

A SARIMA and Adjusted SARIMA Models in a Seasonal Nonstationary Time Series; Evidence of Enugu Monthly Rainfall

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Abstract

The paper compares SARIMA and adjusted SARIMA (ASARIMA) in a regular stationary series where the underlying variable is seasonally nonstationary. Adopting empirical rainfall data and Box-Jenkins iterative algorithm that calculates least squares estimates, Out of 11 sub-classes of SARIMA and 7 sub-classes of ASARIMA models, AIC chose ASARIMA(2,1,1)₁₂ over all sub-classes of SARIMA(p,0,q)x(P,1,Q)₁₂ identified. Diagnostic test indicates absence of autocorrelation up to the 48th lag. The forecast values generated by the fitted model are closely related to the actual values. Hence, ASARIMA can be recommended for regular stationary time series with seasonal characteristics and where parameter redundancy and large sum of square errors are penalized.

Keywords: AIC, ASARIMA model, rainfall, seasonal nonstationary time series.

I. Introduction

The use of seasonal autoregressive integrated moving average (SARIMA) terms for monthly or quarterly data with systematic seasonal movements was recommended by [7]. Technical details can be obtained from the aforementioned citation. Situation could arise when the underlying variable of interest is regularly stationary but it is characterized by cyclical pattern that is seasonally nonstationary and needs seasonal differencing. Time series variables with such characteristics can be better modelled with Adjusted SARIMA(P,D,Q)_s rather than SARIMA(p,d,q)x(P,D,Q)_s model. However, for such time series, SARIMA(p,d,q)x(P,D,Q)_s increases the sum of square residuals due to some redundant parameters and the autocorrelation of the model residuals may be strong in higher lag orders. These are the advantages of Adjusted SARIMA over SARIMA model. Adjusted SARIMA models are frugal in parameter representation. Rainfall is one of the most important natural factors that determine the agricultural production in and across the globe, particularly in Nigeria. The variability of rainfall and the pattern of extreme high or low precipitation are very important for agriculture as well as the economy of the state. Even the global climatic change has increased the quest for more research on the subject matter due to high flood risk disaster at the peak of rainy season.

Enugu State is one of the states in the eastern part of Nigeria located at the foot of the Udi Plateau, a tropical rain forest zone with a derived savannah. The state shares borders with Abia State and Imo State to the south, Ebonyi State to the east, Benue State to the northeast, Kogi State to the northwest and Anambra State to the west. Enugu has good soil-land and climatic conditions all year round, sitting at about 223 metres (732 ft) above sea level, and the soil is well drained during its rainy seasons. Enugu is in the tropical rain forest zone with a derived savannah, with humidity highest between March and November [13]. For the whole of Enugu State the mean daily temperature is 26.7 °C (80.1

°F). The mean temperature in Enugu State in the hottest month of February is about 87.16 °F (30.64 °C), while the lowest temperatures occur in the month of November, reaching 60.54 °F (15.86 °C). The lowest rainfall of about

0.16 cubic centimetres (0.0098 cubic inch) is normal in February, while the highest is about 35.7 cubic centimetres (2.18 cu in) in July. Enugu State had a population of 3,267,837 people at the census held in 2006 (estimated at over 3.8 million in 2012).

A lot of researchers have paid considerable attention towards modelling and forecasting the amount of rainfall pattern in various places. For instance, [14] fitted a SARIMA(0, 1, 1) \times (0, 1, 1)₁₂ monthly rainfall in Tamilnadu, India. [16] fitted the SARIMA models of orders (1, 1, 2) \times (1, 1, 1)₁₂ and (4, 0, 2) \times (1, 0, 1)₁₂ respectively for monthly rainfall in Malaaca and Kuantan in Malaysia. [1] examined the SARIMA model suitable for rainfall prediction in the Brong Ahafo (BA) Region of Ghana using a data from 1975 to 2009. The results revealed that the region experience much rainfall in the months of September and October, and least amount of rainfall in the months of January, December and February. They fitted SARIMA (0,0,0) \times (1,1,1)₁₂ model for predicting monthly average rainfall figures for the Brong Ahafo Region of Ghana.

