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Fuzzy e-paraopen Sets and Maps in Fuzzy Topological Spaces

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Abstract

This article is to study the concepts of fuzzy e-paraopen and fuzzy e-paraclosed sets in fuzzy topological spaces. Further, we extent to study few class of fuzzy maps namely fuzzy e-paracontinuous, -fuzzy e-paracontinuous, fuzzy e-parairresolute, fuzzy minimal e-paracontinuous, fuzzy maximal e-paracontinuous mappings and study their properties.

Keywords: Fuzzy e-paraopen, fuzzy e-paracontinuous, fuzzy minimal e-paracontinuous, fuzzy maximal e-paracontinuous.

I. Introduction

Zadeh [10] established fuzzy sets and since fuzzy topology developed by Chang [2]. The notions of fuzzy minimal(maximal) open and paraopen sets respectively explored by Ittanagi and Wali in [3] and [4]. Subsequently Mukherjee and Bagchi in [1] introduced and showed the notion of mean open set. In section II of current article we introduce the perception of fuzzy e-paraopen set and investigate some comparative results. In section III, we introduce fuzzy e-paracontinuous, -fuzzy e-paracontinuous, fuzzy e-parairresolute, fuzzy minimal e-paracontinuous, fuzzy maximal e-paracontinuous maps and from which we investigate some results with appropriate examples. Throughout this paper following terminologies "fuzzy e-open, fuzzy e-paraopen, fuzzy e-paraclosed, fuzzy minimal e-open, fuzzy minimal e-closed, fuzzy maximal e-open, fuzzy maximal e- closed are respectively abbreviated as Fe-O,Fe-PO,Fe-PC,FMIe-O,FMIe-C,FMAe-O,FMAe-C respectively. Throught this paper F and Y stands for fuzzy topological spaces."

The following terminologies "fuzzy e-continuous, fuzzy e-paracontinuous, fuzzy minimal e-continuous,fuzzy maximal e- continuous, fuzzy minimal e-paracontinuous, fuzzy maximal e-paracontinuous, fuzzy maximal e-parairresolute are respectively abbreviated as f.e-c,f.e-pc,f.mi.e-c,f.ma.e-c,f.mi.e-pc,f.ma.e-pc,f.mi.e-p.i,f.ma.e-p.i respectively"

Definition 1.1 A fuzzy subset ξ of a space *F* is called fuzzy regular open [3] (resp. fuzzy regular closed) if ξ =Int(Cl(ξ)) (resp.ξ =Cl(Int(ξ))).
The fuzzy δ-interior of a fuzzy subset ξ of *F* is the union of all fuzzy regular open sets contained in ξ. A fuzzy subset ξ is called fuzzy δ-open [9] if ξ = Intδ(ξ). The complement of fuzzy δ-open set is called fuzzy δ-closed (i.e., ξ = Clδ(ξ)).
Definition 1.2 A fuzzy subset ξ of a fts *F* is called fuzzy e-open [8] if ξ cl(intδξ) int(clδξ) and fuzzy e-closed set if
ξ cl(intδξ) int(clδξ).
Definition 1.3 [7]A proper nonzero fuzzy e-open set α of *F* is said to be a (i)fuzzy minimal e-open if 1 *F* and α are only fuzzy e-open sets contained in α. (ii)fuzzy maximal e-open1 *F* and α are only fuzzy e-open sets containing α.
Definition 1.4 A map from fts *F* to another fts Y is called,
(i) fuzzy minimal e-continuous[7] if f −1(λ) is a fuzzy e-open set on *F* for any fuzzy minimal e-open set λ on Y. (ii)fuzzy maximal e-continuous[7] if f −1(λ) is a fuzzy e-open set on *F* for any fuzzy maximal e-open set λ on Y.

