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AN APPLICATION OF MEDICAL DIAGNOSIS USING INTUITIONISTIC FUZZY TREE AUTOMATA

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ABSTRACT

This paper is inspired by the Intuitionistic fuzzy Medical diagnosis that involves intuitionistic fuzzy relation, a new approach is proposed as the intuitionistic fuzzy tree automata to using the application of Medical diagnosis.

1 INTRODUCTION:

An Automata is a mathematical theory in investigates behavior, structure and their relationship to discrete systems[2]. Automata has applied in different areas., viz., logic programming, decision making problem, etc[5,10,11]. The theory of Sanchez's method of medical diagnosis using the notion of IFS theory[8,9], the method of intuitionistic medical diagnosis involves the new approach of Intuitionistic fuzzy tree automata. The theory of tree automata and tree languages emerged in the middle of the 1960s from the algebraic approach to finite automata advocated by buchi and wright Tree automata play an important role in computer science. A tree automaton is a type of state machine[4]. Tree automata[3] deal with tree structures, rather then the strings of more conventional state machine.

Using the notion of intuitionistic fuzzy sets [1]. The natural generalization of a fuzzy language as it is characterized by two functions expressing the degree of belongingness and the degree of non- belongingness. An intuitionistic fuzzy language is called intuitionistic fuzzy regular language, if the finite membership and non-membership values between [0,1][6,7].

The paper is organized as follows. In section 2, we recall the definitions of intuitionistic fuzzy automata. In section 3, we introduce the medical diagnosis of intuitionistic fuzzy tree automata.

2 Preliminaries:

Definition 2.1. Let a set 'E'be fixed. An intuitionistic fuzzy set 'A' *in'E'is an object having* the form $A = \{(x, (x), \gamma(x)) | x \in E\}$
where, the functions $\mu_A(x) : E \to [O, 1]$ and $\gamma_A(x) : E \to [O, 1]$ define the degree of membership and the degree of non membership of the element $x \in E$ to the set 'A',

the subset of 'E' respectively, and for every $x \in E$; $0 < \mu_A(x) + \gamma_A(x) \leq 1$.

Definition 2.2. An Intuitionistic fuzzy automata is a five tuple

$IM = (P, X, \delta, p_0, F)$ *where*,

P is a finite non-empty set of states,

X is a finite non-empty set of inputs,

δ: $P \times X \times P \rightarrow [O, 1]$is called the intuitionistic fuzzy transition function.

p_0: is called the initial state and

F: is called the set of final states.

Definition 2.3 *Let* $IM = (P, X, \delta, p_0, F)$ be an intuitionistic fuzzy automaton. Then the language $(IM) = \{(x, (\mu, \gamma)) | x \in X^*, \delta(p_0, w, q_f) = (\mu, \gamma), q_f \in F\}$is called the intuitionistic fuzzy regular language accepted by intuitionistic fuzzy automat on IM

Definition 2.4 FUZZY FINITE TREE AUTOMATA

Let w be a positive integer denoted by $[w]$ the set $\{0, 1, 2, \cdots, w - 1\}$. Let $[w]^*$ be a set of all strings over $[w]$ including the null string ε. A finite subset D of $[w]^*$ is called a finite tree domain if the following conditions hold:

1. $x \in D$ and $x = pq$ implies $p \in D$, where $p,q,x \in [w]^*$;
2. $xw \in D$ and $y \leq w$ implies $xy \in D$, where $x \in [w]^*, y, w \in [w]$.

Let S be a nonempty set of alphabets. A finite w - ary tree is a mapping $t : D \rightarrow S$. We denote IT by the set of all w - ary trees on S. The elements of D are called the nodes of the tree. If $x \in D$ is a node, any node of the form xy for $y \in [w]$ is called a child of x. The height of a finite tree t is the maximal length of the elements of D.

