Exploring the Parameters of Fractional Domination Number in Litact Graph

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Abstract

The aim of this article is to investigate the concept of the fractional domination number of the Litact graph by establishing the bounds and relationships between the fractional domination number and other parameters in Litact graphs. This leads to a significant enhancement in resource allocation. The bounds based on the definitions of the fractional domination number and other parameters are calculated.The relation between the fractional mixed domination number, fractional domination number, and independent domination number of Litact graphs are derived. The primary findings include the effects on the bounds of the fractional domination number and the fractional mixed domination number when vertices or edges are added or removed from the graph.
Keywords: Litact Graph, Domination Number, Fractional domination number, Independence domination number, fractional mixed domination number.

1. **Introduction**

Numerous researchers have explored the domination number and it's associated characteristics, leading to a plethora of papers on domination and its variants [1]. These variants and related outcomes are elaborated upon in several prominent works [2,3]. Fractional domination is a variant that has seen significant advancements in its theoretical framework and applications. Various authors have proposed modifications to the function values. In 1987, Hetetniemi et al. [6,7] formally introduced the fractional domination number of G. A Litact graph L(G) comprises the edges and cut vertices of G. If the edges and cut vertices are incident or adjacent in G, then the two vertices in L(G) are also adjacent [4,5]. Results concerning to domination and the Litact graph are detailed in [4,5]. The fractional domination number of fractal graphs is examined in [11]. Consider the Litact graph of a path, cycle, star graph, and wheel graph with n vertices. The independent domination number i(L(G)) represents the minimum cardinality of the largest independent sets in L(G). A dominating set in L(G) is defined as a collection of vertices such that each vertex in L(G) is either part of the dominating set or adjacent to a vertex within the dominating set. The domination number γ(L(G)) of L(G) is the minimum cardinality of all such dominating sets [14]. For any vertex v in V(L(G)), the open neighborhood of v consists of all vertices adjacent to v, denoted as N(v), while the closed neighborhood N[v] is defined as N(v) ∪ {v}. Similarly, for any edge e_1, the open neighborhood N(e) includes all edges adjacent to e, and the closed neighborhood $M[e_1]$ is defined as $M(e_1) \cup e_1$. A function f: V(L(G)) → [0,1] is referred to as a fractional dominating function (FDF) of L(G) if $f(N[v]) = \sum_{v_1 \epsilon N[v]} f(v_1) \geq 1$ for every vertex v in V(L(G)). The minimum weight among all other FDFs of L(G) is recognized as the fractional domination number (FDN), denoted by γ_f(L(G)) [15]. A fractional mixed dominating function (FMDF) f: V(L(G)) ∪ E(L(G)) → [0,1] satisfies $f(N[w_1]) = \sum_{w_2 \epsilon N*[w_1]} f(w_2) \geq 1$ for all $w_1 \epsilon$ V(L(G))∪L(L(G)). Among all FMDFs of L(G), the function with the minimum weight is termed as the fractional mixed domination number (FMDN), represented as γ_{fm}^{*}(L(G)).

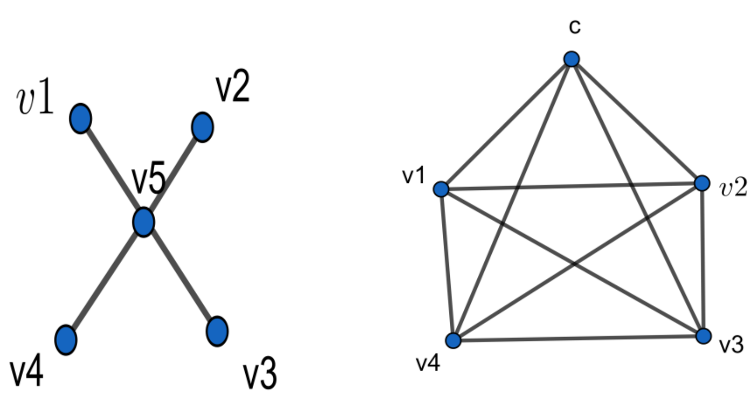
Previous studies on Litact graphs and fractional domination have sparked our interest in investigating the effects and limits of parameters related to fractional domination within Litact graphs. In this research, the neighborhood L[G] of each vertex to ascertain the fractional domination number (L(G)) is analyzed. Assign a weight f(v) to each vertex v, which lies within the range of [0, 1]. The goal is to guarantee that for every v ∈ V (L(G)), the condition $\sum_{v_1 \in L(V)} f(v_1) \geq 1$ holds. Subsequently, L(G(v)) is computed by minimizing the total weight of the FDFs. Moreover, an in-depth analysis is performed utilizing the definitions of various parameters, including the independence domination number, to establish the bounds of these parameters in Litact graphs. Additionally, the limits of the FDN and FMDN are explored by examining how these bounds change when a vertex is either removed or added to the Litact graphs.

