

Modeling Human Heart Blood Circulation Using Hexa Graph Topology

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Abstract: Introducing the concept of heart blood circulation through specific topologies the Right Hexa Graph Topology and the Left Hexa Graph Topology provides an innovative and structured approach to represent the connectivity and flow of blood within the human heart. These topological models abstractly capture the physiological pathways while maintaining the essential features of the circulatory system. In this paper, the human heart's anatomy is modeled as a directed graph, where vertices correspond to key anatomical structures such as the superior vena cava, right atrium, tricuspid valve, pulmonary veins, left atrium, mitral valve, and others, and edges represent the directional flow of blood between them. Two distinct hexa graph frameworks are developed: one for deoxygenated blood flow from the body to the lungs (Right Hexa Graph Topology), and another for oxygenated blood flow from the lungs to the body (Left Hexa Graph Topology). For each pathway, the lower hexa subgraphs, upper hexa subgraphs, and boundary hexa subgraphs are explicitly constructed. The resulting topologies offer a systematic framework to visualize, understand, and analyze the dynamic blood circulation process, emphasizing critical transitions and structural interconnections in the human heart.

Keywords: Hexa graph through lungs to body , Hexa graph through body to lungs

1. Introduction and preliminaries

In graph theory, a hexagon represents a six-node configuration. By applying this concept to heart blood circulation, we can model the flow of blood through the heart and its major vessels using hexagonal graph structures. This approach helps in visualizing how blood circulates through the heart's chambers and the associated major vessels. S.Santhiya, R.Mohanapriya [19] introduced Hexa graph topological space. Based on this paper we create this article.

Blood circulation. Blood circulation in the human body can be viewed through various topological perspectives, often described in terms of flow patterns and pathways. Understanding these topologies helps us grasp how blood circulates through different systems, including the heart, arteries, veins, and capillaries. Here are some key topological aspects of human blood circulation:

Systemic Circulation: Topology Involves a series of arteries and veins arranged in a roughly hierarchical manner. Oxygenated blood flows from the left ventricle of the heart into the aorta, branching into smaller arteries and arterioles, eventually reaching the capillaries. Here, gas and nutrient exchange occurs. Deoxygenated blood then returns through venules and veins to the right atrium of the heart.

Pulmonary Circulation: Blood flows from the right ventricle to the lungs via the pulmonary arteries. In the lungs, blood exchanges carbon dioxide for oxygen in the capillaries surrounding the alveoli. Oxygenated blood then returns to the left atrium of the heart via the pulmonary veins.

Hepatic Portal Circulation: Topology involves Blood from the gastrointestinal tract and spleen is collected by the hepatic portal vein and delivered to the liver. This pathway allows the liver to process nutrients and toxins before blood reaches the systemic circulation. After processing, the blood returns to the systemic circulation via the hepatic veins.

Coronary Circulation: Topology involves the arteries and veins that supply blood to and drain blood from the heart muscle itself. The coronary arteries branch from the aorta and supply the heart muscle with oxygenated blood. Deoxygenated blood is collected by the cardiac veins and returned to the right atrium via the coronary sinus.

Renal Circulation: Topology involves Blood flows from the abdominal aorta into the renal arteries, which supply the kidneys. The kidneys filter the blood, and the filtered blood exits through the renal veins to return to the inferior vena cava.

Cerebral Circulation: Topology involves blood flow to the brain is primarily supplied by the internal carotid arteries and the vertebral arteries. These arteries form a network of interconnected arteries at the base of the brain, known as the Circle of Willis, which ensures consistent blood supply to the brain. Blood is drained from the brain via the venous sinuses and internal jugular veins.

Arteriovenous Shunts: These are direct connections between arteries and veins, bypassing the capillary network. They play roles in thermoregulation and blood flow regulation in certain tissues.

Collateral Circulation: When a major artery is obstructed, collateral circulation can provide alternative routes for blood flow. This system of smaller vessels can expand and create new pathways to ensure adequate blood supply to tissues.