The boundary of t, denoted $Br(t)$ is the set

$$Br(t) = \{x \in D \mid x[w] \cap D = \varphi\}$$

The elements of $Br(t)$ are usually called the leaves of the tree. The outer boundary of t, denoted by $Br^+(t)$ is the set

$$Br^+(t) = D[w] - D$$

formed of all $xy \in D$ such that $x \in D$ and $y \in [w]$ and set $D^+(t) = D \cup Br^+(t)$.
A path P through the tree t is a sequence $v_0, v_1, v_2, \cdots, v_n$ of successive nodes string with the root $v_0 = ε$.

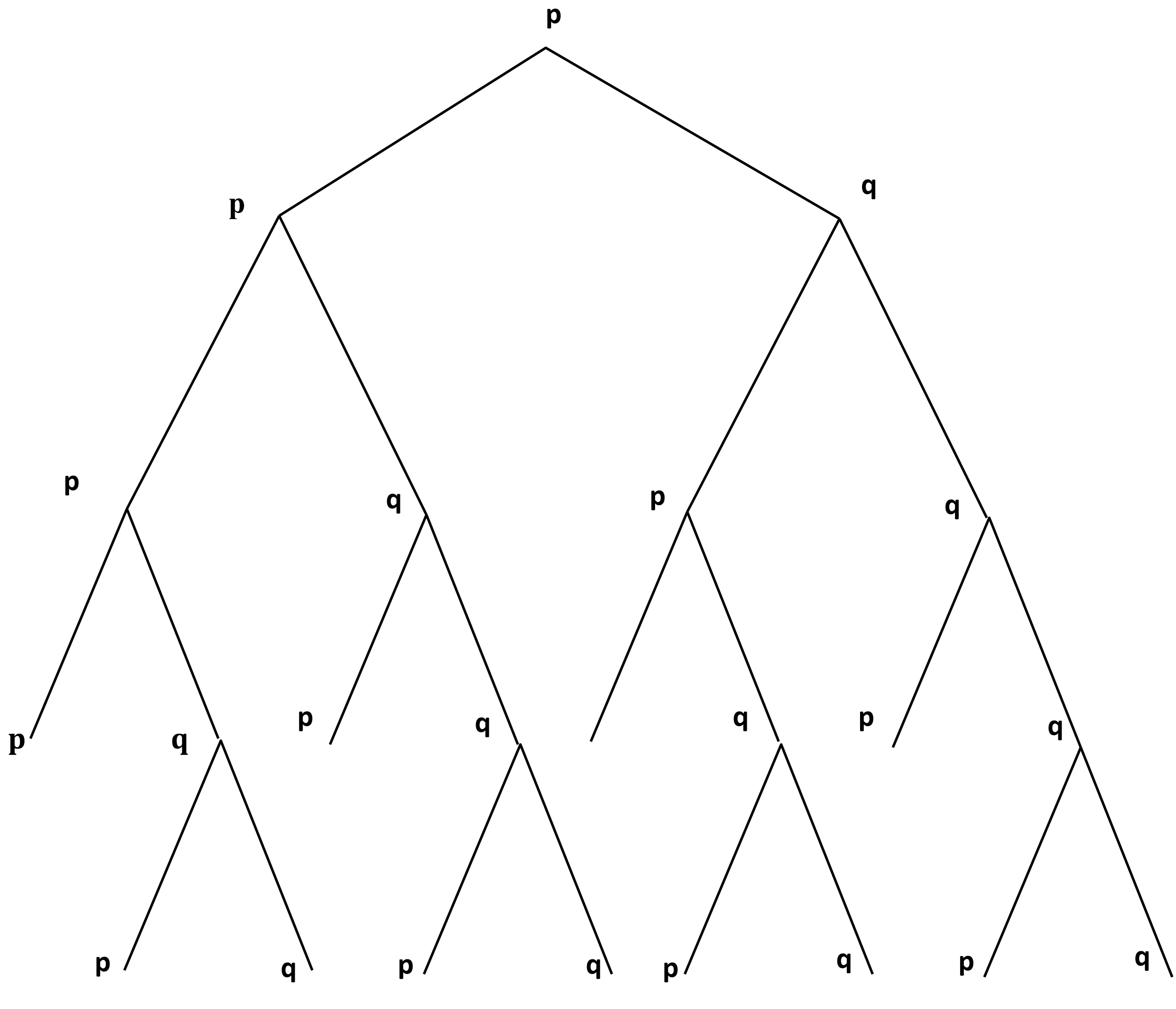

Figure 2 .1: Finite tree t

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Consider the finite binary tree t shown in Figure 2.1, the domain of the tree D = {ε, a, b, aa, ab, ba, bb, aaa, aab, aba, abb, baa, bab, bba, bbb, aaba, aabb, abba, abbb, baba, babb, bbba, bbbb} is presented in Figure 2. The boundary of t is

$Br(t)$ = {aaba, aabb, abba, abbb, baba, babb, bbba, bbbb }

and outer frontier of t is $Br^{+}(t)$ = {aabaa, aabab, aabba, aabbb, abbaa, abbab,
abbba, abbbb, babaa, babab, babba, babbb, bbbaa, bbbab, bbbba, bbbbb }
and

finally, $D^{+}(t)$ = {ε, a, b, aa, ab, ba, bb, aaa, aab, aba, abb, baa, bab, bba, bbb, aaba, aabb, abba, abbb, baba, babb, bbba, bbbb, aabaa, aabab, aabba, aabbb, abbaa, abbab, abbba, abbbb, babaa, babab, babba, babbb, bbbaa, bbbab, bbbba, bbbbb}