This methodology not only deepens our theoretical comprehension of fractional domination in Litact graphs but also highlights their practical applications. Litact graphs, which are employed to model interactions in linear-tactical systems such as supply chains, railway routing, or task scheduling networks, significantly benefit from the implementation of the fractional domination number (FDN) and the mixed fractional domination number (MFDN). The FDN in Litact graphs facilitates the assignment of fractional values to dominating sets, providing flexible management of limited resources while ensuring comprehensive oversight of the network. This is particularly advantageous in constrained settings where partial monitoring or control suffices—for instance, in multi-stage manufacturing lines, where each stage (vertex) necessitates varying levels of supervision. The MFDN further enhances this by taking into account both vertex and edge coverage, making it applicable for scenarios such as optimizing communication links or transport networks, where attention is required for both the nodes and the routes connecting them. These strategies contribute to the design of efficient monitoring systems, reduction of resource consumption, and enhancement of fault tolerance in systems organized using Litact graphs.

2 Definitions :

2.1 Litact Graph[10]

The elements in a litact graph L(G) consist of the edges and cut vertices of G. If the edges and cut vertices are either incident or adjacent in G, then the two vertices in L(G) will also be adjacent.



Graph & Litact GraphL(G)

2.2 Litact Domination number [10]

A dominating set D ⊆ V(L(G)) is referred to as litact dominating set of G if every vertex in V(L(G)) - D is adjacent to at least one vertex v in D. The litact domination number of G is represented as γ_L(G) and is defined by the equation γ_L(G) = min|D|.

2.3 Fractional domination Number [6]

A function f:V(G)→[0,1] is referred to as a fractional dominating function of G if f(v)=$\sum_{v_1 \epsilon N[v]} f(v_1)$ ≥1 , for every v in V(G). The least weight among all other fractional dominating functions (FDFs) of G is termed the fractional domination number (FDN). It is represented by γ_f(G)

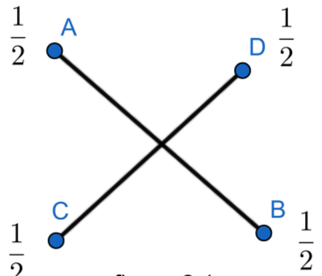
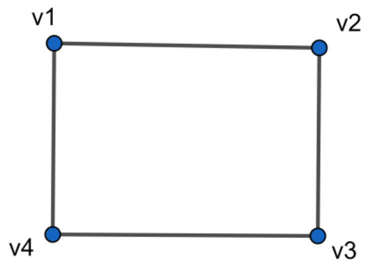


figure 2.1

2.4 Independent Domination Number[8]

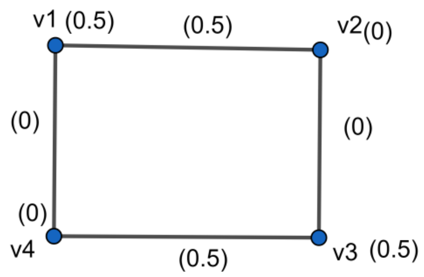
The independent domination number i(G) represents the smallest cardinality of the largest independent sets within G.



Independent Domination Set {v1,v3}

2.5 Fractional Mixed Domination Function(FMDE) [9]

F:V∪E → [0,1] fulfills f[V]= $\sum_{v\epsilon[V]} f[v]$ ≥1 for all V∈V(G)∪E(G). Among all FMDFs of m(G), the function that has the least weight is referred to as the fractional mixed domination number (FMDN), denoted as γ^*_{fm}(G).