Definition 1.1. Let G be an undirected graph with at least 3 edges and let $J_h(S)$, $h = 1, 2, 3, 4, 5, 6$ denotes six graph decomposition of G . Consider six subgraph $S_1, S_2, S_3, S_4, S_5, S_6$ are subsets of the edge set $E(G)$ where each $S_i \subseteq E(G)$ for $i = 1, 2, 3, 4, 5, 6$ and $\zeta_H(S_i) = \{G, \phi, H^-(S_i), H^+(S_i), B_h(S_i)\}$. Then a pair $\zeta_H(S_i)$ is known as Hexa graph topology on G with respect to the subgraph S_i of G provided

that $\zeta_H(S_i)$ satisfies the following axioms

- (1) $G, V_o \in \zeta_H(S_i)$, where $G = \text{full graph}$, $V_o = \text{Null graph}$
- (2) Any union of elements of $\zeta_H(S_i)$ is in $\zeta_H(S_i)$

(3) The finite intersection of the elements of $\zeta_H(S_i)$ is in $\zeta_H(S_i)$.

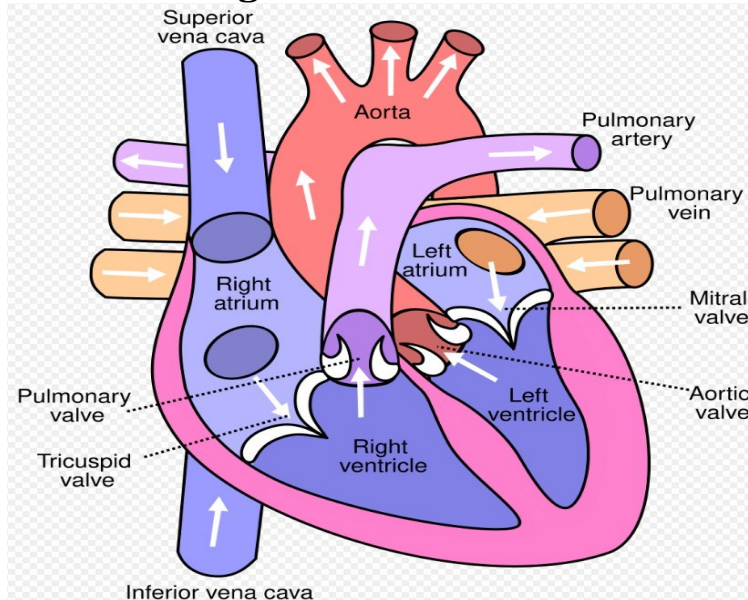
We call the pair $(G, \zeta_H(S_i))$ as the Hexa graph topological space on G . The elements of Hexa graph topological spaces are regard as Hexa open subgraph of G and the complement is the Hexa open subgraph of G induced by the edge set is called Hexa closed subgraph of G . A subgraph which is Hexa open and Hexa closed is called Hexa clopen subgraph.

Summary of Blood Flow Through the Heart:

- (1) Deoxygenated blood returns to the heart via the superior and inferior vena cavae into the right atrium.
- (2) The right atrium contracts, pushing blood through the tricuspid valve into the right ventricle.
- (3) The right ventricle contracts, sending blood through the pulmonary valve into the pulmonary arteries, which transport it to the lungs.
- (4) Oxygenated blood returns from the lungs to the left atrium via the pulmonary veins. The left atrium contracts, pushing blood through the mitral valve into the left ventricle.
- (5) The left ventricle contracts, sending oxygenated blood through the aortic valve into the aorta, distributing it to the rest of the body.

This intricate system of blood flow ensures that the heart muscle itself, along with the rest of the body, receives a continuous and sufficient supply of oxygen and nutrients, while also removing metabolic wastes.