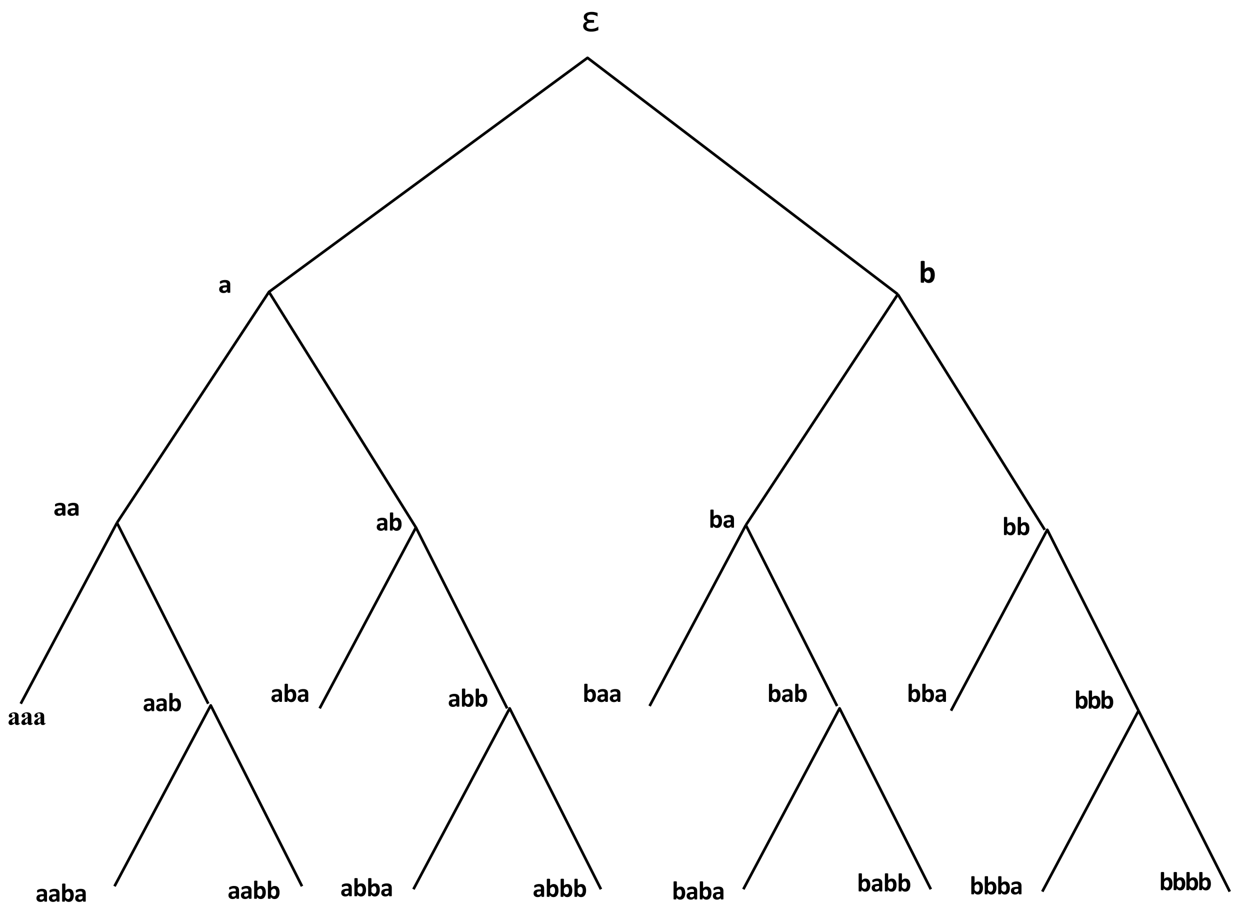

Figure 2.2: Domain of finite tree.

Definition 2.5 INTUITIONISTIC FUZZY FINITE TREE AUTOMATA

An Intuitionistic fuzzy finite tree automaton is a 5 - tuple

ITA = (P, I, q, p, F), where

- P is the finite set of states.
- I is the finite set of input alphabets
- $q : P \times I \times P^n \longrightarrow [0,1]$ is a intuitionistic fuzzy transition function.
- $p : P \rightarrow [0,1]$ is a intuitionistic fuzzy set of initial states.
- $F : P \rightarrow [0,1]$ is a intuitionistic fuzzy set of final states.

A run of the intuitionistic fuzzy finite tree automaton IFA on a tree t is a finite tree $Z : D^+ \rightarrow S$ with $I(Z(\varepsilon)) > (1,0)$ such that $q(Z(x), t(x), Z(x_0), Z(x_1), \cdots, Z(x(n-1))) > (d_1, d_2)$, where $0 < d_1 + d_2 \leq 1$ for all $x \in D$. The run R is successful if $F(Z(v)) > (0,1)$ for all v on the outer frontier $Br^+(t)$ of t.