Fractional Mixed Domination Number

Theorem 2.1. [10]: If G is a graph of order P without isolated vertices then γ(G)≤$\frac{P}{2}$.
Theorem 2.2[10]: For any graph, diam(G)-1≤γ_c(G).

Theorem 2.3[10]: For any graph **G,** $\left\lceil\frac{P}{\Delta(G)+1}\right\rceil$ ≤γ(G).
Theorem 2.4[10]: For any graph **G,** $\alpha_0(G)+\beta_0(G)=P$ and if G has no disconnected vertices, then $\alpha_1(G)+\beta_1(G)=P$.
Theorem 2.5[10]: For each graph G, γ(G)≤ p−Δ(G).
Theorem 2.5[10]: For any graph G, γ(G)≤γ_c(G).
Theorem 2.6[10]: For each graph G, γ(G)≤$\beta_0(G)$.
Theorem 2.7[10]: If G has p vertices and no isolates then γ_t(G)≤P−Δ(G)+1
Theorem 2.8[10]: For any connected graph, then $\left\lceil\frac{diam(G)+1}{3}\right\rceil$ ≤ γ(G).
Theorem 2.9[10]: For any graph G, p-q ≤ γ(G), further more γ(G) =p-q iff each components of G is a star.
Corollary 2.1[10]: Litact graph of star graph is regular.

3.Bounds and effect of fractional domination and related parameters of Litact graphs

In this section, the limits of the fractional domination number, independent domination number, and fractional mixed domination number of the Litact graph are determined. Also the connections between these parameters are illustrated and significant insights into their structural characteristics are examined.

Theorem 3.1

For any Ladder graph G(V,E) then γ_f L((G)) = $\frac{P}{2}$

Proof :

Let G(V,E) be a ladder graph, which is an planar graph with 2P vertices and 3P-2 edges. Therefore, $|V|$=2P and $|E|$ =3P-2.
Now construct the litact graph of ladder graph. That is each edge of the ladder graph becomes a vertex in the litact graph.
By the definition of litact graph two vertices in G are cut vertices. Then these vertices are edges in the litact graph. Now compute fractional domination number , Assign each vertex f(v)= 1/2 .Most vertices in this litact have degree ≥2 or 3.So their closed neighborhood contains atleast 3 vertices.

Hence $\sum f(v)$≥1 , that is γ_fL((G)) = $\frac{P}{2}$

Theorem 3.2

(a)For any cycle graph C_P, P>3 vertices γ_f (L(C_P))= P.

(b)Every path graph P_P, P>3 vertices γ_f (L(P_P))= $\left\lceil\frac{P}{2}\right\rceil$.

(C) Every wheel graph W_P, P≥4 vertices, γ_f (L(P_P))= $\left\lceil\frac{P+1}{3}\right\rceil$.

(d) Every complete graph K_P, P>3 vertices γ_f (L(K_P))= 2.

(d) Every star graph $K_{1,P}$, P>3 vertices γ_f (L($K_{1,P}$))= 1.

Proof:

(a) Let C_P be a cycle graph on P≥ 3. Each vertex is connected .
Convert this cycle graph is converted into litact graph, here 2 vertices are cut vertices.then these vertices are converted into edges. Then again form a cycle graph. Then compute domination number in fractional value and satisfies $\sum f(v) \geq 1$.
Example:
Consider C_4, L(C_4) again form a cycle that is triangle.
Then γ_f (L(C_P))= $\sum_{v\in V(L(Cp))} f(v)$= 3n. $\frac{1}{3}$= 1

(b) In path graph P_P P>3 , To prove: γ_f (L(P_P))= $\left\lceil\frac{P}{2}\right\rceil$.

Let P_p = v_1, v_2,…,v_n.Assume the litact path graph has the same structure Define a function f as f(vi)=1/2.Since each vertex has 1/2.

Hence γ_f (L(P_P))= n.1/2= $\frac{n}{2}$= $\left\lceil\frac{P}{2}\right\rceil$

(c) In wheel graph W_P, P≥4 , To prove γ_f (L(W_P))= $\left\lceil\frac{P+1}{3}\right\rceil$.

This proof is similar of cycle graph. Henceγ_f (L(W_P))= $\left\lceil\frac{P+1}{3}\right\rceil$.

Similar proof for γ_f (L(K_P))= 2 & γ_f (L($K_{1,P}$))= 1.

