ldots (3.20)$$

Utilising (2) in Theorem 3.1 found in (3.20), we obtain

$$"k^{\ l} + l^{\ \ k} = D(l, s)\alpha(k) + \beta(l)D(k, s) + D(k, s)\alpha(l) + \beta(k)D(l, s) - D(k, s)\alpha(l) - D(l, s)\alpha(k) - \beta(k)D(l, s) - \beta(l)D(k, s)" \ldots\ldots (3.21)$$

When we solve (3.21), we obtain

$$k^{\ l} + l^{\ \ k} = 0$$

Assume "$k, l_1, l_2 s \in S$." Next, based on Definition 3.2, we possess

$$"k^{l_1+l_2} = \mathrm{D}(l_1 + l_2, \mathrm{s})\alpha(\mathrm{k}) + \beta(l_1 + l_2)\mathrm{D}(\mathrm{k}, \mathrm{s}) - \mathrm{D}(\mathrm{k}(l_1 + l_2), \mathrm{s})$$
$$= \mathrm{D}(l_1, \mathrm{s})\alpha(\mathrm{k}) + \mathrm{D}(l_2, \mathrm{s})\alpha(\mathrm{k}) + \beta(l_1)\mathrm{D}(\mathrm{k}, \mathrm{s}) + \beta(l_2)\mathrm{D}(\mathrm{k}, \mathrm{s}) - \mathrm{D}(\mathrm{k}\, l_1, \mathrm{s}) - \mathrm{D}(\mathrm{k}l_2, \mathrm{s}) \ldots\ldots\ldots (3.22)"$$

This relationship may be expressed as

$$"k^{l_1+l_2} = D(l_1, s)\alpha(k) + \beta(l_1)D(k, s) - D(k\, l_1, s) + D(l_2, s)\alpha(k) + \beta(l_2)D(k, s) - D(kl_2, s)" \ldots\ldots (3.23)$$

Applying Definition 3.2 from (3.23) reveals that

$$k^{l_1+l_2} = k^{l_1} + k^{l_2}. \ldots\ldots\ldots\ldots\ldots\ldots (3.24)$$

(3) Assume that "$k_1, k_2, l, s \in S$." Next, based on Definition 3.2, we've got

$$"(k_1 + k_2)^l = D(l, s)\alpha(k_1 + k_2) + \beta(l)D(k_1 + k_2, s) - D\big((k_1 + k_2)l, s\big)$$
$$= D(l, s)\alpha(l) + D(l, s)\alpha(k_2) + \beta(l)D(k_1, s) + \beta(l)D(k_2, s) - D(k_1 l + k_2 l, s)" \ldots\ldots\ldots\ldots (3.25)$$

One way to rewrite the relationship above is as

$$"(k_1 + k_2)^l = D(l, s)\alpha(k_1) + \beta(l)D(k_1, s) - D(k_1 l, s) + D(l, s)\alpha(k_2) + \beta(l)D(k_2, s) - D(k_2 l, s)" \ldots\ldots (3.26)$$

Applying Definition 3.2 from (3.26) reveals that

$$(k_1 + k_2)^l = k^{l_1} + k^{l_2} \ldots\ldots\ldots (3.27)$$

It is not always true that the opposite is true, considering that any symmetric (α, β) bi-derivation of S is likewise "a symmetric (α, β) Jordan bi-derivation of S." In the following theorem, we will show that certain conditions applied to ring S may change "a symmetric (α, β) Jordan bi-derivation of S" into a symmetric (α, β) bi-derivation of S.

Theorem 3.4: Let S be a "non-commutative prime ring" with automorphisms α and β. A "symmetric (α, β) Jordan bi-derivation D of S" is defined as a function that fulfils characteristics (2) and (3) of Theorem 3.1, with the condition that the characteristic of S is not equal to 2. In this case, D may be seen as "a symmetric (α, β) bi-derivation of S."

