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Generating Functions For The Modified Hermite Polynomials By Using Truesdell Method

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Abstract: In this paper, two generating functions are derived for the Modified Hermite polynomials $H_m(x;u)$ applying the Truesdell's Method by giving a proper explanation to the index m. Two differential recurrence relations, which are linearly independent are utilized to obtain the generating functions for $H_m(x;u)$. The main aim is to attribute the effective and suitable way application of the Truesdell's method for obtaining the generating functions from the ascending and descending differential recurrence relations of $H_m(x;u)$.

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1. Introduction

In the study of Applied Mathematics and Mathematical physics generating functions [2-3] play a vital role. These are used to investigate some different important properties of sequences and are important in the study of orthogonal polynomials. Hermite polynomials are an important class of orthogonal polynomials which play vital role in the study of pure and applied mathematics, Mathematical physics and engineering and have lot of applications in physics and engineering. Quantum mechanical behavior of some systems in physics is analyzed by using Hermite polynomials.

The goal of this paper is to obtain the generating functions for the Modified Hermite polynomials by applying the Truesdell's method [2,4,6], giving a proper explanation to the index m. Two generating functions are obtained for the Modified Hermite polynomials from ascending differential recurrence relation and descending differential recurrence relation. In Truesdell's method to derive the generating functions, two functional equations called as $F-$type function and $G-$type function are utilized and have been given as

$$\frac{\partial}{\partial z}F(z,\alpha)=F(z,\alpha+1) \text{ and } \frac{\partial}{\partial z}G(z,\alpha)=G(z,\alpha-1).$$

To obtain an ascending generating function for the set $\{\varphi_{k+m}(x)\}$, where k is fixed. Truesdell [7] made use of the following differential difference equation of the ascending type,

$$\frac{d}{dx}\varphi_m(x)=A(x,m)\ \varphi_m(x)+B(x,m)\ \varphi_{m+1}(x),$$

where the coefficients $A(x,m)$ and $B(x,m)$ are suitably restricted. Also for obtaining a descending generating function for the set $\{\varphi_{k-m}(x)\}$,where k is fixed. The following differential difference equation of the descending type is used

$$\frac{d}{dx}\varphi_m(x)=C(x,m)\ \varphi_m(x)+D(x,m)\ \varphi_{m-1}(x),$$

where the coefficients $C(x,m)$ and $D(x,m)$ are suitably restricted. In this paper, by utilizing Truesdell method the generating functions are obtained for the Modified Hermite polynomial by giving a proper explanation to the index m and with its applications.

1.1 Definition: M. A. Khan, A. H. Khan and N. Ahmad [1] defined the Modified Hermite Polynomials,

$$u^{2xt-t^2}=\sum_{m=0}^{\infty}\frac{H_m(x;u)t^m}{m!},u>0. \tag{1}$$

where, $H_m(x;u)=\sum_{k=0}^{\left[\frac{m}{2}\right]}\frac{(-1)^k\ m!(2x)^{m-2k}\ (\log u)^{m-k}}{(m-2k)!\ k!}$

for $u=e$, Eq.1 transforms to Hermite Polynomials, we get

$$e^{2xt-t^2}=\sum_{m=0}^{\infty}\frac{H_m(x)t^m}{m!}.$$

1.2 Differential Recurrence Relations:

Using definition given in Eq. 1, we write the ascending differential recurrence relation and descending differential recurrence relation

$$H'_m(x;u)=2m\log u\ H_{m-1}(x;u) \tag{2}$$

and

$$xH'_m(x;u)=m\ H'_{m-1}(x;u)+mH_m(x;u) \tag{3}$$

using Eq. 2 into Eq. 3, we write

$$2mx\ \log u\ H_{m-1}(x;u)=mH'_{m-1}(x;u)+m\ H_m(x;u) \tag{4}$$

Dividing Eq. 4 by m, we get

$$2x\log u\ H_{m-1}(x;u)=H'_{m-1}(x;u)+H_m(x;u) \tag{5}$$

replacing m by $m-1$ in the Eq. 5,

$$2x\log u\ H_m(x;u)=H'_m(x;u)+H_{m+1}(x;u) \tag{6}$$

$$H'_m(x;u) = 2x \log u \; H_m(x;u) - H_{m+1}(x;u) \tag{7}$$

dividing Eq. 7 by $m!$,

$$\frac{H'_m(x;u)}{m!} = 2x \log u \; \frac{H_m(x;u)}{m!} - \frac{H_{m+1}(x;u)}{m!} \tag{8}$$