[12] modelled monthly rainfall in Port Harcourt, Nigeria, using seasonal SARIMA (5, 1, 0) \times (0, 1, 1)₁₂ model. The time- plot shows no noticeable trend. The known and expected seasonality is clear from the plot. Seasonal (i.e. 12-point) differencing of the data is done, then a nonseasonal differencing is done of the seasonal differences. The correlogram of the resultant series reveals the expected 12- monthly seasonality, and the involvement of a seasonal moving average component in the first place and a nonseasonal autoregressive component of order 5. Hence the model mentioned above. The adequacy of the modelled has been established. [15] modelled quarterly rainfall in Port Harcourt, Nigeria, as a SARIMA(0, 0, 0) \times (2, 1, 0)₄ model.

[3] examined the time series analysis on rainfall in Oshogbo Osun State, Nigeria, using monthly data of rainfall between 2004-2015. The time plot reveals that the rainfall data show high level of volatility characterized by seasonal and irregular variations. And the logistic model applied showed to be better and then used to forecast the rainfall for the next 2 years. [5] examined the modelling of mean annual rainfall pattern in Port Harcourt, Nigeria using ARMA(p,q) model.. The data on rainfall used covered the period of 1981 to 2016. Sum of squares deviation forecast criteria (SSDFC) was adopted to select the best performing sub-classes of ARMA(p, q) that fits

the data. Among ARMA(1, 1), ARMA(1, 2) ARMA(2, 1) and ARMA(2, 2) models estimated, SSDFC chose ARMA(1, 2) as the best performing model. The selected model were supported by AIC and BIC respectively. And concluded that ARMA(1, 2) can be used to predict long term quality of water for agriculture and hydrological purpose and to create long term awareness against flood and control strategy for Port Harcourt.

[6] modelled monthly rainfall pattern in Imo state using seasonal autoregressive integrated moving average (SARIMA) model with univariate monthly rainfall data spanning from 1981M1-2017M12. Sum of square deviation forecast criterion (SSDFC) was used to compare nine (9) different sub-classes of SARIMA(p,d,q)x(P,D,Q)12 models identified. And the result indicated that SARIMA(0,0,0)x(1,1,1)12 is more appropriate in predicting monthly rainfall in the state.

The modelling of monthly rainfall in any state is essential in understanding the temporal and spatial variability which is very important in flood risk management, irrigation and surface water management and so on. Moreover, the need to diversify the economy towards agricultural base in Nigeria has made it necessary to model seasonal pattern of rainfall in the state for agricultural planning. Hence, this study examines the best fitted model between seasonal ARIMA (SARIMA) model and adjusted seasonal ARIMA (ASARIMA) model for rainfall forecast in Enugu State.

The study presents a simple analytical model adjusted from SARIMA process. The remaining part of the paper is arranged as follows; section two presents the materials and methods, section three presents data analysis and results and section four deals with conclusion.

II. MATERIALS AND METHODS

This section highlights the methods and sources of data collection, variable measurement, method of unit root test, model specification, and model identification, method of data analysis, model comparison techniques and diagnostic checks.

A. Source of Data and Variable Measurement

The monthly rainfall data was obtained from central bank of Nigeria (CBN) (2018) statistical bulletin. The univariate time series data collected covered the period of 1981M1- 2017M12 (432 observations of monthly rainfall data). Rainfall is usually measured in millimetre using rain gauge.

B. SARIMA Model Specification

If the time series $\{X_t\}$ is nonstationarity due to the presence of one or several of five conditions: outliers, random walk, drift, trend, or changing variance, it is conventional that first or second differencing (d) is necessary to achieve stationarity. Hence, the original series is said to follow an autoregressive integrated moving average model or orders p, d and q denoted by ARIMA(p, d, q) of the form

$$A(L)\nabla^d X_t = B(L)u_t \quad (1)$$

If the series $\{X_t\}$ exhibits seasonal patterns of nonstationarity, this may be detected using time plot, correlograms or even unit root test. And according to [7] Seasonal ARIMA