Diagram of Human Heart:



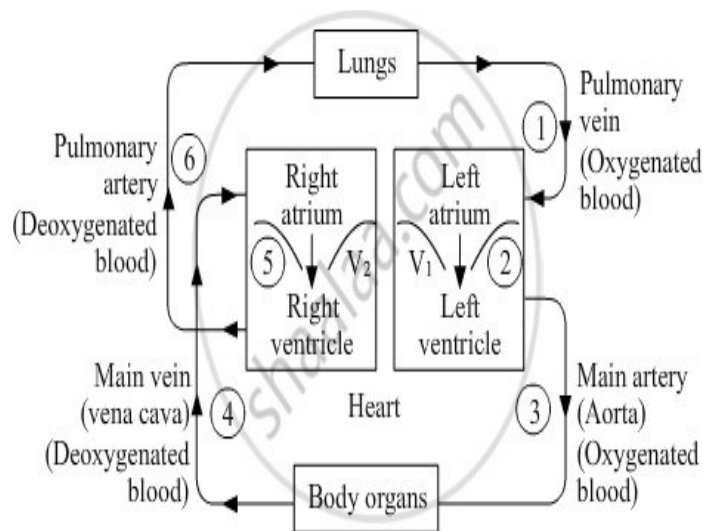
Representation of Hexa graph topology For Blood Circulation To Human Heart

Let G be a graph of Human Heart with vertex are $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and edges of Human Heart are SP(Superior vena cava),IV(Inferior vena cava),RA(Right atrium),Tricuspid valve(TV),RV(Right ventricle),PV(Pulmonary Valve),PA(pulmonary

arteries),PVE(Pulmonary veins),LA(Left atrium),MV(Mitral valve),LV(Left ven- tricle),AV(Aortiv valve),Aorta.

deoxygenated blood	oxygenated blood
inferior/superior vena cava	pulmonary veins
right atrium	left atrium
tricuspid valve	mitral or bicuspid valve
right ventricle	left ventricle
pulmonary arteries	aortic valve
lungs	aorta

Working of Human Blood Circulation Heart:



Representation of Hexa graph topology from lungs to body

Each of these elements will be considered as vertices in the graph, and we'll connect them in a sequence that reflects the blood flow:

Vertices: L, PV, LA, MV, LV, AV, B

Edges:

- (L, PV) — Blood flows from the lungs to the pulmonary veins.
- (PV, LA) — Blood flows from the pulmonary veins to the left atrium.
- (LA, MV) — Blood flows from the left atrium through the mitral valve.
- (MV, LV) — Blood flows from the mitral valve to the left ventricle.
- (LV, AV) — Blood flows from the left ventricle through the aortic valve.
- (AV, B) — Blood flows from the aortic valve to the body.

then its Hexa graph topology through lungs to body is defined by

C	$J_h(c), h = 1, 2, 3, 4, 5, 6$	$H-(C)$
$\{L, PV\}$	$\{\{L, PV, LA\}, \{MV, LV\}, \{AV, B\}\}$	ϕ
$\{PV, LA\}$	$\{\{L, LA, MV\}, \{PV, LV\}, \{AV, B\}\}$	ϕ
$\{LA, MV\}$	$\{\{L, LA\}, \{PV, MV\}, \{LV\}, \{AV, B\}\}$	ϕ
$\{MV, LV\}$	$\{\{L, MV\}, \{PV, LA\}, \{LV\}, \{AV, B\}\}$	$\{LV\}$
$\{LV, AV\}$	$\{\{L, PV, MV\}, \{LA, LV\}, \{AV, B\}\}$	ϕ
$\{AV, B\}$	$\{\{L, PV\}, \{LA, MV\}, \{LV\}, \{AV\}, \{B\}\}$	$\{AV, B\}$

- $H-(C)$
 $\{L, PV, LA\}$
 $\{L, LA, MV, PV, LV\}$
 $\{L, LA, PV, MV\}$
 $\{L, MV, LV\}$
 $\{LA, LV, AV, B\}$
 $\{AV, B\}$

Here C be the set of edges of the blood circulation through lungs - pulmonary veins, pulmonary veins-left atrium, left atrium-mitral valve, mitral valve - left ventricle, left ventricle-aortic valve, aortic valve to body of G . Hence the Lower Hexa subgraph and Upper Hexa subgraph and Boundary hexa subgraph is $H-(N_i) = \{LV, AV, B\}$, $H^-(N_i) = \phi$, $B_h(N_i) = \{LV, AV, B\}$ then $\zeta_H(N_i) =$

$\{G, \phi, \{LV, AV, B\}\}$ is a Hexa graph topology on G .