$$\frac{H'_m(x;u)}{m!} = 2x \log u \; \frac{H_m(x;u)}{m!} - \frac{(m+1)H_{m+1}(x;u)}{(m+1)m!} \tag{9}$$

putting $\log u = k$ into Eq.2 and Eq.9, we have

$$H'_m(x;u) = 2mk \; H_{m-1}(x;u) \tag{10}$$

$$\frac{H'_m(x;u)}{m!} = 2xk \; \frac{H_m(x;u)}{m!} - \frac{(m+1)H_{m+1}(x;u)}{(m+1)!} \tag{11}$$

Eq.10 and Eq.11 are descending and ascending differential recurrence relations, respectively of the Modified Hermite polynomials.

2. Generating relations deduced from the ascending differential recurrence relation

To derive the generating function from the differential difference equation of the ascending type, $F-$type function is used,

$$\frac{\partial}{\partial z} F(z,\alpha) = F(z,\alpha+1) \tag{12}$$

The Truesdell's Method has been used to find a generating function for class of polynomials $\{H_{\alpha+m}(x;u)\}$.

The ascending differential recurrence relation for the polynomial $H_m(x;u)$ is,

$$\frac{H'_m(x;u)}{m!} = 2xk \; \frac{H_m(x;u)}{m!} - \frac{(m+1)H_{m+1}(x;u)}{(m+1)!} \tag{13}$$

Let $\varphi_m(x) = \dfrac{H_m(x;u)}{m!}$, then the Eq.13 can be written as,

$$\frac{d}{dx}\varphi_m(x) = 2xk\,\varphi_m(x) - (m+1)\,\varphi_{m+1}(x) \tag{14}$$

By using the mathematical symbols used in Truesdell's method ,we have,

Let , $f(y,\alpha) = \varphi_\alpha(y) = \dfrac{H_\alpha(y;u)}{\Gamma(\alpha+1)}$, α is an integer. (15)

by Eq.14 and Eq.15, we get

$$\frac{\partial}{\partial y} f(y,\alpha) = 2ky\, f(y,\alpha) - (\alpha+1)\, f(y,\alpha+1) \tag{16}$$

This type of equation is called as $f-$ type equation and can be written as,

$$\frac{\partial}{\partial y} f(y,\alpha) = A(y,\alpha)\, f(y,\alpha) + B(y,\alpha)\, f(y,\alpha+1)$$

with $A(y,\alpha) = 2ky$ and $B(y,\alpha) = -(\alpha+1)$.

Transforming $f(y,\alpha)$ into $g(y,\alpha)$, so that

$$\frac{\partial}{\partial y} g(y,\alpha) = C(y,\alpha)\ g(y,\alpha+1).$$

Let $$g(y,\alpha) = C(y,\alpha)\exp\left\{-\int_{y_0}^{y} A(v,\alpha)\, dv\right\}$$

therefore, $$g(y,\alpha) = f(y,\alpha)\exp\left\{-\int_{y_0}^{y} 2kv\, dv\right\}$$

$$= f(y,\alpha)\exp\left\{-k\,(y^2 - {y_0}^2)\right\}$$

by choosing $y_0 = 0$, we obatin

$$g(y,\alpha) = f(y,\alpha)\exp\left\{-k\,y^2\right\} \tag{17}$$

Now differentiating Eq.17 partially w.r. to y, we have

$$\frac{\partial}{\partial y} g(y,\alpha) = -(\alpha+1)\ e^{-ky^2} f(y,\alpha+1)$$

$$\frac{\partial}{\partial y} g(y,\alpha) = -(\alpha+1)\ g(y,\alpha+1) \tag{18}$$

Thus, we get that $g-$type equation is satisfied.

Let $C(y,\alpha)$ denote the coefficient of $g(y,\alpha+1)$ which is factorable in Eq.18,

therefore, $C(y,\alpha) = (-1)(\alpha+1)$ with $C(y,\alpha) = A(\alpha)Y(y)$

where, $A(\alpha) = \alpha+1$ and $Y(y) = -1$.