models sometimes called SARIMA models has the general form SARIMA(p, d, q) \times (P, D, Q)_s and it is given as

$$A(L)\Phi(L^s)\nabla^d\nabla_s^D X_t = B(L)\Theta(L^s)u_t \quad (2)$$

where A(L) is the autoregressive (AR) operator, given by $A(L)=1-\alpha_1L-\dots-\alpha_pL^p$ and B(L) is the moving average (MA)operator, given by $B(L)=1-\beta_1L-\dots-\beta_qL$. For L denotes the backshift operator. $\Phi(L^s)$ and $\Theta(L^s)$ are lagged seasonal AR and MA operators of order P and Q respectively. The operator ∇^d denotes the difference operator defined by $\nabla^d = 1 - L$ and $d \leq 2$. The ∇_s^D represents the seasonal difference operator defined by $\nabla = 1 - L^s$ and D is the seasonal differencing order. The seasonal differencing $(1 - L^s)$ is called the simplifying operator, which renders the residual series stationary and amenable to further analysis.

C. Adjusted SARIMA Model

The SARIMA model in (2) is the combination of nonseasonal AR and MA operators of order p and q and seasonal AR and MA operators of order P and Q. If a univariate time series is stationary in non-seasonal component (where d=0) and exhibits a purely seasonal pattern that is nonstationary (where D=1). It could be parsimoniously better to only fit the seasonal AR and MA operators of order P and Q. In such cases, it is appropriate to assume that A(L) \square 1, B(L) \square 1 and d=0 so that (2) can be of the form;

$$\Phi(L_s)\nabla_s X_t = \Theta(L_s)u_t \quad (3)$$

where $\Phi(L_s)$ is the seasonal autoregressive (SAR) operator, given by $\Phi(L_s)=1-\phi_1L_{s \times 1}-\dots-\phi_P L_{s \times P}$ and $\Theta(L_s)$ is the seasonal moving average (SMA) operator, given by $\Theta(L_s)=1-\theta_1L_{s \times 1}-\dots-\theta_Q L_{s \times Q}$. Generally, the Adjusted SARIMA(P,D,Q)_s model which hereafter is known as ASARIMA(P,D,Q)_s model with the inbuilt constant term is specifically of the form;

$$\begin{aligned} \nabla_s X_t = \omega + \phi_1 \nabla_s X_{t-(s \times 1)} + \dots + \phi_P \nabla_s X_{t-(s \times P)} \\ + \theta_1 u_{t-(s \times 1)} + \dots + \theta_Q u_{t-(s \times Q)} \end{aligned} \quad (4)$$

where ω is the constant parameter and s is the seasonal index. ASARIMA(P,D,Q)_s model is special case of SARIMA(p, d, q) \times (P, D, Q)_s model.

D. Model Identification

The ACF of an MA(q) model cuts off after lag q whereas that of an AR(p) model is a combination of sinusoidals dying off slowly. On the other hand, the PACF of an MA(q) model dies off slowly whereas that of an AR(p) model cuts off after lag p. The AR and MA models are known to exhibit some duality relationships. Parametric parsimony consideration in model building entails the use of the mixed ARMA fit in preference to either the pure AR or the pure MA fit.

Note that SARIMA can be fitted irrespective of whether the underlying variable is seasonally stationary or not. The differencing operators $d \neq 0$ for stationary series and for nonstationary series d could be 1 or 2 depending on the order of integration of the variable under study. The seasonal difference D may be chosen to be at most equal to 1. The nonseasonal and seasonal AR orders p and P are fitted by the nonseasonal and the seasonal cut-off lags of the partial autocorrelation function (PACF) respectively. Similarly the nonseasonal and the seasonal MA orders q and Q are fitted respectively by the nonseasonal and seasonal cut-off points of the ACF.