II. FUZZY e-PARAOPEN AND SOME of THEIR PROPERTIES

Definition 2.1 A Fe-O set β of a fts *F* is said to be a Fe-PO set if is neither FMIe-O nor FMAe-O set. The complement of Fe-PO set is Fe-PC set.
Remark 2.2 It could be clear from definitions that every Fe-PO set is a Fe-O set and every Fe-PC set is a Fe-C set converse is not true as shown in the succeeding example.
Example 2.3 Let β1,β2, β3 and β4 be fuzzy sets on *F* = {a, b, c} . Then β1 = 0.5/a + 0.8/b + 0.8/c, β2 = 0.5/a + 0.8/b + 0.9/c, β3 = 1.0/a + 0.9/b + 0.8/c and β4 = 1.0/a + 0.9/b + 0.9/c be fuzzy sets with $\mathfrak{F}_1 = \{0_F, \beta_1, \beta_2, \beta_3, \beta_4, 1_F\}$, Then $FM_iO(F) = \{\beta_1\}, FM_aO(F) = \{\beta_4\}, FM_iC(F) = \{\beta_4^c\}$. $FM_aC(F) = \{\beta_1^c\}$ $, FP_aO(F) = \{\beta_2, \beta_3\}, FP_aC(F) = \{\beta_2^c, \beta_3^c\}$. Here β1 is a Fe-O set but not a Fe-PO set and βc is a fuzzy e-closed set but not a Fe-PC set.
Remark 2.4 The succeeding example revealed that union and intersection of Fe-PO (resp. Fe-PC) sets need not be a Fe-PO (resp. Fe-PC).
Example 2.5 In example 2.3, fuzzy sets β2, β3 are Fe-PO sets but β2 ∨ β3 = β4 and β2 ∧ β3 = β1 which are not Fe-PO sets. Similarly for the Fe-PC sets βc2,βc3 but βc2 ∨ βc3 = βc1 and βc2 ∧ βc3 = βc4 which are not Fe-PC sets.
Theorem 2.6 Let α be a nonzero proper Fe-PO subset of *F*. Then there exists a FMIe-O set β such that β < α.
Proof. Since the definition of FMIe-O set, we can conclude that β < α.
Theorem 2.7 Let α be a nonzero proper Fe-PO subset of *F*. Then there exists a FMAe-O set P such that α < P.
Proof. Since the definition of FMAe-O set, we can conclude that α < P.
Theorem 2.8 (i)Let α be a Fe-PO and β be a FMIe-O set in *F*. Then α^β = 0 *F* or β < α.

(ii)i)Let α be a Fe-PO and τ1 be a FMAe-O set in *F*. then αντ1 = 1 *F* or α < τ1.
(iii)Intersection of Fe-PO sets is either Fe-PO or FMIe-O set.
Proof. (i) Let α be a Fe-PO and β be a FMIe-O set in *F*. Then α ^ β = 0 *F* or α ^ β ≠ 0 *F*. Suppose α ^ β = 0 *F*, then we need not prove anything. Assume α β Ç 0 *F*. Then we get α β is a Fe-O set and α β < β. Hence β < α.
(ii) Let α be a Fe-PO and γ be a FMAe-O set in *F*. Then α *v* γ = 1 *F* or α *v* β ≠ 1 *F*. Assume α γ = 1 *F*, then we need not prove anything. Suppose α γ Ç 1 *F*. Then we get α γ is a Fe-O set and γ < α γ. Since γ is a FMAe-O set, α *v* γ = γ which implies α < γ.
(iii) Let α and ηbe a Fe-PO sets in *F*. As α ∧ η is a Fe-PO set then we need not prove anything. Assume α ∧ η is not a Fe-PO set. Since definition, α∧η is a FMIe-O or FMAe-O set. If α∧η is a f.mi. e-open set then we need not prove anything. Suppose α ∧ η is a FMAe-O set. Now α ∧ η < α and α ∧ η < η which contradicts the fact that α and η are Fe-PO sets. Therefore α ∧ η is not a FMAe-O set. That is α ∧ η must be a FMIe-O set.
Theorem 2.9 A subset τ1 of *F* is Fe-PC iff it is neither FMAe-C nor FMIe-C set.
Proof. Since the definition of FMAe-C set and the fact that the complement of FMIe-O set is FMAe-C set and the complement of FMAe-O set is FMIe-C set.
Theorem 2.10 Let *F* be a fts and τ1 be a nonzero Fe-PC subset of *F*. Then there exists a f.mi.e-c set P such that P < τ1.
Proof. Since the definition of FMIe-C set we can conclude that P < τ1.
Theorem 2.11 Let *F* be a fts and τ1 be a nonzero Fe-PC subset of *F*. Then there exists a f.ma. closet set Q such that τ1 < Q. Proof. Since the definition of FMAe-C set we can conclude that τ1 < Q.
Theorem 2.12 Let *F* be a fts.
(i)Let δ be a Fe-PC and τ be a FMIe-C set. Then $\delta \wedge \tau = 0_F$ or $\tau < \delta$.
(ii)Let δ be a Fe-PC and γ be a FMAe-C set. Then $\delta \vee \gamma = 1_F$ or $\delta < \gamma$.
(iii)Intersection of Fe-PC sets is either Fe-PC or FMIe-C set.
Proof. (i) Let δ be a Fe-PC and τ be a FMIe-C set in *F*. Then (1 *F* − δ) is Fe-PO and (1 *F* − τ) is FMAe-O set in *F*. By Theorem 2.8(ii) we have (1 *F* − δ) ∨ (1 *F* − τ) = *F* or (1 *F* − δ) < (1 *F* − τ) which implies 1 *F* − (δ ∧ τ) = 1 *F* or τ < δ. Therefore δ ∧ τ = 0 *F* or

