Admissible Relation on Plus Weighted Finite State Mealy Machine

Dr. K. CHITRA
Head, Assistant Professor,
Department of Mathematics, Karuppannan Mariappan College,
Muthur - 638105,Thiruppur District, Tamilnadu, India.
(Affiliated to Bharathiar University, Coimbatore.)
chitra.mahathi86@gmail.com

The major goal of this work is to introduce the admissible(weak admissible, strong admissible) relation on the set of Plus Weighted Finite State Mealy Machine (pwfmm).

key words: Plus Weighted Finite State Mealy Machine, admissible relation,weak admissible relation, strong admissible relation.

1 Introduction

The Mealy machine, a kind of limited state transducer, was created by American mathematician and PC researcher George H. Coarse. A Mealy machine is a limited state machine whose result values are resolved both by its current status and the ongoing data sources in the hypothesis of calculation. He was also a pioneer of secluded programming, one of the primary creators of the IPL-V programming language, and an early supporter of large scale processors in low level computing construct programming. In contrast, a Moore machine's (Moore) yield values are determined solely by its current state. A Coarse machine is a deterministic restricted state transducer: One change is possible for each state and contribution. Plus Weighted Finite state Mealy machine is a generalisation of mealy machine, as seen in [5, 6]. This can be further extended in the field of Plus weighted grammar for mealy machines cited as [8, 9, 10]. Algebraic properties of Plus Weighted Finite state Mealy machine with weighted output capabilities are investigated.The first section covers some fundamental terms. Section 2 gives pwfmm and discusses string concatenation in pwfmm.The topic of admissible relation on pwfmm is covered in Section 3.

2 Preliminaries

The definition of Plus weighted finite state automata pwfa, a specific instance of semiring automata. The semiring $W = ([0,\infty), +, \cdot)$ is explored here, with the normal addition and multiplication.

Definition 1 *A Plus weighted finite state automaton* $(pwfa)$ *is a sextuplet* $P = (Q_P, \Sigma, W, \gamma_P, \pi_P, \eta_P)$, *where*

(a) Q_P*, a finite non-empty set of states,*

(b) Σ*, a finite non-empty set of input symbols,*

(c) W*, a weighting space,i.e., weighting space* $W = ([0,\infty), +, \cdot)$*, where `` + ", represents normal addition and `` · ", represents normal multiplication,*

(d) The weighted subset $\gamma: Q_P \times \Sigma \times Q_P \to [0,\infty)$*, a function called the weighted transition function,*

(e) π_P*, a weighted subset of* Q_P*, i.e.,* $\pi_P: Q_P \to [0,\infty)$*, called the weighted subset of initial states.*

(f) η_P *is a weighted subset of* Q_P*, i.e.,* $\eta: Q_P \to [0,\infty)$ *, called the weighted subset of final states.*

Definition 2 *Let* $P = (Q_P, \Sigma, W, \gamma_P, \pi_P, \eta_P)$ *be a* $pwfa$*, the extended weighted function for* P *is the weighted subset* $\gamma^*: Q_P \times \Sigma^* \times Q_P \rightarrow [0,\infty)$ *with ordinary addition* $"+"$ *and multiplication* $"\cdot"$ *has been defined as follows:*

For all $s, t \in Q_P,\ a \in \Sigma,\ x \in \Sigma^*$

$$(s,\lambda,t)\gamma^* = \begin{cases} 1, & if\ \ s = t \\ 0, & if\ \ s \neq t \end{cases}$$

$$(s, xa, t)\gamma_P^* = \sum_{s \in Q_P} (s,x,p)\gamma_P^* \cdot (p,a,t)\gamma_P$$

Definition 3 *Let* $P = (Q_P, \Sigma, W, \gamma_P, \pi_P, \eta_P)$ *be a* $pwfa$*. Let* $x \in \Sigma^*$*. Then* x *is said to be recognized by* P *if* $L(x) > 0$*, where*

$$L(x) = \sum_{s,t \in Q_P} \{\pi_P(p) \cdot (p,x,q)\gamma_P^* \cdot \eta_P(q)\} > 0$$

Example 1 *Let* $P = (Q_P, \Sigma, W, \gamma_P, \pi_P, \eta_P)$ *be a* $pwfa$*, where* $Q_P = \{q_1, q_2, q_3\}$*,* $\Sigma = \{0,1\}$*,* $\gamma_P: Q_P \times \Sigma \times Q_P \rightarrow [0,\infty)$ *is defined as follows:*

$$(q_1,0,q_1)\gamma_P = 2 \qquad (q_1,1,q_2)\gamma_P = 1 \qquad (q_1,0,q_2)\gamma_P = 2$$
$$(q_1,1,q_1)\gamma_P = 3 \qquad (q_2,0,q_3)\gamma_P = 3$$

we omit the weight values which are zero.