Let Z denote the set of all different runs of F on a tree t. The weight of an accepted tree $t(d_1,d_2)$ is calculated as follows:

$$W(d1) = \vee_{Z\in Z}\{\wedge\{\{I(Z(\varepsilon))\}, \wedge\{q\ (Z(x), t(x), Z(x_0), Z(x_1), \cdots, Z(x_{(n-1)})\},$$

$$\wedge\{F(Z(v)) | v \in Br^+(t)\}\}.$$

$$W(d2) = \wedge_{Z\in Z}\{\vee\{\{I(Z(\varepsilon))\}, \vee\{q(Z(x), t(x), Z(x_0), Z(x_1), \cdots, Z(x_{(n-1)})\},$$

$$\vee\{F(Z(v)) | v \in Br^+(t)\}\}.$$

The set T of finite tree languages recognized by an intuitionistic fuzzy finite tree automaton is formed of all trees t such that there is a successful run R on t. An intuitionistic fuzzy set $d : T \longrightarrow (d_1, d_2)$, where $0 < d_1 + d_2 \leq 1$ of intuitionistic fuzzy finite tree languages is recognizable if there is a intuitionistic fuzzy tree automaton IF such that $W(t) = d(t)$ where d(t)=$0 < d_1 + d_2 \leq 1$.