E. Conditions for ASARIMA(P,D,Q)s Model

The following conditions should lead to the adoption of ASARIMA(P,D,Q)s model;

- 1) The underlying univariate time series must be non- seasonally stationary ($d=0$) and exhibits cognizable seasonal pattern. Note, seasonal differencing(D) may be 0 or 1.
- 2) The ACF must reveal seasonal oscillation with significant spikes at every k th lag, here $k = s \times i$ and $i = 1, 2, \dots, K$.
- 3) The PACF tends to cut-off at every k th lag and cut-in.
- 4) If the spikes in (iii) tails off at every k th lag consider fitting ASARIMA(P,D,0)s
- 5) If the spikes in (iii) do not tails off at every k th lag consider fitting ASARIMA(0,D,Q)s
- 6) If the spikes indicate mixture of (iv) and (v) consider fitting ASARIMA(P,D,Q)s
- 7) Use some information criteria such SSDFC, AIC BIC, SC etc to select the best fitted model.

F. ADF Unit Root Test

ADF unit root test helps to check the order of integration of the variables under study. The unit root test here, is based on Augmented Dickey Fuller (ADF) test and is of the form

$$\nabla y_t = \alpha + \alpha_1 t + \beta y_{t-1} + \sum_{i=1}^k \xi_i \nabla y_{t-i} + a_t \quad (5)$$

where k is the number of lag variables. In (5) there is intercept term, the drift term and the deterministic trend. The non deterministic trend term removes the trend term in (5). And it can be carried out with the choice of removing both the constant and deterministic trend term in the above regression.

ADF unit root test null hypothesis $H_0 : \beta = 0$ and alternative $H_a : \beta < 0$. According to [7], if the ADF test statistic is greater than 1%, 5% and 10% critical values, the null hypothesis of a unit root test is accepted. ERS unit root test will used to consolidate the result provided by ADF test. See the technical details in [11].

G. Model Comparison

There are several model selection criteria in literature such as; Bayesian information criterion(BIC),Aikaike information criterion(AIC), residual sum of squares and so on. If n is the sample size and RSS is the residual sum of squares, then, BIC and AIC are given as follows;

$$BIC = 2k + \ln(RSS / n) + k(\ln n / n) \quad (6)$$

$$AIC = 2k + n \ln(RSS / n) \quad (7)$$

where, n is the sample size, k is the number of estimated parameters (for the case of regression, k is the number of regressors) and RSS is the residual sum of squares based on the estimated model. However, it is good to note that both BIC and AIC are affected by the number of parameters included to be estimated in a model. For the case of BIC , it penalizes free parameters while AIC becomes smaller as the number of free parameters to be estimated increases. But for this study, model selection will be based on AIC . The sum of squares deviation forecast criterion introduced by [4] will be used to check models output performance for 150 forecast lead time . And it is of the form;

$$SSDFC = \frac{1}{m} \sum_{i=1}^m (y_{t(l,i)} - \hat{y}_{t(l,i)})^2 \quad (8)$$

Where l is the lead time, m is the number of forecast values to be deviated from the actual values (m should be reasonably large), $y_{t(l,i)}$ is the actual values of the time series corresponding to the i th position of the forecast values and $\hat{y}_{t(l,i)}$ is the forecast values corresponding to the i th position of the actual values. In comparison, the model with the smallest value of $SSDFC$ is the best output performing model that can describe, to the closest precision, the behavior of the underlying fitted model.

H. Model Estimation

The coefficients are estimated using an iterative algorithm that calculates least squares estimates. At each point of iteration, the back forecasts are computed and sum of squares error (SSE) is calculated. For more details, see [8].

III. DATA ANALYSIS AND RESULTS

This section presents the time series plot of Enugu monthly rainfall data, results of ADF unit root test, plots of ACF and PACF and estimates of SARIMA(p,d,q)×(P,D,Q)_s model.