$\pi_P: Q_P \rightarrow [0,\infty)$ *is defined by* $\pi_P(q_1) = 3,\ \pi_P(q_2) = \pi_P(q_3) = 0.$

$\eta_P: Q_P \rightarrow [0,\infty)$ *is defined by* $\eta_P(q_3) = 3,\ \eta_P(q_1) = \eta_P(q_2) = 0.$

Consider $x = 000,$

$$L(x) = L(000)$$
$$= \pi_p(q_1) \cdot (q_1,0,q_1)\gamma_P \cdot (q_1,0,q_2)\gamma_P \cdot (q_2,0,q_3)\gamma_P \cdot \eta_P(q_3)$$
$$= 3 \cdot 2 \cdot 2 \cdot 3 \cdot 3 = 108$$

Definition 4

A Mealy machine is a six tuple M= (Q, X, Y, q_0 *,T, G), where*

(a) Q is a finite non-empty set of states.

(b) X is a finite non-empty set of input symbols.

(c)Y is a finite non-empty set of output symbols.

(d) $q_0 \in Q$ *is the initial state.*

(e) $T: Q \times X \rightarrow Q$ *is called a transition function.*

(f) $G : T: Q \times X \rightarrow Y$ *is called a output function.*

3 Plus Weighted Finite State Mealy Machine

In this section the idea of Plus Weighted Finite State Mealy Machine is introduced

Definition 5 A Plus Weighted Finite Mealy Machine(pwfmm) is a quintuple $\mathcal{M}_{\mathrm{m}} = (\mathrm{Q_s}, \mathrm{X_{in}}, \mathrm{Y_{out}}, \mathrm{W}, \alpha_{\mathrm{f}})$ where,

(a)Q_s *, a non-empty finite set of states.*

(b)X_{in} *,non-empty finite set of input symbols.*

(c)Y_{out} *, a non-empty finite set of output symbols.*

(d) W, a weighting space, $i.e., W = [0,\infty), +, \cdot)$ *with ordinary addition* $+$ *and* $\cdot$ *multiplication.*

(e)$\alpha_f: Q_s \times X_{in} \times Q_s \times Y_{out} \rightarrow [0,\infty)$*, called a weighting output function.*

Definition 6 Let $\mathcal{M}_m = (Q_s, X_{in}, Y_{out}, W, \alpha_f)$ be a Plus Weighted Finite State Mealy Machine. Define $\alpha_f^*: Q_s \times X_{in}^* \times Q_s \times Y_{out}^* \rightarrow [0,\infty)$ as follows:

$\forall a, b \in Q_s, \quad x \in X^*_{in}, \; p \in X_{in}, \; y \in Y_{out}^*, \; q \in Y_{out}$ *and* $|x| = |y|$

(i) $(a, p, b, q)\alpha_f^* = (a, p, b, q)\alpha$

(ii) $(a, \lambda, b, \lambda)\alpha_f^* = \begin{cases} 1 & if\ a = b \\ 0 & if\ a \neq b \end{cases}$

(iii)$(a, p, b, \lambda)\alpha_f^* = (a, \lambda, b, q)\alpha_f = 0$ *and*

(iv)$(a, xp, b, yq)\alpha_f^* = \sum_{c \in Q_s} (a, x, c, y)\alpha_f^* \cdot (c, p, b, q)\alpha_f.$

4 Admissible Relation on pwfmm

This section deals with admissible relation over Plus weighted finite state mealy machines.