Where,

$$W(d1) = \vee_{Z\in Z}\{\wedge\{\{I(Z(\varepsilon))\}, \wedge\{q\ (Z(x), t(x), Z(x_0), Z(x_1), \cdots, Z(x_{(n-1)})\}, \wedge\{F(Z(v)) | v \in Br^+(t)\}\}.$$

$$W(d2) = \wedge_{Z\in Z}\{\vee\{\{I(Z(\varepsilon))\}, \vee\{q(Z(x), t(x), Z(x_0), Z(x_1), \cdots, Z(x_{(n-1)})\}, \vee\{F(Z(v)) | v \in Br^+(t)\}\}.$$

3 Medical diagnosis to using Intuitionistic fuzzy Tree automata

In this section we present an application of intuitionistic fuzzy set theory in Sanchez's approach for medical diagnosis. In a given pathology, suppose

S is a set of symptoms, D a set of diagnoses, and P a set of patients. Analogous to Sanchez's notion of "Medical Knowledge" we dene "Intuitionistic Medical Knowledge" as an intuitionistic fuzzy tree automata from the set of symptoms S to the set of diagnoses D

(i.e., on S → D) which reveals the degree of association and the degree of non-association between symptoms and diagnosis.

Now let us discuss intuitionistic fuzzy medical diagnosis.

The methodology involves mainly the following three jobs:

1. Determination of symptoms.
2. Formulation of medical knowledge based on intuitionistic fuzzy set.
3. Determination of diagnosis on the basis of intuitionistic fuzzy tree automata.

Let IFS of the set S; and be an Intuitionistic fuzzy tree automata from S to D. Then the transition of B of IFS with the IFTA (S → D)

An Intuitionistic fuzzy finite tree automaton is a 5 - tuple

ITA = (S, D, q, p, F), where

- S is the finite set of symptoms.
- D is the finite set of diagnosis
- $q : P \times I \times P^n \longrightarrow [0,1]$ is a intuitionistic fuzzy transition function.
- $p : P \rightarrow [0,1]$ is a intuitionistic fuzzy set of initial states.
- $F : P \rightarrow [0,1]$ is a intuitionistic fuzzy set of final states.

intuitionistic fuzzy finite tree automaton IFA on a tree t is a finite tree $R : S^+ \rightarrow D$ with such that $q(S,W) > (d_1, d_2)$, for all W=w_1a where $0 < d_1 + d_2 \leq 1$ for all $x \in S$. Then

$F_w = \mu_R - \nu_R \cdot \pi_R$ is greatest, $> (0,1)$.

Let Z denote the set of all different runs of F on a tree t. The weight of an accepted tree t (d_1,d_2) is calculated as follows:

$W(d1) = \vee_{w\in W}\{\{S\}, \wedge\{q(s,d) \wedge q(d,p)\}, \wedge\{F(w)\}\}$.

$W(d2) = \wedge_{w\in W}\{\vee\{S\}, \vee\{q(s,d) \vee q(d,p)\}, \vee\{F(w)\}\}$.

If the state of a given patient P is described in terms of an IFS A of S, then P is assumed to be assigned diagnosis in terms of IFS B of D, through an IFR R of "Intuitionistic Medical Knowledge" from S to D which is assumed to be given by a doctor who is able to translate his own perception of the intuitionism involved in degrees of association and non-association respectively between symptoms and diagnosis.

Now let us extend this concept to a finite number of patients. Let there be n patients p_i, i = 1,2,…,n, in a hospital. Thus $p_i \in P$. Let IFTA be an $(S \to D)$ and the state of patients p_i in terms of the diagnosis as an IFTA from P to D given by the membership function.

4 Case study

The intuitionistic fuzzy relation $Q(P \to S)$ is given as in (hypothetical) Table:1.

Let the set of Diagnosis be D = {Viral Fever, Malaria, Typhoid, Stomach Problem, Heart Problem}.