(ii) Let δ be a Fe-PC and γ be a FMAe-C set in *F*. Then (1 *F* − δ) is Fe-PO and (1 *F* − γ) is FMIe-O sets in *F*. By Theorem 2.8(i) we have (1 *F* − δ) ∧ (1 *F* − γ) = 0 *F* or 1 *F* − γ < 1 *F* − δ which implies 1 *F* − (δ ∨ γ) = 0 *F* or δ < γ. Therefore δ ∨ γ = 1 *F* or

(iii) Let δ and η be a Fe-PC sets in *F*. As δ ∧ η is a Fe-PC set then nothing to prove. Assume δ ∧ η is not a Fe-PC set. By definition, δ ∧ η is a FMIe-C or FMAe-C set. If δ ∧ η is a f.mi. e-closed set, then nothing to prove. Suppose δ ∧ η is a FMAe-C set. Now δ < δ ∧ η and η < δ ∧ η which contradicts the fact that δ and η are Fe-PC sets. Therefore δ ∧ η is not a FMAe-C set. That is δ ∧ η must be a FMIe-C set.

III. FUZZY E-PARACONTINUOUS MAPS AND SOME of THEIR PROPERTIES

Definition 3.1 A map ψ from fts *F* to another fts Δ is called

(i) f.e-pc if $\psi^{-1}(\alpha)$ is a Fe-O set on *F* for every Fe-PO set α on Δ.

(ii) *-f.e-pc if $\psi^{-1}(\alpha)$ is a Fe-PO set on *F* for every Fe-O set α on Δ.

(iii) f.e-p.i if $\psi^{-1}(\alpha)$ is a Fe-PO set on *F* for every Fe-PO set α on Δ.

(iv) f.mi.e-pc if $\psi^{-1}(\alpha)$ is a Fe-PO set on *F* for every FMIe-O set α on Δ.

(v) f.ma.e-pc if $\psi^{-1}(\alpha)$ is a Fe-PO set on *F* for every FMAe-O set α on Δ.

Theorem 3.2 Every f.e-c map is f.e-pc but not conversely.

Proof. Let $\psi : F \to \Delta$ be a f.e-c map. We have to prove ψ is f.e-pc. Let α be any Fe-PO set in Δ. Since every Fe-PO set is a Fe-O set, α is Fe-O set in Δ. Since ψ is a f.e-c, $\psi^{-1}(\alpha)$ is Fe-O set in *F*. Hence ψ is a f.e-pc.

Example 3.3 Let $\alpha_1, \alpha_1^c, \alpha_2, \alpha_3, \alpha_4$ and α_5 be fuzzy sets on *F* = {a, b, c} with

$$\alpha_1 = \frac{0.3}{a} + \frac{0.4}{b} + \frac{0.4}{c},\ \alpha_2 = \frac{0.3}{a} + \frac{0.4}{b} + \frac{0.5}{c},\ \alpha_3 = \frac{0.6}{a} + \frac{0.5}{b} + \frac{0.4}{c},\ \alpha_4 = \frac{0.6}{a} + \frac{0.5}{b} + \frac{0.5}{c},\ \alpha_5 = \frac{0.7}{a} + \frac{0.6}{b} + \frac{0.4}{c} \text{ and } \alpha_1{}^c = \frac{0.7}{a} + \frac{0.6}{b} + \frac{0.5}{c}.$$

Let $\tau_1 = \{0_F, \alpha_1, \alpha_2, \alpha_3, \alpha_4, 1_F\}$ and $\tau_2 = \{0_F, \alpha_1, \alpha_1^c, \alpha_2, \alpha_3, \alpha_4, \alpha_5, 1_F\}$ be fuzzy topologies on *F*. Consider the fuzzy identity mapping $\psi : (F, \tau_1) \to (F, \tau_2)$. Then ψ is f.e-pc but not f.e-c mapping because for a Fe-O set α_5 on (F, τ_2), $\psi^{-1}(\alpha_5) = \alpha_5$ which is not a Fe-O set on (F, τ_1).

Theorem 3.4 Every ∗-f.e-pc is f.e-c but not conversely.