Corollary 3.1

For every p≥3 , γ_f(L(G))≤ γ(L(G))

Proof :

The inequality γ_f(L(G))≤ γ(L(G)) is obtained for each p_3 by comparing the fractional domination number to the domination number of a Litact graph G.

The following theorem gives the bounds for the independent domination number of Litact graph.

Theorem 3.4

Let G be a simple, connected graph with minimum degree at least 1. Then the independent domination number L(G) satisfies i(L(G) ≤ i(G) +i L(G) + i T(G)

4.Independent domination Number i(G):

Theorem 4.1

If G be litact path graph L_{P_p} then i(L_{P_p})=$\left\lceil\frac{p}{3}\right\rceil$.

Proof :

Let G= L_{P_p} be a litact path with p vertices. Suppose the Litact graph of P_P adds edge between some non-adjacent vertices or combines vertex/edge relations.

i(L_{P_p})=$\left\lceil\frac{p}{3}\right\rceil$, P_3: i (L_{P_3})= $\left\lceil\frac{3}{3}\right\rceil$=1, P_3: i (L_{P_5})= $\left\lceil\frac{5}{3}\right\rceil$=1.

Theorem 4.2

If G be a litact wheel graph $L_{W_p}$with p≥4 then i(L_{W_p})=$\left\lceil\frac{p}{3}\right\rceil$.

Proof :

Let G= L_{W_p}be a litact Wheel graph is formed by connecting a single central vertex to all vertices of a cycle C_{p-1}.Hence smallest cardinality of the largest independent sets is $\left\lceil\frac{p}{3}\right\rceil$

i (L_{W_p})=$\left\lceil\frac{p}{3}\right\rceil$ for p≥4, i (L_{W_4})=$\left\lceil\frac{4}{3}\right\rceil$=1, i ($L_{W_p}$)=$\left\lceil\frac{5}{3}\right\rceil$=1

Theorem 4.3

If G be a litact star graph $L_{K_{1,p-1}}$with p≥2 ,then i($L_{K_{1,p-1}}$)=1.

Proof :

Let G be a litact star graph $L_{K_{1,p-1}}$with p≥2. A central vertex connected to n-1 leaves,here smallest cardinality of the largest independent sets is 1, Hence i ($L_{K_{1,p-1}}$)=1

Theorem 4.4

If G be a litact cycle graph $L_{C_p}$with p≥3 ,then i(L_{C_p})=$\left\lceil\frac{p}{3}\right\rceil$.

Proof :

A cycle graph C_p is a connected graph with P≥3 vertices where each vertex is connected to exactly two other vertices i (C_p)=$\left\lceil\frac{p}{3}\right\rceil$

Corollary 4.1

For every n≥3, γ_f (L(G))≤ i(L(G)

Proof :

It follows that the results from fractional domination number of Litact graph and independent domination number of Litact graph.

5. Fractional Mixed Domination number of Litact Graph in both vertices and edges in the graph.

Theorem 5.1

For any litact cycle C_r with r≥3 we have γ^*_{fm}(L(C_r)) =$\frac{2r}{5}$

Proof :

Let v_1,v_2,...,v_r and e_1,e_2,...e_r represents L(c_r)(vertices and edges, respectively).
Let x∈L(C_r) be either an edge or a vertex. For every V, we define the function
f:V(L(c_r))∪E(L(c_r)) →[0,1] by f(x_i)=$\frac{1}{5}$.for all x_i ∈ V∪E
For each i in this graph, f(R[x])=1, and
Its boundage is the smallest of all L(C_r)'s FMXDFs.
P(f) = γ^*_{fm}(m(c_r))= $\frac{r}{5}$ +$\frac{r}{5}$= $\frac{2r}{5}$ is the poundage of f.