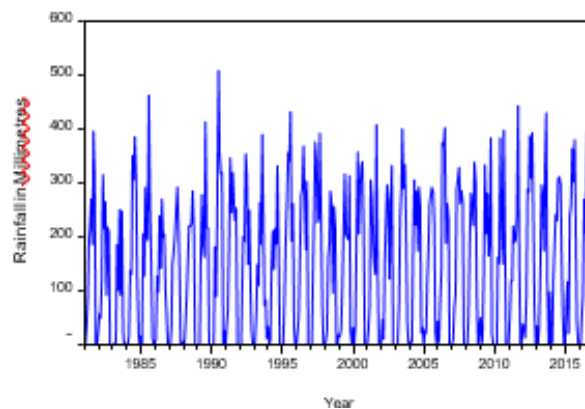


Fig.1. Time plot of EMR (1981M1 – 2016M12)

The plot of monthly rainfall in Figure1 exhibits seasonal nonstationary pattern. It is also observable that the time series plot lacks trend with the highest precipitation of 508.3 Millimeters in July 1990 and lowest precipitation of 0.5 Millimeters in January and February the same year.

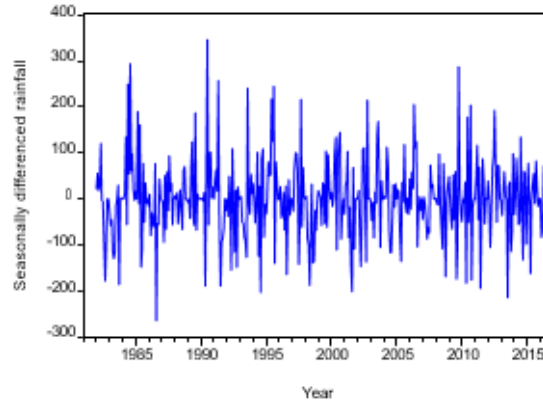


Fig.2. Time plot of Seasonally differenced EMR (1981M1 – 2016M12)

The seasonally differenced EMR data in Fig.2 is seasonally stationary with most of the data concentrated around zero.

TABLE I. ANALYSIS OF ORDER OF INTEGRATION OF EMR

Test	Rainfall	DT	Lags	Test Value	Critical Values	Remark
					1% 5% 10%	
ADF	EMR	C	11	-3.8511	-3.4457 -2.8682 -2.5704	I(0) significant under 5%
ERS	EMR	C	5	2.4747	1.9900 3.2600 4.4800	I(0) significant at 5%

Note that DT represents ‘deterministic term’.

The results of ADF and ERS unit root tests in Table I above generally indicate that EMR variable is integrated order zero I(0), significant at 5% level. Hence, the monthly rainfall under investigation is stationary. Having the EMR variable exhibiting stationarity, then, it will be modeled using seasonal autoregressive moving average SARIMA (p, 0, q)×(P,D,Q)s model.

A. Correlogram

The correlogram presents the plots of autocorrelation function (ACF) and the partial autocorrelation function (PACF) for model identification as presented in Figure3 and Figure4 below.

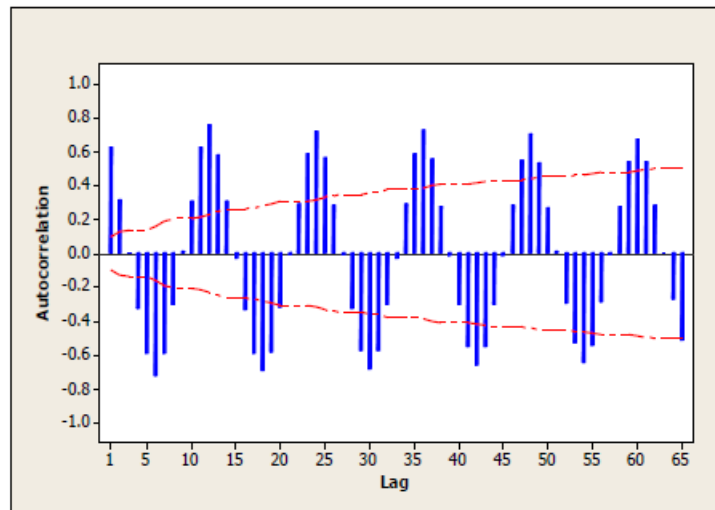


Fig.3. Plots of ACF EMR (with 5% significance limits for the correlogram)