Again take body(B), IV(inferior/superior vena cava), RA(Right atrium), TV(Tricuspid valve), RV(Right ventricle), PA(pulmonary arteries), lungs(L) be the edges of the blood circulation graph through body to lungs then its Hexa graph topology through body to lungs is defined by

C	$J_h(c), h = 1, 2, 3, 4, 5, 6$	$H-(C)$
$\{B, IV\}$	$\{\{B, IV, RA\}, \{TV, RV\}, \{PA, L\}\}$	ϕ
$\{IV, RA\}$	$\{\{B, RA, TV\}, \{IV, RV\}, \{PA, L\}\}$	ϕ
$\{RA, TV\}$	$\{\{B, RA\}, \{IV, TV\}, \{RV\}, \{PA, L\}\}$	ϕ
$\{TV, RV\}$	$\{\{B, TV\}, \{IV, RA\}, \{RV\}, \{PA, L\}\}$	$\{RV\}$
$\{RV, PA\}$	$\{\{B, IV, TV\}, \{RA, RV\}, \{PA, L\}\}$	ϕ
$\{PA, L\}$	$\{\{B, IV\}, \{RA, TV\}, \{RV\}, \{PA\}, \{L\}\}$	$\{PA, L\}$

- $H-(C)$
 $\{B, IV, RA\}$
 $\{B, RA, TV, IV, RV\}$
 $\{B, RA, IV, TV\}$
 $\{B, TV, RV\}$
 $\{RA, RV, PA, L\}$
 $\{PA, L\}$

Given these vertices and edges, the graph can be visualized as a directed path: $B \rightarrow IV \rightarrow RA \rightarrow TV \rightarrow RV \rightarrow PA \rightarrow L$. Hence the Lower Hexa subgraph and Upper Hexa subgraph and Boundary hexa subgraph is $H-(N_i) = \{RV, PA, L\}$, $H^-(N_i) = \phi$, $B_h(N_i) = \{RV, PA, L\}$ then $\zeta_H(N_i) = \{G, \phi, \{RV, PA, L\}\}$ is a Hexa graph topology on G .

Conclusion: The study successfully demonstrates that human blood circulation through the heart can be effectively modeled using hexa graph topology. By defining vertices and edges corresponding to physiological structures and blood

flow paths, both the lungs-to-body (oxygenated blood) and body-to-lungs (de-oxygenated blood) circulations are represented through structured hexa graphs. The identification of lower hexa subgraphs, upper hexa subgraphs, and boundary hexa subgraphs further refines the understanding of critical junctures in blood movement. This topological representation not only aids in the mathematical modeling of biological systems but also offers a promising approach for future applications in medical diagnostics, simulation studies, and educational tools.

ACKNOWLEDGEMENTS

we would like to thank editors and the unidentified reviewers for their valuable comments and ideas which led to significant improvements in the paper

DECLARATIONS

- Consent to participate -Not Applicable
- Consent for publication -Not Applicable
- Author's contributions :
 Authour 1 - Review and Editing the Manuscript
 Authour 2 - Wrote the Manuscript, Prepared the content, and typed. Authour 3 - Wrote the Manuscript, Prepared the content, and typed. Authour 4 - Wrote the Manuscript, Prepared the content, and typed. Authour 5 - Wrote the Manuscript, Prepared the content, and typed. Authour 6 - Wrote the Manuscript, Prepared the content, and typed.
- Compliance with Ethical Standards:
 This article does not contain any studies with human participants or animals performed by any of the authors.
- Conflict of interest/Competing interests:
 Author 1 declares that she has no conflict of interest.
 Author 2 declares that she has no conflict of interest
 Author 3 declares that she has no conflict of interest
 Author 4 declares that she has no conflict of interest
 Author 5 declares that she has no conflict of interest
 Author 6 declares that she has no conflict of interest
- Funding -Not Applicable

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