Theorem 5.2

For r≥2, γ^*_{fm}(L($K_{1,r}$))=1

Proof :

Let L($K_{1,r}$) be the litact star graph, let V , V_i(1≤i≤r) be its vertices, V_0 be its central vertex and V_1, V_2,…v_n are leaf vertices of star graph $L(K_{1,r})$, each connected only to v_0.
Give the function f_1 : V (L($K_{1,r}$))∪E (L($K_{1,r}$)) →[0,1] by f_1(v)=1, others are zero.. This suggests that P(f_1)= $\sum_{x\epsilon V\cup E} f_1(x)$ = 1 is the poundage of. It is evident that P(f_2)=f_2(v)+f_2(v_i)+f_2(V V_i)>1= P(f_1) if define the additional function f_2 : V (L($K_{1,r}$))∪E (L($K_{1,r}$)) →[0,1] by f_2(v)=1,f_2(V_i)=1 and f_2(VV_i)=0 for every i. Consequently, P(f_1) is the least in relation to P(f2). Thus, γ^*_{fm}(L($K_{1,r}$))=1.

Theorem 5.3

If P_r is a non trivial path with r≥2, then γ^*_{fm}(L(P_r))= $\left\lfloor\frac{r}{2}\right\rfloor$.

Proof :

Let G be the path graph and L(G) denote the litact path graph with r, where v_2,…v_r represent the vertices of L(G) and e_1,e_2,...e_r be the edges of L(G).
Let x∈L(G) be either a vertex or an edge. Define the function f:V(L(P_r))∪E(L(P_r)) →[0,1] by f(x_i)=$\frac{1}{2}$ for all x_i ∈ V∪E.
In this graph, f(R[x]) = 0 for every i, and its boundage is the minimum among all the FMXDF's of L(G).
The poundage of f is P(f) = γ^*_{fm}(L(G)) = $\sum_{x\epsilon V\cup E} f(x)$= $\left\lfloor\frac{1}{2}\right\rfloor$.r=$\left\lfloor\frac{r}{2}\right\rfloor$.

Theorem 5.4

If W_r is a Wheel graph with r≥2, then γ^*_{fm}(L(W_r))= $\left\lfloor\frac{r}{2}\right\rfloor$.

Proof :

Let W_r be the wheel graph and L(W_r) denote the litact wheel graph with r≥2, where v_1, v_2,…v_r signify the vertices of L(W_r) and e_1,e_2,...e_r represent the edges of L((W_r). Here, v_1 is the central vertex while the remaining vertices are rim vertices.

Let x be either a vertex or an edge. Define the function f:V(L(w_r))∪E(L(w_r)) →[0,1] by f(x_i)=$\frac{1}{2}$. for all V.

In this graph, f(R[x])=0 for every i, and its boundage is the minimum among all the FMXDF's of L((W_r).

The poundage of f is P(f) =γ^*_{fm}(L(P_r))= $\sum_{x\epsilon V\cup E} f(x)$= $\left\lfloor\frac{r}{2}\right\rfloor$.

The following theorem explains how adding a new vertex to Litact graph affects the FMDN.

Theorem: 5.5

For every r≥3 of the Litact graph L(G) , γ^*_{fm}(L(G+V))=$\frac{4r+4}{5}$ where G+V is adding a new vertex v to other vertex in G.

Proof :

Let the vertices x of the graph L(G) r≥3 be denoted as {v_1, v_2,…v_r }. By adding the vertex v to any of the vertices in G, we obtain G+v. Define a function f: V(L(G)∪E(L(G)) →[0,1] that maps to the interval [0,1], such that f(v_1)=$\frac{1}{5}$ equals a specific value for all v_1 in V(L(G)), and f(e_{3i}) equals 0 for i=1,2,3,…,p.

Except for these edges, all other remaining edges are assigned $\frac{1}{5}$, and the new vertex v is assigned a value of f(v) =$\frac{4}{5}$ a specific value, with the corresponding edges assigned a value of 0. Alternatively, f(v) can be defined for all v in L(V(G)), and f(ei) = 0 for i=1,2,3,…,p. Again, except for these edges, all other remaining edges are assigned a certain value $\frac{1}{5}$, and the new vertex v is assigned a value of f(v) = $\frac{1}{5}$, with the corresponding edges assigned a certain value $\frac{3}{5}$. In either case, the total weight is computed by summing these function values, resulting in f being equal to $\frac{2r}{5}$ + $\frac{2r}{5}$+$\frac{4}{5}$ = $\frac{4r+4}{5}$. Since the function holds, it follows that f(M[w_1])=$\sum_{w_2\in M[w_1]} f(w_2)$≥1 for all vertices in $w_1\in$ V(L(G)∪E(L(G)). Thus, it is a minimum FMDF, and the weight of the function γ^*_{fm} (L(G+V)) is established. Now, suppose that γ^*_{fm}(L(G+V))≠ $\frac{4r+4}{5}$. This indicates that there exists an edge or vertex$w_1\in$ V(L(G)∪E(L(G)) in such that f(M[w_1])=$\sum_{w_2\in M[w_1]} f(w_2)$<1. Consequently, this implies that f is not a minimum FMDF, and the weight of the function f does not correspond to γ^*_{fm}(L(G+V)). Therefore, for every r, r≥3,γ^*_{fm}(L(G+V)) = $\frac{4r+4}{5}$. this holds true.