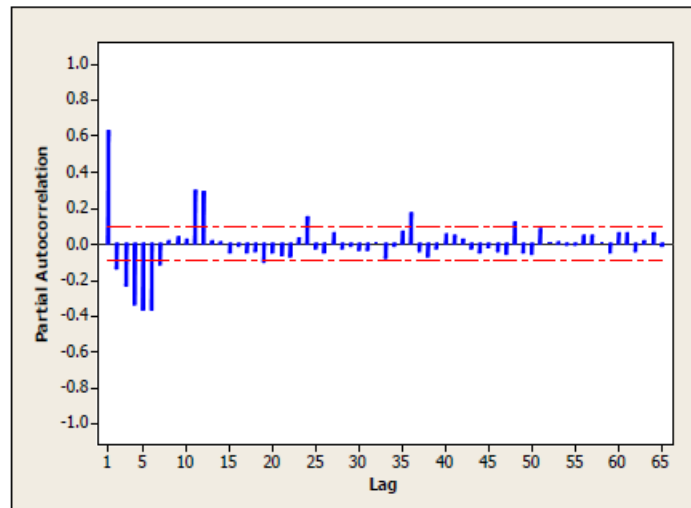


Fig.4. Plots of PACF for EMR (with 5% significance limits for the correlogram)

The plot of autocorrelation function in Figure3 exhibits presence of seasonal effect. The cyclical correlogram with a seasonal frequency suggests fitting a seasonal ARMA model to the rainfall data. The result indicates the need for seasonal differencing in the model. The time plot revealed seasonality in the rainfall variable. But where this is not too clear via time plot, the autocorrelation function (ACF) could reveal the value of s , as the significant lag of the ACF.

There appear to be annual or 12-month spikes in the ACF and PACF as shown in Figure3 and Figure4. The ACF clearly exhibits this prima facie evidence of seasonal nonstationarity. The PACF reveals the seasonal spikes at lags 12, 24 36 and

48. Slow attenuation of the seasonal peaks in the Figure4 ACF signifies seasonal nonstationarity. The 12-month PACF periodicity can be seen in the periodic peaks at every 12th lag up to 48th lag evocating of seasonal differencing at lag 12.

B. Model Comparison

This section presents a comparison of 27 possible models using SSDFC as presented in Table II below;

TABLE II. MODEL SELECTION USING AIC

Model	AIC	BIC	SSDFC
SARIMA(1,0,0)×(1,1,1) ₁₂	3557.36	16.2723	3114.06*
SARIMA(2,0,0)×(1,1,1) ₁₂	3559.35	18.2863	3116.75
SARIMA(3,0,0)×(1,1,1) ₁₂	3560.40	20.2982	3123.90
SARIMA(1,0,1)×(1,1,1) ₁₂	3558.85	18.2852	3118.40
SARIMA(2,0,1)×(1,1,1) ₁₂	3560.18	20.2977	3126.79
SARIMA(3,0,1)×(1,1,1) ₁₂	3559.91	22.3065	3123.78
SARIMA(1,0,3)×(1,1,1) ₁₂	3559.78	22.3061	3123.79
SARIMA(1,0,0)×(2,1,1) ₁₂	3555.24	18.2768	3238.72
SARIMA(0,0,1)×(1,1,1) ₁₂	3557.34	16.2722	3117.09
SARIMA(0,0,2)×(1,1,1) ₁₂	3559.31	18.2862	3119.80
SARIMA(0,0,3)×(1,1,1) ₁₂	3560.48	20.2984	3121.29
ASARIMA(0,1,1) ₁₂	3553.90	12.2455*	3116.94
ASARIMA(0,1,2) ₁₂	3551.61	14.2496	3186.91
ASARIMA(0,1,3) ₁₂	3554.10	16.2647	3218.03
ASARIMA(1,1,1) ₁₂	3555.91	14.2595	3116.73
ASARIMA(2,1,1) ₁₂	3550.12*	16.2555	3229.91
ASARIMA(1,1,2) ₁₂	3551.88	16.2596	3119.28
ASARIMA(2,1,2) ₁₂	3551.24	18.2676	3157.69

The 18 models in Table II above showed at least no serial correlation in the model residuals up to 12th lag using Modified Box-Pierce statistic. Model comparison using AIC indicates that ASARIMA(2,1,1)₁₂ is preferred to the other sub-classes of SARIMA(p,d,q)×(P,D,Q)₁₂ and ASARIMA (P,D,Q)₁₂ models since it has the smallest value of AIC. Though the chosen information criterion is AIC, BIC also preferred ASARIMA to SARIMA. However, based on output performance such as forecast (for 150 lead time), SSDFC prefers and SARIMA(1,0,0)×(1,1,1)₁₂ followed by ASARIMA(1,1,1)₁₂.