Proof. Let $\psi : F \to \Delta$ be a -f.e-pc map. We have to prove ψ is f.e-c. Let α be a Fe-O set in Δ. Since ψ is *-f.e-pc, $\psi^{-1}(\alpha)$ is Fe-PO set in *F*. Since every Fe-PO set is a Fe-O set, $\psi^{-1}(\alpha)$ is Fe-O set in *F*. Hence ψ is a f.e-c.

Example 3.5 Let β_1, β_2 and $\beta_3$be fuzzy sets on *F* = Δ = {a, b, c}. Then β_1 = 1.0/a + 0.0/b + 0.0/c, β_2 = 1.0/a + 0.6/b + 0.0/c and β_3 = 1.0/a + 0.6/b + 0.5/c are defined as follows: Consider $\mathfrak{F}_1 = \{0_F, \beta_1, \beta_2, \beta_3, 1_F\}$, let $\psi : F \to \Delta$ be an identity mapping. Then ψ is f.e-c but not ∗-f.e-pc mapping since for the Fe-O set β_3 on Δ, $\psi^{-1}(\beta_3) = \beta_3$ which is not a Fe-PO set on *F*.

Theorem 3.6 Every ∗-f.e-pc is f.e-pc but not conversely.

Proof. The proof follows from Theorems 3.2 and 3.4.

Example 3.7 In Example 3.5, "ψ is f.e-pc map but it is not *-f.e-pc map."

Theorem 3.8 Every f.e-p.i map is f.e-pc but not conversely.

Proof. Let $\psi : F \to \Delta$ be a f.e-p.i map. We have to prove that ψ is f.e-pc. Let α be any Fe-PO set in Δ. Since ψ is f.e-p.i, $\phi^{-1}(\alpha)$ is Fe-PO set in *F*. Since every Fe-PO set is a Fe-O set, $\psi^{-1}(\alpha)$ is Fe-O set in *F*. Hence ψ is a f.e-pc map.

Example 3.9 As described in Example 3.5, consider $\mathfrak{F}_3 = \{0_F, \beta_2, \beta_3, 1_F\}$ and $\mathfrak{F}_1 = \{0_\Delta, \beta_1, \beta_2, \beta_3, 1_\Delta\}$. Let $\psi : F \to \Delta$ be an identity mapping. Then ψ is f.e-pc but not f.e-p.i mapping since for the Fe-PO set β2 on Δ, ψ−1(β2) = β2 which is not a Fe-PO set on *F*.

Theorem 3.10 Every ∗-f.e-pc is f.e-p.i but not conversely.

Proof. Let $\psi : F \to \Delta$ be a f.e-pc map. We have to prove that ψ is f.e-p.i. Let α be a Fe-PO set in Δ. Since every Fe-PO set is a Fe-O set, α is a Fe-O set. Since ψ is ∗-f.e-pc, ψ−1(α) is Fe-PO set in *F*. Hence ψ is a f.e-p.i map.

Example 3.11 In Example 3.5," ψ is f.e-p.i map but it is not -f.e-pc map."

Remark 3.12 Fuzzy e-p.irresolute and f.e-c maps are independent of each other.

Example 3.13In Example 3.3, ψ is f.e-p.i map but it is not f.e-c map because for the Fe-O set β5 on Δ, ψ−1(β5) = β5 which is not a Fe-O set on *F*.

Let β1,β2, β3 be fuzzy sets on ▪ = {a, b, c} and let α1,α2,α3 be fuzzy sets on Δ = {x, y, z}. Then β1 = 0.2/a + 0.2/b + 0.2/c, $\beta_2 = \frac{0.3}{a} + \frac{0.3}{b} + \frac{0.3}{c}$, $\beta_3 = \frac{0.7}{a} + \frac{0.7}{b} + \frac{0.7}{c}$, $\alpha_1 = \frac{0.2}{x} + \frac{0.0}{y} + \frac{0.2}{z}$, $\alpha_2 = \frac{0.7}{x} + \frac{0.0}{y} + \frac{0.7}{z}$, $\alpha_3 = \frac{0.7}{x} + \frac{0.7}{y} + \frac{0.7}{z}$ are defined as follows:

Consider $\mathfrak{F}_1 = \{0_F, \beta_1, \beta_2, \beta_3, 1_F\}, \mathfrak{F}_2 = \{0_\Delta, \alpha_1, \alpha_1, \alpha_3, 1_\Delta\}$. Let $\psi : F \to \Delta$be a fuzzy mapping defined as f (a) = f (b) = f (c) = y. Then ψ is f.e-c but not fuzzy e-parairreolute because for the Fe-PO set α2 on Δ, ψ−1(α3) = 0 *F* which is not a Fe-PO set on *F*.