Definition 7 *Let* $\mathcal{M}_m = (Q_s, X_{in}, Y_{out}, W, \alpha_f)$ *be a* $pwfmm$. *and let* $\approx$ *be an equivalence relation on* Q_s. *Then,*

(i) the equivalence relation $\approx$ *is said to be weak admissible if and only if*
$\forall \; a, b, s \in Q_s, \; \forall\, p \in X_{in}, \; \forall\, q \in Y_{out},$ *if* $a \approx b$ *and* $(a, p, s, q)\alpha_f > 0$ *then* $\exists t \in Q_s$ *such that* $(b, p, t, q)\alpha_f > 0.$

(ii) the equivalence relation $\approx$ *is said to be admissible if and only if*
$\forall \; a, b, s \in Q_s, \; \forall p \in X_{in}, \; \forall q \in Y_{out},$ *if* $a \approx b$ *and* $(a, p, s, q)\alpha_f > 0$ *then* $\exists\, t \in Q_s$ *such that* $(a, p, t, q)\alpha_f > 0$ *and* $t \approx s.$

(iii) the equivalence relation $\approx$ *is said to be strong admissible if and only if*
$\forall\, a, b, s \in Q_s, \; \forall\, p \in X_{in}, \; \forall\, q \in Y_{out},$ *if* $a \approx b$ *and* $(a, p, s, q)\alpha_f > 0$ *then* $\exists\, t \in Q_s$ *such that* $(a, p, t, q)\alpha_f \geq (a, p, s, q)\alpha_f$ *and* $t \approx s.$

Lemma 1 *Let* $\mathcal{M}_m = \left(Q_s, X_{in}, Y_{out}, W, \alpha_f\right)$ *be a* $pwfmm$ *and let* $\approx$ *be an equivalence relation on* Q_s. *Then the equivalence relation* $\approx$ *is admissible if and only if* $a, b, s \in Q_s,$
$\forall\, x \in X_{in}, \; \forall\; y \in Y_{out},$ *if* $a \approx b$ *and* $(a, x, s, b)\alpha_f^* > 0$ *then* $\exists\, t \in Q_s$ *such that* $(b, x, t, y)\alpha_f^* > 0$ *and* $t \approx s$

Proof. Suppose $\approx$ *is an admissible relation on* Q_s.
Let $a, b \in Q_s, a \approx b$. *Let* $x \in X_{in}^*, \; y \in Y_{out}^*, \; s \in Q_s$ *and suppose* $(a, x, s, y)\alpha_f^* > 0.$
We prove the result by induction on $|x| = n$. *If* $n = 0$, *then* $x = y = \lambda.$
Therefore $(a, x, s, y)\alpha_f^* = (a, \lambda, s, \lambda)\alpha_f > 0$.*Implies* $p = s$ *and* $(a, \lambda, a, \lambda)\alpha_f = 1,$
Now, $(b, \lambda, b, \lambda)\alpha_f = 1 > 0.$ *Further* $a \approx b.$
Thus by selecting t *as* b, *the result holds for the base case.*
Suppose the result is true for all $w \in X_{in}^*$ *such that* $|w| = n - 1, \; n > 0.$
Let $x = wp, \; y = zq$ *where* $w \in X_{in}^*, \; p \in X_{in} \;\; z \in Y_{out}^*, q \in Y_{out}$ *and* $|w| = |z| = n - 1.$
Now, $(a, x, s, y)\alpha_f^* = (a, wp, s, z, q)\alpha_f^*$

$$= \sum_{c \in Q_s} \{(a,w,c,z)\alpha_f^* \cdot (c,p,s,q)\alpha_f > 0.$$

Since Q is finite $\exists c' \in Q_s$ such that

$$(a,w,c',z)\alpha_f^* \cdot (c',p,s,q)\alpha_f = \sum_{c \in Q_s} \{(a,w,c,z)\alpha_f^* \cdot (c,p,s,q)\alpha_f > 0.$$

Thus $(a,w,c',z)\alpha_f^* > 0$ and $(c',p,s,q)\alpha_f > 0$

By induction hypothesis, $\exists\ t' \in Q_s$ such that $(b,w,t',z)\alpha_f^* > 0$ and $t' \approx r'$.

Now, $(c',p,s,q)\alpha_f > 0$ and $t' \approx r'$.

Hence by induction hypothesis, $\exists t \in Q_s$ such that $(t',p,t,q)\alpha_f > 0$ and $t \approx s$.

Thus $\exists t \in Q_s$ such that $(b,w,t',z)\alpha_f^* \cdot (t',p,t,q)\alpha_f > 0$ and $t \approx s$.

Thus $(b,x,t,y)\alpha_f^* > 0$ and $t \approx s$.

The converse is trivial.