TABLE III. FINAL ESTIMATES OF ASARIMA(2,1,1)₁₂ PARAMETERS

Type	Coef	SECoef	T	P
SAR 12	0.0047	0.0518	0.09	0.928
SAR 24	24	0.0519	-1.89	0.060
SMA 12	12	0.0234	40.66	0.000
Constant	1.0529	0.2195	4.80	0.000

Differencing: 0 regular, 1 seasonal of order 12, Number of observations: Original series 432, after differencing 420, Residuals: SS = 1571863 (backforecasts excluded), MS = 3779 DF = 416

The model result in Table III shows that the parameters SMA lag 12 and SAR at lag 24 are significant under 5% and 10% respectively. The ASARIMA(2,1,1)12 model is of the form;

$$\nabla_{12}X_t = 1.0529 + 0.0047\nabla_{12}X_{t-12} - 0.0981\nabla_{12}X_{t-24} + 0.9513u_{t-12} \quad (9)$$

TABLE IV. MODIFIED BOX-PIERCE (LJUNG-BOX) CHI-SQUARE STATISTIC

Lag	12	24	36	48
Chi-Square	13.4	24.2	45.5	53.4
DF	8	20	32	44
P-Value	0.099	0.058	0.058	0.0156

The result of Table IV shows that the probability of Modified Box-Pierce (Ljung-Box) Chi-Square statistic are all greater than 5% significant level, this indicates that the residuals of the ASARIMA(2,1,1)12 are not correlated up to 48th lag. Hence the model is adequate.

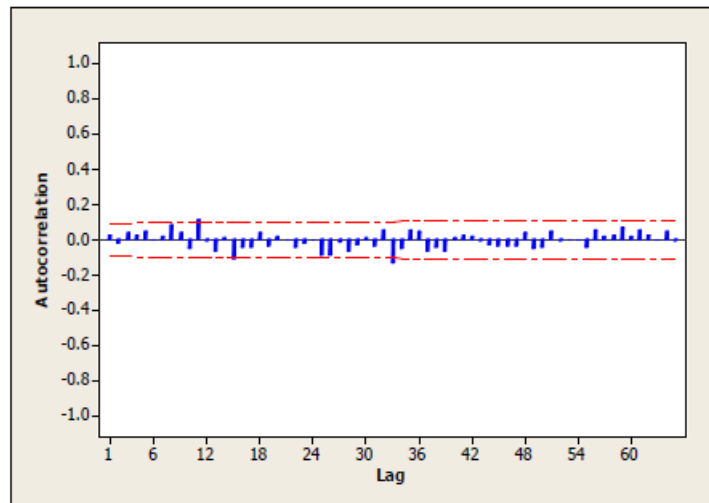


Fig.5. Plot of ACF of Residuals (with 5% significance limits for autocorrelations)