Theorem 3.14 Every f.mi.e-pc map is f.mi. e-continuous but not conversely.

Proof. Let $\psi : F \to \Delta$ be a f.mi.e-pc map. We have to prove that ψ is f.mi. e-continuous. Let τ1 be any FMIe-O set in Δ. Since ψ is f.mi.e-pc, ψ−1(τ1) is Fe-PO set in *F*. Since every Fe-PO set is a Fe-O set, ψ−1(τ1) is a Fe-O set in *F*. Hence ψ is a fuzzy minimal e-continuous.

Example 3.15From Example 3.2, ψ is f.mi. e-continous but it is not a f.mi. e-p.continuous, since for the FMIe-O β1 on Δ, ϕ −1(β1) = β1 which is not a Fe-PO set on *F*.

Remark 3.16Fuzzy minimal e-p.continuous and f.e-pc(resp. f.e-c) are independent of each other.

Example 3.17 Let β1,β2 be fuzzy sets on *F* = {a, b, c} and let α1,α2,α3 be fuzzy sets on Δ = {x, y, z}. Then β1 = 0.5/a + 0.0/b + 0.0/c, β2 = 0.5/a + 0.7/b + 0.0/c, β3 = 0.5/a + 0.7/b + 0.1/c, $\beta_2 = \frac{0.5}{a} + \frac{0.7}{b} + \frac{0.0}{c}$, $\beta_3 = \frac{0.5}{a} + \frac{0.7}{b} + \frac{0.1}{c}$, $\alpha_1 = \frac{0.5}{x} + \frac{0.7}{y} + \frac{0.0}{z}$, $\alpha_2 = \frac{0.5}{x} + \frac{0.7}{y} + \frac{0.9}{z}$, $\alpha_3 = \frac{0.5}{x} + \frac{0.8}{y} + \frac{0.0}{z}$ and $\alpha_4 = \frac{0.5}{x} + \frac{0.8}{y} + \frac{0.9}{z}$ are defined as follows: Consider $\mathfrak{F}_1 = \{0_F, , \beta_1, \beta_2, \beta_3, 1_F\}, \mathfrak{F}_2 = \{0_\Delta, \alpha_1, \alpha_1, \alpha_3, \alpha_4, 1_\Delta\}$. Let $\psi : F \to \Delta$ be an identity maping. Then ψ is f.mi.e-pc but not f.e-pc(resp. f.e-c) map because for the Fe-PO set α3 on Δ, ψ−1(α3) = α3 which is not a Fe-O set on *F*. In Example 3.2, ψ is f.e-pc but not f.mi.e-pc.

Theorem 3.18 Every f.ma.e-pc is f.ma.e-c but not conversely.

Proof. Let $\psi : F \to \Delta$ be a f.ma.e-pc map. To prove ψ is f.mi. e-continuous. Let δ be any FMAe-O set in Δ. Since ψ is f.ma.e-pc, ψ−1(δ) is Fe-PO set in *F*. Since every Fe-PO set is a Fe-O set,ψ−1(δ) is a Fe-O set in *F*. Hence ψ is a f.ma.e-c.

Example 3.19 In Example 3.2, "ψ is f.ma.e-c but it is not f.ma.e-pc map."

Remark 3.20 Fuzzy maximal e-p.continuous and f.e-pc(resp. f.e-c) are independent of each other.

Example 3.21 Let β1,β2 be fuzzy sets on *F* = a, b, c, d and let α1,α2,α3 be fuzzy sets on Δ = {x, y, z, w}. Then β1 = 0.0/a + 0.0/b + 0.0/c + 0.9d , β2 = 0.0/a + 0.0b + 0.7/c + 0.9/d , β3 = 0.0/a + 0.5/b + 0.7/c + 0.9/d , β4 = 0.2/a + 0.5/b + 0.7/c + 0.9/d , α1 = 0.0/a + 0.0/b + 0.3/c + 0.0/d , α 2 = 0.0/a + 0.0/b + 0.3/c + 0.9/d , α3 = 0.0/a + 0.5/b + 0.7/c + 0.9/d , are defined as follows: Consider F1 = {0 *F*, β1, β2, β3, β4, 1 *F* },F2 = {0Δ, α1, α2, α3, 1Δ}.

Let $\psi : F \rightarrow \Delta$ be an identity maping. Then ψ is f.ma.e-pc but not f.e-pc(resp. f.e-c) map because for the Fe-PO set α2 on Δ, ϕ −1(α2) = α2 which is not a Fe-O set on *F*. In Example 3.2, ψ is f.e-pc(resp. f.e-c) but not f.ma.e-pc.