Now transforming $g(y,\alpha)$ into $F(z,\alpha)$ by letting,

$$z = \int_{y_1}^{y} Y(v)\, dv = \int_{y_1}^{y} (-1)\, dv = -y + y_1 \tag{19}$$

and $$F_0\, F(z,\alpha) = g(y,\alpha)\exp\left\{\int_{\alpha_0}^{\alpha} \log A(v)\, \Delta v\right\}$$

$$= g(y,\alpha)\exp\left\{\int_{\alpha_0}^{\alpha} \log A(1+v)\, \Delta v\right\}$$

$$F_0\, F(z,\alpha) = g(y,\alpha)\exp\left\{\int_{-1}^{\alpha} \log(1+v)\, \Delta v\right\} \tag{20}$$

by choosing $\alpha_0 = -1, y_0 = 0$, $F_0 = \frac{1}{\sqrt{2\pi}}$ and using the relation

$$\log\Gamma(x) = \int_0^x \log z\Delta z + \log\sqrt{2\pi} \tag{21}$$

$$\int_0^x \log z\Delta z = \log\Gamma(x) - \log\sqrt{2\pi} = \log\left(\frac{\Gamma(x)}{\sqrt{2\pi}}\right)$$

we get, $$\left(\frac{1}{\sqrt{2\pi}}\right)F(z,\alpha) = g(y,\alpha)\exp\left\{\log\frac{\Gamma(\alpha+1)}{\sqrt{2\pi}}\right\}$$

$$\left(\frac{1}{\sqrt{2\pi}}\right)F(z,\alpha) = g(y,\alpha)\left\{\frac{\Gamma(\alpha+1)}{\sqrt{2\pi}}\right\}$$

$$F(z,\alpha) = g(y,\alpha)\Gamma(\alpha+1) \tag{22}$$

From Eq. 19, by choosing $y_1 = 0$, we have $z = -y$.

Therefore, Eq. 22 becomes,

$$F(z,\alpha) = g(-z,\alpha)\Gamma(\alpha+1) \tag{23}$$

Now to verify that $F-$equation is satisfied by $F(z,\alpha)$, we find

$\frac{\partial}{\partial z}F(z,\alpha)$,

therefore, $$\frac{\partial}{\partial z}F(z,\alpha) = \Gamma(\alpha+1)\,\frac{\partial}{\partial z}g(-z,\alpha)$$

$$\frac{\partial}{\partial z}F(z,\alpha) = \Gamma(\alpha+1)\left[(-(\alpha+1))g(-z,\alpha+1)\right]\ (-1)$$

$$\frac{\partial}{\partial z}F(z,\alpha) = \Gamma(\alpha+1)\left[((\alpha+1))g(-z,\alpha+1)\right]$$

$$\frac{\partial}{\partial z}F(z,\alpha) = \Gamma(\alpha+2)\,g(-z,\alpha+1)$$

$$\frac{\partial}{\partial z}F(z,\alpha) = F(z,\alpha+1)$$

now writing $F(z,\alpha)$ in the following form by using Eq. 17 and Eq. 23 as,

$$F(z,\alpha) = \Gamma(\alpha+1)\ f(-z,\alpha)\exp(-k\,z^2) \tag{24}$$

$$F(z,\alpha) = H_\alpha(-z,u)\exp(-k\,z^2) \tag{25}$$

Now using Truesdell's generating function theorem[2,p.82, Theorem 14.1], which states that if a function $F(z,\alpha)$ satisfies the F equation $F(z+y,\alpha)$and if possesses a Taylor series, then this series may be put in the form,

$$F(z+y,m) = \sum_{m=0}^{\infty}\frac{y^m}{m!}F(z,\alpha+m).$$

from Eq. 25, we have

$$F(z+y,\alpha)=H_{\alpha}(-z-y;u)\left[\exp\left\{-k(z+y)^{2}\right\}\right] \qquad (26)$$

and

$$F(z,\alpha+m)=\exp\left(-k\,z^{2}\right)H_{\alpha+m}(-z;u) \qquad (27)$$

from Eq. 26 and Eq.27, and by above theorem, we get

$$H_{\alpha}(-z-y;u)\exp\left\{-k(z+y)^{2}\right\}=\sum_{m=0}^{\infty}\frac{y^{m}}{m!}\exp\left(-kz^{2}\right)H_{\alpha+m}(-z;u) \quad (28)$$

cancelling the factor $\exp\left(-k\,z^{2}\right)$ from both sides of equation (28),

$$H_{\alpha}(-z-y;u)\left[\exp\left(-ky^{2}-2kyz\right)\right]=\sum_{m=0}^{\infty}\frac{y^{m}}{m!}H_{\alpha+m}(-z;u) \qquad (29)$$

replacing $-z=x$ in Eq.29, we get

$$H_{\alpha}(x-y;u)\exp\left(-ky^{2}+2kyx\right)=\sum_{m=0}^{\infty}\frac{y^{m}}{m!}H_{\alpha+m}(x;u) \qquad (30)$$