The subsequent corollary demonstrates how FMDN is affected when a new edge is added to the Litact graph

The next corollary illustrates how FMDN change when adding a new edge in the Litact graphs.

Corollary 5.1

For every r ≥3 of the Litact graphs L(G), γ^*_{fm}(L(G+e)) = $\frac{4r+4}{5}$ where a new edge e= v_iv is added, where vi is defined as a specific value $v_{i\in L(V(G))}$ and v is a new vertex formed by adding an edge.

The following theorem provides the bounds of FMDN for removing any one of the vertices in the Litact graphs.

Theorem 5.6

For every r≥3,γ^*_{fm}(L(G−V)) = $\frac{4r-1}{5}$ where G−V it is removing any vertex v from L(V(G)).

Proof :

To derive L(G-v), one must remove any vertex from L(G). Assume we remove v_1 from L(V(G)), and the corresponding edges are also eliminated. Let f: V(L(G))∪E(L(G)) →[0,1] be defined such that f(v_i)=$\frac{1}{5}$ for all V(L(G-v_1)) and f(e_{3i}) = 0 where i = 1, 2, 3, …, p, except for the edges f(e_{2r-2}) = f(e_3) = $\frac{1}{5}$ and all other remaining edges are assigned to be $\frac{1}{5}$. Concerning the possible values listed, the total weight of the function f is $\frac{4r-1}{5}$. Since this function holds, f[L(V(w_1)] = $\sum_{w_2 \epsilon v[w_1]} f(w_2)$ ≥1 for all $w_1 \epsilon$ V(L(G)∪E(L(G)). Therefore, it represents a minimum FMDF, and the weight of the function is γ^*_{fm}(L(G−V)). Now, suppose that γ^*_{fm}(L(G−V)) ≠ $\frac{4r-1}{5}$. Then there exists an edge or vertex in $w_1 \epsilon$ V(L(G)∪E(L(G)) such that f(V[w_1])<1.. This indicates that f is not a minimum FMDF, and the weight of the function f is not a γ^*_{fm}(L(G−V)). Hence γ^*_{fm}(L(G−V))= $\frac{4r-1}{5}$..

The subsequent theorem delineates the bounds of FMDN when vertices are removed in the Litact graphs.

Theorem: 5.7

For every r≥3 ,γ^*_{fm}(L(G−e)) = $\frac{4r}{5}$ if removable any edge e∈L(E(G)).

Proof :

The relation of bounds of the FDN and the FMDN when a vertex or edge is added or removed from the litact graph.

Remark 5.1

For every r≥3, γ^*_{fm}(L(G+v)) =γ^*_{fm}(L(G+e))>γ^*_{fm}(L(G−e))≥γ^*_{fm}(L(G))γ^*_{fm}(L(G−v)) >γ^*_{f}(L(G))

6. Conclusion

In this work, the fractional domination number (FDN) of litact graphs, a class of transformed graphs derived from classical structures like paths, stars, cycles, regular, and wheel graphs are investigated. This Analysis focused on establishing bounds and relational properties of the FDN, particularly in connection with other domination parameters such as the independent domination number and the fractional mixed domination number (FMDN). How the fractional domination number is influenced by both the topological structure of the underlying litact graph and by operations such as the addition or removal of vertices and edges are examined. Our study highlights notable correlations between FDN and other domination parameters, offering insight into how domination strength shifts in the fractional sense when compared to stricter, classical parameters. These findings not only enrich the mathematical landscape of graph domination theory but also point toward practical implications in network analysis, resource allocation, and optimization, where fractional domination can represent cost-effective control or coverage in systems modeled by litact graphs.

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