Remark 3.22 Fuzzy minimal e-p.continuous and f.ma.e-pc are independent of each other.

Example 3.23 In Example 3.17, "ψ is f.mi.e-pc map but it is not f.ma.e-pc map. From Example III, ψ is f.ma.e-pc map but it is not f.mi.e-pc map."

Theorem 3.24 Let *F* and Δ be ftss. A map $\psi : F \rightarrow \Delta$ is a f.e-pc iff the inverse image of each Fe-PC set in Δ is a fuzzy e-closed set in *F*.

Proof. Obvious.

Theorem 3.25 Let A be a nonzero fuzzy subset of *F*. If $\psi : F \rightarrow \Delta$is f.e-pc then the restriction map ψA : A → Δ is a f.e-pc.

Proof. Let $\psi : F \rightarrow \Delta$ be a f.e-pc map and A ⊂ *F*. To prove ψA is a f.e-pc. Let α be a Fe-PO set in Δ. Since ψ is f.e-pc, ψ−1(α) is a Fe-O set in *F*. By the definition of relative topology fA−1(α) = A ^ ψ−1(α). Therefore A ^ ψ−1(α) is a Fe-O set in A. Hence ψA is a f.e-pc.

Remark 3.26 The composition of f.e-pc maps need not be f.e-pc.

Example 3.27 Let *F* = Δ = Φ = {a, b, c, d} and the fuzzy sets β1 = 0.0/a + 0.0/b + 0.2/c + 0.0/d , β2 = 0.0/a + 0.0/b + 0.2/c + 0.5/d, β3 = 0.0/a + 0.7/b + 0.2/c + 0.5/d and β4 = 0.3/a + 0.7/b + 0.2/c + 0.5/d are defined as follows: consider $\mathfrak{F}_1 = \{0_F, \beta_1, \beta_2, 1_F\}$, $\mathfrak{F}_2 = \{0_\Delta, \beta_1, \beta_2, \beta_3, 1_\Delta\}$ and F3 = {0Φ, β1, β3, β4, 1Φ} . Let $\psi : F \rightarrow \Delta$and ξ : Δ → Φ be identity mappings. Then ψ and ξ are f.e-pc maps $\xi \circ \psi : F \rightarrow \Phi$ is not f.e-pc, since for the Fe-PO set β3 in Φ, ψ−1(β3) = β3 which is not Fe-O set in *F*.

Theorem 3.28 If ψ : *F* → Δ is f.e-c and ξ : Δ → Φ is f.e-pc. Then ξ◦ : *F* → Φ is a f.e-pc.

Proof. Let τ1 be any Fe-PO set in Φ. As ξ is f.e-pc, ξ−1(τ1) is a Fe-O set in Δ. Again since ψ is f.e-c, ψ−1(ξ−1(τ1)) = (ξ◦ψ)−1(τ1) is a Fe-O set in *F*. Hence ξ ◦ ψ is a f.e-pc. □

Theorem 3.29 Let *F* and Δ be ftss. A map $\psi : F \rightarrow \Delta$ is -f.e-pc iff the inverse image of each fuzzy e-closed set in Δ is a Fe-PC set in *F*.

Proof. Obvious.

Remark 3.30 Let *F* and Δ be fts. If $\psi : F \rightarrow \Delta$is ∗-f.e-pc, then the restriction map ψA : A → Δ need not be ∗-f.e-pc.

Example 3.31 Let $F = \Delta = \Phi = \{a, b, c\}$ and the fuzzy sets β1 = 0.7/a + 0.0/b + 0.0/c , β2 = 0.7/a + 0.3/b + 0.0/c and β3 = 0.7/a + 0.3/b + 0.5/c are defined as follows: Consider F = {0 *F* , β , β , β , 1 *F* } and $\mathfrak{F}_1 = \{0_\Delta, \beta_2, 1_\Delta\}$. Let $\delta = \frac{0.0}{a} + \frac{0.3}{b} + \frac{0.9}{c}$ be a fuzzy set with Fδ = {0δ, β4, β5, β6, δ} where β4 = 0.0/a + 0.3/b + 0.0/c and β5 = 0.0/a + 0.3/b + 0.5/c. Let $\psi : F \to \Delta$ be an identity map. Then ψ is ∗-f.e-pc but fδ : Fδ → Δ is not a ∗-f.e-pc, since for the Fe-O set β2 in Δ, ψ−1(β2) = β2 which is not a Fe-PO set in Fδ.