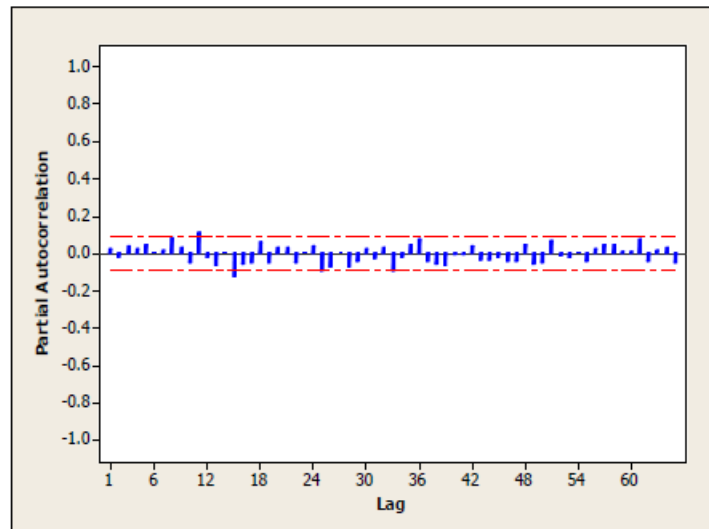


Fig.6. Plot of PACF of Residuals (with 5% significance limits for autocorrelations)

The ACF and PACF of residuals in Figure5 and Figure6 respectively for the Enugu rainfall data showed no significant spikes (the spikes are within the confidence limits) indicating that the residuals are uncorrelated. Therefore, the ASARIMA(2,1,1)12 model appears to fit well and can be used to make forecasts for Enugu monthly rainfall.

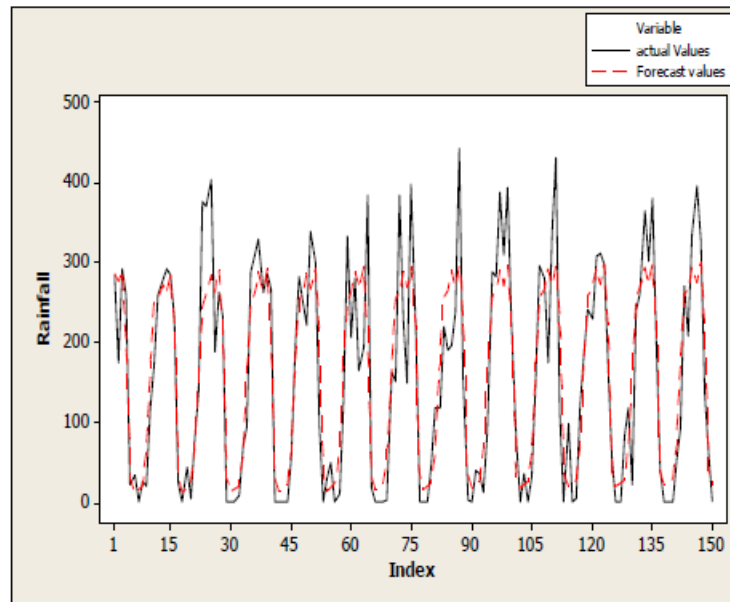


Fig.7. Time plot of forecast and actual values

The generated forecast values in Figure7 above showed close relation with the actual values. Hence, it can be said that the fitted model has performed pretty good.

C. Discussion of Results

In a regular stationary time series variable with seasonal nonstationary behaviour, such as that of Enugu monthly rainfall (EMR) pattern, the comparison study using AIC reveals that

ASARIMA(2,1,1)₁₂ performed better than all the sub-classes of SARIMA(p,d,q)×(P,D,Q)_s model. Modified Box-Pierce (Ljung-Box) Chi-Square statistic indicates that the residuals of the ASARIMA(2,1,1)₁₂ are not correlated up to 48th lag. Again, the ACF and PACF of the model residuals are uncorrelated too and the forecast values are very close, indicating the adequacy of the fitted model.

However, unlike past studies by researchers have clustered on the application of SARIMA model introduced by Box and Jenkins(1979), the Adjusted SARIMA introduced here, has spiced up a new dimension in the modeling of seasonal behaviour of variables that are adjudged to be regularly stationary (where $d = 0$) and seasonally nonstationary (where $D = 1$).

IV. CONCLUSION

The paper compared SARIMA and Adjusted SARIMA models in a regular stationary time series with seasonal nonstationary behaviour such as Enugu monthly rainfall data, and the finding indicates that ASARIMA(2,1,1)₁₂ subclass is better than all SARIMA sub-classes as reported by AIC.

Therefore, it can be recommended that for such pattern of time series, ASARIMA is preferred due to its ability to reduce parameter redundancy and sum of square errors in the model.

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