Lemma 2 *Let $\mathcal{M}_m = (Q_s, X_{in}, Y_{out}, W, \alpha_f)$ be a $pwfmm$ and let $\approx$ be an equivalence relation on Q_s.Then the equivalence relation $\approx$ is strong admissible if and only if $a,b,s \in Q_s$, $\forall\ x \in X*_{in}$, $\forall\ y \in Y_{out}^*$, if $a \approx b$ and $(a,x,s,b)\alpha_f^* > 0$ then $\exists\ t \in Q_s$ such that $(b,x,t,y)\alpha_f^* \geq (a,x,s,y)\alpha_f^*$ and $t \approx s$*

Proof. Suppose $\approx$ is a strong admissible relation Q_s.

Let $a,b \in Q_s$, $a \approx b$. Let $x \in X_{in}^*$, $y \in Y_{out}^*$, $s \in Q_s$ and suppose $(a,x,s,y)\alpha_f^* > 0$.

We prove the result by induction on $|x| = n$. If $n = 0$, then $x = y = \lambda$.

Therefore $(a,\lambda,a,\lambda)\alpha_f^* = (a,\lambda,s,\lambda)\alpha_f^* > 0$, implies $p = s$ and $(a,\lambda,a,\lambda)\alpha_f^* = 1$.

Now, $(a,\lambda,a,\lambda)\alpha_f^* = 1 = (b,\lambda,b,\lambda)\alpha_f^*$(since $a \approx b$.)

Thus by selecting t as b, the result holds for the base case.

Suppose the result is true for all $w \in X_{in}^*$ such that $|w| = n-1, n > 0$.

Let $x = wp$, $y = zq$, where $w \in X_{in}^*$, $p \in X_{in}$, $z \in Y^*, q \in Y_{out}$ and $|w| = |z| = n-1$.

Now, $(a,x,s,y)\alpha_f^* = (a,wp,s,zq)\alpha_f^*$

$$= \sum_{c \in Q_s} \{(a,w,c,z)\alpha_f^* \cdot (c,p,s,q)\alpha_f > 0.$$

Since Q_s is finite, $\exists\ c' \in Q_s$ such that

$$(a,w,c',z)\alpha_f^* \cdot (c',p,s,q)\alpha_f = \sum_{c \in Q_s} \{(a,w,c,z)\alpha_f^* \cdot (c,p,s,q)\alpha_f > 0.$$

Thus $(a,w,c',z)\alpha_f^* > 0$ and $(c',p,s,q)\alpha_f > 0$.

By induction hypothesis, $\exists\ t' \in Q_s$ such that $(b,w,t',z)\alpha_f^* \geq (a,w,c',z)\alpha_f^*$ and $t' \approx c'$

Now, $(c',p,s,q) > 0$ and $t' \approx c'$. Hence by induction hypothesis, $\exists t \in Q_s$ such that $(t',p,t,q)\alpha_f \geq (c',p,s,q)\alpha_f$ and $t \approx s$.Thus $\exists\ t \in Q_s$ such that $(b,w,t',z)\alpha_f^* \cdot (t',p,t,q)\alpha_f \geq (a,w,c',z)\alpha_f^* \cdot (c',p,s,q)\alpha_f$ and $t \approx s$

Thus, $(a,x,s,y)\alpha_f^* = (a,wp,s,zq)\alpha_f^*$

$$= \sum_{c\in Q_s} \{(a,w,c,z)\alpha_f^* \cdot (c,p,s,q)\alpha_f\} > 0.$$

$$= (a,w,c',z)\alpha_f^* \cdot (c',p,s,q)\alpha_f > 0.$$

$$\leq (b,w,t',z)\alpha_f^* \cdot (t',p,t,q)\alpha_f$$

$$\leq (b,wp,t,zq)\alpha_f^* \leq (b,x,t,y)\alpha_f^*$$

Therefore $(b,x,t,y)\alpha_f^* \geq (a,x,s,y)\alpha_f^*$ *and* $t \approx s$

The converse is trivial.

Theorem 1 *Let* $\mathcal{M}_m = (Q_s, X_{in}, Y_{out}, W, \alpha_f)$ *be a* $pwfmm$ *and let* $\simeq (\sim, \equiv)$ *be an equivalence relation on* Q_s*. Then* $\simeq (\sim, \equiv)$ *is a weak admissible(admissible, strong admissible) relation on* Q_s*.*

Proof. Let $a_1, a_2 \in Q_s$*, if* $a_1 \simeq a_2$*, then if* $\forall\ p \in X_{in},\ q \in Y_{out}\ \ b \in Q_s$*,* $(a_1,p,b,q)\alpha_f > 0 \Leftrightarrow \exists\ c \in Q_s,\ (a_2,p,c,q)\alpha_f > 0$ *which implies that* $a_1 \approx a_2$.