Theorem 3.32 If $\psi : F \to \Delta$ and ξ : Δ → Φ is ∗-f.e-pc, then ξ ◦ ψ : *F* → Φ is a ∗-f.e-pc.

Proof. Let τ1 be any Fe-PO set in Φ. As every Fe-PO set is a Fe-O set, ξ−1(τ1) is a Fe-PO set in Δ. Again since ψ is fuzzy
∗-f.e-pc, ψ−1(ξ−1(τ1)) = (ξ ◦ ψ)−1(τ1) is a Fe-PO set in *F*. Hence ξ ◦ ψ is a ∗-f.e-pc. □

Theorem 3.33 If $\psi : F \to \Delta$ is f.e-pc and ξ : Δ → Φ is ∗-f.e-pc, then ξ ◦ ψ : *F* → Φ is a f.e-pc(resp. f.e-c).

Proof. Let τ1 be any Fe-PO(resp. Fe-O) set in Φ. As every Fe-PO set is a Fe-O set, ξ−1τ1 is a FePO set in Φ. Since ξ is a -f.e-pc, ξ−1(τ1) is a Fe-PO set in Δ. Again since ψ is f.e-pc, ψ−1(ξ−1(τ1)) = (ξ ψ)−1(τ1) is a Fe-O set in *F*. Hence ξ ψ is f.e-pc(resp. f.e-c) map.

Theorem 3.34 A map ψ : *F* → Δ is f.e-p.i iff the inverse image of each fuzzy are e-paraclosed set in Δ is a Fe-PC set in *F*.

Proof. Straightforward.

Remark 3.35 If ψ : *F* → Δ is f.e-p.i. Then the restriction map ψA : A → Δ need not be f.e-p.i.

Example 3.36 In Example 3.2, let δ = 0.0/a + 0.0/b + 0.6/c be a fuzzy set with Fδ = {0δ, β4, δ} where β4 = 0.0/a + 0.0/b + 0.5/c. Let $\psi : F \to \Delta$ be an identity map. Then ψ is f.e-p.i but fδ : F▸Δ is not a f.e-p.i, since for the Fe-PO set β2 in Δ, ψ−1(β2) = β2 which is not a Fe-PO set in Fδ.

Theorem 3.37 If ψ : *F* → Δ is f.e-pc and ξ : Δ → Φ is f.e-p.i, then ξ ◦ ψ : *F* → Φ is a f.e-pc.

Proof. Let τ1 be a Fe-PO set in Φ. As ξ is a f.e-p.i ξ−1(τ1) is a Fe-PO set in Δ. Again since ψ is f.e-pc, ψ−1(ξ−1(τ1)) = (ξ◦ψ)−1(τ1) is a Fe-O set in *F*. Hence ξ ◦ ψ is f.e-pc. □

Theorem 3.38 If ψ : *F* → Δ and ξ : Δ → Φ are f.e-p.i, then ξ ◦ ψ : *F* → Φ is a f.e-p.i.

Proof. Let τ1 be a Fe-PO set in Φ. Since ξ is a f.e-p.i ξ−1(τ1) is a Fe-PO set in Δ. Again since ψ is f.e-p.i, ψ−1(ξ−1(τ1)) =
(ξ ◦ ψ)−1(τ1) is a Fe-PO set in *F*. Hence ξ ◦ ψ is f.e-pc.

Theorem 3.39 If ψ : *F* → Δ is ∗-f.e-pc and ξ : Δ → Φ is f.e-p.i. Then ξ ◦ ψ : *F* → Φ is a f.e-p.i.

Proof. Let τ1 be a Fe-PO set in Φ. As ξ is a f.e-p.i, ξ−1(τ1) is a Fe-PO set in Δ. Since every Fe-PO set is a Fe-O set, we have
ξ −1(τ1) is a Fe-O set in Δ. Again since ψ is ∗-f.e-pc, ψ−1(ξ−1(τ1)) = (ξ ◦ψ)−1(τ1) is a Fe-PO set in *F*. Hence ξ ◦ψ is f.e-p.i. □

Theorem 3.40 If ψ : *F* → Δ is f.e-p.i and ξ : Δ → Φ is ∗-f.e-pc, then ξ ◦ ψ : *F* → Φ is a f.e-p.i.