Let there be four patients Paul, Jadu, Kundu and Rohit in a hospital at Calcutta. Their symptoms

are Temperature, Headache, Stomach-Pain, Cough and Chest-Pain. Clearly,

P = {Paul, Jadu, Kundu, Rohit} and the set of symptoms

S = {Temperature, Headache, Stomach-Pain, Cough, Chest-Pain}

The intuitionistic fuzzy relation $R(S \to D)$ is given as in (hypothetical) Table:2.

Therefore, the transition $T = R \times Q$ is as given in Table:3.

In Final state, we calculate S_R as given in Table:4.

Table:1 Q (P → S)

Q	Temperature	Headache	Stomach-Pain	Cough	Chest-Pain
Paul	(0.3, 0.7)	(0.1,0.6)	(0.8,0.4)	(0.4,0.7)	(0.3,0.7)
Jadu	(0.5,0.2)	(0.1,0.3)	(0.7,0.1)	(0.1,0.8)	(0.1,0.3)
Kundu	(0.2,0.6)	(0.6,0.8)	(0.8,0.1)	(0.2,0.3)	(0.1,0.5)
Rohit	(0.4,0.1)	(0.2,0.4)	(0.5,0.1)	(0.2,0.7)	(0.9,0.5)

Table:2 R (S → D)

R	Viral Fever	Malaria	Typhoid	Stomach Problem	Heart Problem
Temperature	(0.1,0.5)	(0.8,0.2)	(0.5,0.1)	(0.3,0.1)	(0.9,0.7)
Headache	(0.7,0.2)	(0.8,0.3)	(0.6,0.3)	(0.4,0.2)	(0.5,0.3)
Stomach-Pain	(0.3,0.1)	(0.9,0.5)	(0.2,0.1)	(0.3,0.7)	(0.6,0.1)
Cough	(0.6,0.4)	(0.1,0.7)	(0.7,0.9)	(0.5,0.1)	(0.8,0.5)
Chest-Pain	(0.2,0.8)	(0.3,0.2)	(0.1,0.6)	(0.1,0.2)	(0.4,0.3)

Table:3 → T = R x Q

T	Viral Fever	Malaria	Typhoid	Stomach Problem	Heart Problem
Paul	(0.1,0.8)	(0.1,0.7)	(0.1,0.9)	(0.1,0.7)	(0.1,0.7)
Jadu	(0.1,0.8)	(0.1,0.8)	(0.1,0.9)	(0.1,0.8)	(0.1,0.8)
Kundu	(0.1,0.8)	(0.1,0.8)	(0.1,0.9)	(0.1,0.8)	(0.1,0.8)
Rohit	(0.1,0.8)	(0.1,0.7)	(0.1,0.9)	(0.1,0.7)	(0.2,0.7)

Table:4 → $S_R = \frac{\mu x\, Q}{\pi}$

S_R	Viral Fever	Malaria	Typhoid	Stomach Problem	Heart Problem
Paul	0.08	0.07	0.09	0.07	0.07
Jadu	0.08	0.08	0.09	0.08	0.08
Kundu	0.08	0.08	0.09	0.08	0.08
Rohit	0.08	0.07	0.09	0.07	0.14

⇒ Paul, Jadu, Kundu →Typhoid (Score 0.09)

⇒ Rohit → Heart Problem (Score 0.14)

From Table:4 it is obvious that, if the doctor agrees, then Paul, Jadu, Kundu suffer from Typhoid

whereas Rohit faces Heart Problem.

In case of medical diagnosis, there is a fair chance of the existence of a non-zero hesitation part at each moment of evaluation of any unknown object.

CONCLUSION

We have studied that an application of medical diagnosis is intuitionistic fuzzy set and then discussed Sanchez's approach for medical diagnosis concept is generalized to intuitionistic fuzzy tree automata. The non-membership functions have more important roles here in comparison to the membership function corresponding to the complement of fuzzy set because of fact that in decision making problems, particularly in case of medical diagnosis, there is a fair chance of the existence of a non-zero hesitation part at each moment of evaluation of any unknown object.

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