Now simplifying the expression on the left side,

$$\exp\left(-ky^{2}+2kxy\right)=\exp\left[-k\left(y^{2}-2xy\right)\right]=\exp\left[-\left(y^{2}-2xy\right)\log a\right]=\exp\left[\log a^{-\left(y^{2}-2xy\right)}\right]$$

Eq. 30 can be written as,

$$a^{-\left(y^{2}-2xy\right)}H_{\alpha}(x-y;u)=\sum_{m=0}^{\infty}\frac{y^{m}}{m!}H_{\alpha+m}(x;u) \qquad (31)$$

$$a^{\left(2xy-y^{2}\right)}H_{\alpha}(x-y;u)=\sum_{m=0}^{\infty}\frac{y^{m}}{m!}H_{\alpha+m}(x;u) \qquad (32)$$

Eq. 32 is a generating relation [1, p.273, (10.1)] for the modified Hermite polynomials $H_{m}(x;u)$.

3. Generating function derived from Descending Equation

By using Truesdell's $G-$equation,

$$\frac{\partial}{\partial z}G(z,\alpha)=G(z,\alpha-1) \qquad (33)$$

we obtain a generating relation for the class of the polynomials $\{H_{\alpha-m}(x,u)\}$, which satisfies the descending differential recurrence relation.

Now from Eq. 10, we have

$H'_{m}(x;u)=2mk\,H_{m-1}(x;u)$

Let $\varphi_{m}(x)=H_{m}(x;u)$, then the above equation can be written as,

$$\frac{d}{dx}\varphi_{m}(x)=2mk\,\varphi_{m-1}(x) \qquad (34)$$

By using the notations in Truesdell's method ,we have

Let $f(y,\alpha)=\varphi_{\alpha}(y)=H_{\alpha}(y;u)$, where α is an integer.

therefore Eq. 34 can be written as,

$$\frac{\partial}{\partial y} f(y,\alpha) = 2k\alpha\, f(y,\alpha-1) \tag{35}$$

Eq. 35 called $f-$ type equation and can be written as

$$\frac{\partial}{\partial y} f(y,\alpha) = A(y,\alpha)\, f(y,\alpha) + B(y,\alpha)\, f(y,\alpha-1)$$

with $A(y,\alpha)=0$ and $B(y,\alpha)=2k\alpha$.

now transforming $f(y,\alpha)$ into $g(y,\alpha)$, so that

$$\frac{\partial}{\partial y} g(y,\alpha) = C(y,\alpha)\ g(y,\alpha-1).$$

Let us assume that, $g(y,\alpha) = f(y,\alpha)\exp\left\{-\int_{y_0}^{y} A(v,\alpha)\,dv\right\}$

therefore, $g(y,\alpha) = f(y,\alpha)\exp\left\{-\int_{y_0}^{y} 0\,dv\right\}$

$$g(y,\alpha) = f(y,\alpha)\exp(a_1) \tag{36}$$

where, a_1 is a constant of integration.

differentiating Eq.36 partially w.r. to y and using Eq. 35, we have

$$\frac{\partial}{\partial y} g(y,\alpha) = 2k\alpha \exp(a_1)\, f(y,\alpha-1)$$

by using Eq. 36, we write

$$\frac{\partial}{\partial y} g(y,\alpha) = 2k\alpha\, g(y,\alpha-1) \tag{37}$$

Let $C(y,\alpha)$ denote the coefficient of $g(y,\alpha-1)$ which is factorable in Eq. 37.

Then, $C(y,\alpha) = 2k\alpha$, where $C(y,\alpha)$ satisfies the factorability condition $C(y,\alpha) = A(\alpha)Y(y)$.

therefore, $A(\alpha) = 2k\alpha$ and $Y(y) = 1$.