Therefore $\simeq$ *is a weak admissible relation on* Q_s*. The proofs of other cases are similar.*

Theorem 2 *Let* $\mathcal{M}_m = (Q_s, X_{in}, Y_{out}, W, \alpha_f)$ *be a* $pwfmm$ *and let* $\simeq$ *is a weak admissible(admissible, strong admissible) relation* Q_s*. Then* $\approx$ *is a refinement of* $\simeq (\sim, \equiv)$.

Proof. Let $\approx$ *be a weak admissible relation on* Q_s*.By definition of weak admissible* $\forall\ a,b,s \in Q_s,\ p \in X_{in}, q \in Y_{out}$ *if* $a \approx b$ *and* $(a,p,s,q)\alpha_f > 0$*, then* $\exists\ t \in Q_s$ *such that* $(a,p,t,q)\alpha_f > 0$ *which implies* $p \simeq q$.

Therefore $\approx$ *is a refinement of* $\simeq$

The proof of other cases are similar.

4 conclusion

This work introduces and investigates the concept of pwfmm. In addition, several of the features of pwfmm are investigated. The homomorphism of strings to pwfmm has been verified. The concept can be extended to admissible relations, which have been explored by numerous academics in the field of fuzzy finite state automata since their inception.

5 bibliography

References

[1] Aho, A. and Ullman, J., Foundations of Computer Science, Computer Science Press, New York, 1994.

[2] Aho, A. and Ullman, J., Theory of Parsing, Translation and Compiling, **1**, Prentice-Hall, Englewood, N.J., 1972.

[3] Chitra, K., Rajaretnam, T., “A Study On Recognizability of Max Weighted Finite

State Automaton", Journal Of Computer and Mathematical Sciences, Volume 8, Issue 11, December 2017, Pg. No. 658-673, Impact Factor: 4.655, ISSN NO: 0976-5727. 2018 – 2019

[4] Chitra, K., Rajaretnam, T., "Some Products On Max Weighted Finite State Automaton", International Journal Of Applied Engineering Research, Volume 13, Number 22, 2018. Pg. No. 15806 - 15803 , Impact Factor:2.38, ISSN No: 0973-4562.

[5] Chitra, K., Rajaretnam, T., "Homomorphism On Max Weighted Finite State Mealy Machine", Journal Of Applied Science And Computations, Volume 5, Issue XII, December 2018. Pg. No. 96-103, Impact Factor:5.8, ISSN No: 1076-5131

[6] Chitra, K., Rajaretnam, T., "Algebraic Properties Of Max Weighted Finite State Mealy Machines", International Journal Of Information And Computing Science, Volume 6, Issue 1, January 2019. Pg. No. 69-83, Impact Factor: 5.2, ISSN No: 0972-1347.

[7] K. Chitra, Some Algebraic Properties of Plus Weighted Finite state Mealy Machine, Samdharshi, e-ISSN: 2581-3986, Vol.16, Issue 4, September 2023, 2335-2340.

[8] Saridha. S. and Haridha Banu. S, "A New Direction Towards Plus Weighted Grammar", Volume 11, Issue II, International Journal for Research in Applied Science and Engineering Technology (IJRASET), February 2023, Page No: 845-850, ISSN: 2321-9653.

[9] Saridha. S, Jothika. T, "Construction of Derivation Trees of Plus Weighted Context Free Grammars", Volume 11, Issue III, International Journal for Research in Applied Science and Engineering Technology (IJRASET), March 2023, Page No: 1821-1827, ISSN: 2321-9653.

[10] Saridha. S, Jothika. T, "An Introduction to Plus Weighted Dendro language and its properties", Volume 15, Issue I, Aryabhatta journal of mathematics and Informatics (AJMI), Jan-June 2023, Page No: 93-100, ISSN: 2394-9309.

[11] Saridha. S., Haridha Banu. S "An innovative Ideas on Plus Weighted Linear Grammar",Volume 15, Issue 2, Aryabhatta journal of mathematics and Informatics (AJMI), Jan-June 2023, Page No: 173-178, ISSN: 2394-9309.