Proof: By replacing the variable r with kl in equation (3) of Theorem 3.1, we obtain

$$"D(klkl + kllk, s) = D(kl, s)\alpha(l)\alpha(k) + D(k, s)\alpha(l)\alpha(kl) + \beta(kl)\beta(l)D(k, s) + \beta(k)\beta(l)D(kl, s) + \beta(kl)D(l, s)\alpha(k) + \beta(k)D(l, s)\alpha(kl)" \ldots. (3.28)$$

We will answer this problem by dividing it into two separate sections. In the first step, we address the right-hand side of equation (3.28), and subsequently, we tackle the left-hand side of equation (3.28). By solving the right side, we obtain

$$"D(kl, s)\alpha(l)\alpha(k) + D(k, s)\alpha(l)\alpha(k)\alpha(l) + \beta(k)\beta(l)\beta(l)D(k, s) + \beta(k)\beta(l)D(kl, s) + \beta(k)\beta(l)D(l, s)\alpha(k) + \beta(k)D(l, s)\alpha(k)\alpha(l)" \ldots\ldots\ldots (3.29)$$

Upon calculating the left side of equation (3.28), we determine that

$$D(klkl + kllk, s) = D\left((kl)^2 + kl^2\, k, s\right) = D\left(((kl)^2, s + D(kl^2\, k, s\right) \ldots\ldots (3.30)$$

As we can see from (2) of Theorem 3.1 in (3.30) and the formulation of the "(α, β) Jordan bi-derivation,"

$$"D(kl,s)\alpha(k)\alpha(l)+\beta(k)\beta(l)D(kl,s)+\ D(k,s)\alpha(l)\alpha(l)\alpha(k)+\beta(k)\beta(l)\beta(l)D(k,s) + \ \beta(k)\big(D(l^2,s)\big)\alpha(k)" \ldots\ldots\ldots (3.31)$$

One way to rewrite the relationship above is as

$$"D(kl,s)\alpha(k)\alpha(l)+\ \beta(k)\beta(l)D(kl,s)+\ D(k,s)\alpha(l)\alpha(l)\alpha(k)+\beta(k)\beta(l)\beta(l)D(k,s) + \ \beta(k)D(l,s)\alpha(l)\alpha(k)+\ \beta(k)\beta(l)D(l,s)\alpha(k)" \ldots.\ldots\ldots (3.32)$$

When we compare (3.29) with (3.32), we see that

$$"D(k,s)\alpha(l)\alpha(k)\alpha(l)\ +\ \beta(k)D(l,s)\alpha(k)\alpha(l)\ +\ D(kl,s)\alpha(l)\alpha(k) = D(kl,s)\alpha(k)\alpha(l)\ +\ D(k,s)\alpha(l)\alpha(l)\alpha(k)\ +\ \beta(k)D(l,s)\alpha(l)\alpha(k)."$$

One way to rearrange the equation above is as

$$"D(k,s)\alpha(l)[\alpha(k),\alpha(l)]\ +\ \beta(k)D(l,s)[\alpha(k),\alpha(l)]\ -\ D(kl,s)[\alpha(k),\alpha(l)]\ =\ 0" \ldots\ldots (3.33)$$

This indicates that

$$"(D(k,s)\alpha(l)\ +\ \beta(k)D(l,s)\ -\ D(kl,s)\ \ [\alpha(k),\alpha(l)]\ =\ 0" \ldots\ldots\ldots (3.34)$$

Applying Lemma 2.1 from equation (3.34), we derive

$$"(D(k,s)\alpha(l)\ +\ \beta(k)D(l,s)\ -\ D(kl,s)\ \ =\ 0" \ldots\ldots. (3.35)$$

In other words,

$$"D(kl,s) =\ D(k,s)\alpha(l)+\ \beta(k)D(l,s)" \ldots\ldots\ldots.. (3.36)$$

The equation mentioned above represents “a symmetric (α, β) bi-derivation of S,” indicating that the task is complete.

4. Conclusion

A prime ring S is studied in particular with respect to "symmetric (α, β) Jordan bi-derivations D" on this ring, where α and β are automorphisms of S. Theorem 3.1 defines the features of "the (α, β) Jordan bi-derivation D on a ring S." In the end, we provided the conditions under which the symmetric (α, β) Jordan bi-derivation D becomes a symmetric (α, β) bi-derivation on a prime ring S.

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