Now transforming the function $g(y,\alpha)$ into $g(z,\alpha)$ by letting,

$$z = \int_{y_1}^{y} Y(v)\,dv = \int_{y_1}^{y} (1)\,dv = y - y_1$$

by choosing $y_1 = 0$, we get $z = y$.

now, $G_0\, G(z,\alpha) = g(z,\alpha)\exp\left\{-\int_{\alpha_0}^{\alpha+1} \log A(v)\,\Delta v\right\}$

$$= g(z,\alpha)\exp\left\{-\int_{\alpha_0}^{\alpha+1}\log(2kv)\Delta v\right\}$$

$$= g(z,\alpha)\exp\left\{-\int_{\alpha_0}^{\alpha+1}[\log(2k)+\log v]\Delta v\right\}$$

$$= g(z,\alpha)\exp\left\{-\int_{\alpha_0}^{\alpha+1}\log(2k)\Delta v-\int_{\alpha_0}^{\alpha+1}\log v\,\Delta v\right\}$$

$$= g(z,\alpha)\exp\left\{-\int_{\alpha_0}^{\alpha+1}\log(2k)\Delta v-\int_{\alpha_0}^{\alpha+1}\log v\,\Delta v\right\}$$

$$= g(z,\alpha)\exp\left\{-[\log(2k)][\alpha+1-\alpha_0]-\int_{\alpha_0}^{\alpha+1}\log v\,\Delta v\right\}$$

Now, if we choose $\alpha_0 = 0$ in the above equation, then we get

$$= g(z,\alpha)\exp\left\{-[\log(2k)][\alpha+1]-\int_{0}^{\alpha+1}\log v\,\Delta v\right\}$$

$$G_0\, G(z,\alpha) = g(z,\alpha)\exp\left\{[\log(2k)^{-(\alpha+1)}]-\int_{0}^{\alpha+1}\log v\,\Delta v\right\}$$

using the relation, $\log\Gamma(x) = \int_0^x \log z\Delta z + \log\sqrt{2\pi}$ and choosing $G_0 = \sqrt{2\pi}$,

We get, $$\sqrt{2\pi}\, G(z,\alpha) = g(z,\alpha)\left\{(2k)^{-(\alpha+1)}\frac{\sqrt{2\pi}}{\Gamma(\alpha+1)}\right\}$$

$$G(z,\alpha) = g(z,\alpha)\left\{\frac{1}{(2k)^{(\alpha+1)}\Gamma(\alpha+1)}\right\}$$

$$G(z,\alpha) = \exp(a_1)\, f(z,\alpha)\left\{\frac{1}{(2k)^{(\alpha+1)}\Gamma(\alpha+1)}\right\}$$

$$G(z,\alpha) = \frac{1}{(2k)^{(\alpha+1)}\Gamma(\alpha+1)}\exp(a_1)\, f(z,\alpha)$$

Now to show that $G-$equations is satisfied by $G(z,\alpha)$, we determine $\frac{\partial}{\partial z}G(z,\alpha)$

Therefore , $\frac{\partial}{\partial z}G(z,\alpha) = \frac{1}{(2k)^{(\alpha+1)}\Gamma(\alpha+1)}\exp(a_1)\,\frac{\partial}{\partial z}f(z,\alpha)$

$$\frac{\partial}{\partial z}G(z,\alpha) = \frac{1}{(2k)^{(\alpha+1)}\Gamma(\alpha+1)}\exp(a_1)\; 2k\alpha\, f(z,\alpha-1)$$

$$\frac{\partial}{\partial z}G(z,\alpha)=\frac{1}{(2k)^{(\alpha+1)}\,\alpha\,\Gamma(\alpha)}\exp(a_1)\;2k\alpha\,f(z,\alpha-1)$$

$$\frac{\partial}{\partial z}G(z,\alpha)=\frac{1}{(2k)^{\alpha}\;\Gamma(\alpha)}\exp(a_1)\;\;f(z,\alpha-1)$$

$$\frac{\partial}{\partial z}G(z,\alpha)=G(z,\alpha-1)$$

Now expressing $G(z,\alpha)$ in the form as below

$$G(z,\alpha)=\frac{\exp(a_1)}{(2k)^{(\alpha+1)}\Gamma(\alpha+1)}\;H_\alpha(z;u)$$

therefore, $$G(z+y,\alpha)=\frac{\exp(a_1)}{(2k)^{(\alpha+1)}\Gamma(\alpha+1)}\;H_\alpha(z+y;u)$$

and $$G(z,\alpha-m)=\frac{\exp(a_1)}{(2k)^{(\alpha-m+1)}\Gamma(\alpha-m+1)}\;H_{\alpha-m}(z;u)\,.$$

by using Taylor's series expansion in powers of y,

$$G(z+y,\alpha)=\sum_{m=0}^{\infty}\frac{y^m}{m!}\;G(z,\alpha-m) \tag{38}$$

Therefore, we have

$\dfrac{\exp(a_1)}{(2k)^{(\alpha+1)}\Gamma(\alpha+1)}\;H_\alpha(z+y;u)=\sum_{m=0}^{\infty}\dfrac{y^m}{m!}\dfrac{\exp(a_1)}{(2k)^{(\alpha-m+1)}\Gamma(\alpha-m+1)}\;H_{\alpha-m}(z;u)$ cancelling the factor $\dfrac{\exp(a_1)}{(2k)^{(\alpha+1)}}$ on both sides and simplyfying, we write

$$\frac{1}{\Gamma(\alpha+1)}\;H_\alpha(z+y;u)=\sum_{m=0}^{\infty}\frac{(2ky)^m}{m!}\frac{1}{\Gamma(\alpha-m+1)}\;H_{\alpha-m}(z;u)$$

$$H_\alpha(z+y;u)=\sum_{m=0}^{\infty}\frac{(2ky)^m}{m!}\frac{\Gamma(\alpha+1)}{\Gamma(\alpha-m+1)}\;H_{\alpha-m}(z;u)$$

$$H_\alpha(z+y;u)=\sum_{m=0}^{\infty}\frac{(-2ky)^m}{m!}(-\alpha)_m\;\;H_{\alpha-m}(z;u)$$

$$H_\alpha(z+y;u)=\sum_{m=0}^{\infty}\frac{(-2y)^m(\log u)^m}{m!}(-\alpha)_m\;\;H_{\alpha-m}(z;u) \tag{39}$$

replacing y by $y-z$, we have

$$H_\alpha(y;u)=\sum_{m=0}^{\infty}\frac{(-2)^m(\log u)^m(y-z)^m}{m!}(-\alpha)_m\;\;H_{\alpha-m}(z;u) \tag{40}$$

and replacing y by lz in Eq.40, we have

$$H_\alpha(lz;u)=\sum_{m=0}^{\infty}\frac{(-2)^m(\log u)^m(l-1)^m}{m!}z^m(-\alpha)_m\ H_{\alpha-m}(z;u) \tag{41}$$

Equations (39), (40) and (41) are new generating relations for the Modified Hermite polynomials.

4. Applications

The following results are derived if we put $u=e$ and $\log u=1$into equations (32),(39), (40) and (41),respectively.

by using Eq. 32,We get

$$e^{(2xy-y^2)}H_\alpha(x-y)=\sum_{m=0}^{\infty}\frac{y^m}{m!}H_{\alpha+m}(x)\ . \tag{42}$$

This result is discussed in Rainville [3; p.197, (1)], Truesdell [7,p.85,(10)], Weisner [6,p. 144,(4.2)].

by using Eq. 39,We get

$$H_\alpha(z+y)=\sum_{m=0}^{\infty}\frac{(-2y)^m}{m!}(-\alpha)_m\ H_{\alpha-m}(z) \tag{43}$$

This result is discussed in Weisner [6, p. 144,(4.1)].

by using Eq. 40 and 41, we get

$$H_\alpha(y)=\sum_{m=0}^{\infty}\frac{(-2)^m(y-z)^m}{m!}(-\alpha)_m\ H_{\alpha-m}(z) \tag{44}$$

and

$$H_\alpha(lz)=\sum_{m=0}^{\infty}\frac{(-2)^m(l-1)^m}{m!}z^m(-\alpha)_m\ H_{\alpha-m}(z)\ . \tag{45}$$

above results, Eq.44 and Eq.45 are discussed in Rainville [3; p.197, (1)], Truesdell [7,p.85,(10)],

4. Conclusion

Two new generating relations are obtained for the Modified Hermite polynomials $H_m(x;u)$ by using the Truesdell's method. In applications certain generating relations of Modified Hermite polynomials are discussed. This method is a powerful technique to obtain generating relations from the differential recurrence relations of the ascending and the descending type for the polynomial $H_m(x;u)$.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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