Proof. Let τ1 be a Fe-PO set in Φ. As every Fe-PO set is a Fe-O set, τ1 is a Fe-O set in Φ Since ξ is a f.e-pc, ξ−1(τ1) is a Fe-PO set in Δ. Again Since ψ is f.e-p.i, ψ−1(ξ−1(τ1)) = (ξ ∘ ψ)−1(τ1) is a Fe-PO set in *F*. Hence ξ ∘ ψ is f.e-p.i mapping. □

Theorem 3.41 A map ψ : *F* → Δ is f.mi. f.e-pc iff the inverse image of each FMAe-C set in Δ is a Fe-PC set in *F*.

Proof. Obvious.

Remark 3.42 The composition of f.mi.e-pc maps need not be a f.mi.e-pc.

Example 3.43 Let *F* = Δ = Φ = {a, b, c, d} and the fuzzy sets τ1 = 0.0/a + 0.0/b + 0.2/c + 0.4/d, τ2 = 0.0/a + 0.7/b + 0.2/c + 0.4/d, τ3 = 0.2/a + 0.7/b + 0.2/c + 0.4/d and τ4 = 0.3/a + 0.7/b + 0.2/c + 0.4/d are defined as follows: consider $\mathfrak{F}_1 = \{0_F, \tau_1, \tau_2, \tau_3, 1_F\}$, $\mathfrak{F}_2 = \{0_\Delta, \tau_2, \tau_3, \tau_4, 1_\Delta\}$ and $\mathfrak{F}_3 = \{0_\Phi, \tau_3, \tau_4, 1_\Phi\}$. Let $\psi : F \to \Delta$and ξ : Δ ⟶ Φ be identity mappings. Then ψ and ξ are f.mi.e-pc maps ξ ψ : *F* ⟶ Φ is not f.mi.e-pc, since for the FMIe-O set τ3 in Φ, ψ−1(τ3) = τ3 which is not Fe-PO set in *F*.

Theorem 3.44 If ψ : *F* → Δ is f.e-p.i and ξ : Δ → Φ is f.mi.e-pc, then ξ ∘ ψ : *F* → Φ is a f.mi.e-pc.

Proof. Let η be a FMIe-O set in Φ. As ξ is f.mi.e-pc, ξ−1(η) is a Fe-PO set in Δ. Again since ψ is f.e-p.i, ψ−1(ξ−1(η)) = (ξ∘ψ)−1(η) is a Fe-PO set in *F*. Hence ξ ∘ ψ is f.mi.e-pc map.
□

Theorem 3.45 If ψ : *F* → Δ is f.e-pc and ξ : Δ → Φ is f.mi.e-pc, then ξ ∘ ψ : *F* → Φ is a f.mi.e-pc.

Proof. Let η be a FMIe-O set in Φ. Since ξ is f.mi.e-pc, ξ−1(η) is a Fe-PO set in Δ. Again since ψ is f.e-pc, ψ−1(ξ−1(η)) =
(ξ ∘ ψ)−1(η) is a Fe-O set in *F*. Hence ξ ∘ ψ is f.mi.e-pc mapping.

Theorem 3.46 If ψ : *F* → Δ is f.e-p.i and ξ : Δ → Φ is ∗-f.e-pc, then ξ ∘ ψ : *F* → Φ is a f.mi.e-pc.

Proof. Let η be a FMIe-O set in Φ. As every f.mi. e-open set is a Fe-O set, η is an e-open set in Φ. Since ψ is ∗-f.e-pc, ξ−1(η) is a Fe-PO set in Δ. Again since ψ is f.e-p.i ψ−1(ξ−1(η)) = (ξ ∘ ψ)−1(η) is a Fe-PO set in *F*. Hence ξ ∘ ψ is f.mi.e-pc.

Theorem 3.47 Let *F* and Δ be fts. A map $\psi : F \to \Delta$ is f.ma.e-pc iff the inverse image of each FMIe-C set in Δ is a Fe-PC set in *F*.

Proof. Sraightforward.

Remark 3.48 The composition of f.ma.e-pc maps need not be a f.ma.e-pc.

Example 3.49 Let *F* = Δ = Φ = {a, b, c, d} and the fuzzy sets τ1 = 0.0/a + 0.1/b + 0.0/c + 0.0/d, τ2 = 0.0/a + 0.1/b + 0.7/c + 0.0/d, τ3 = 0.0/a + 0.1/b + 0.7/c + 0.2/d and τ4 = 0.3/a + 0.1/b + 0.7/c + 0.2/d are defined as follows: consider $\mathfrak{F}_1 = \{0_F, \tau_1, \tau_2, \tau_3, 1_F\}$, $\mathfrak{F}_2 = \{0_\Delta, \tau_2, \tau_3, \tau_4, 1_\Delta\}$ and $\mathfrak{F}_3 = \{0_\Phi, \tau_3, \tau_4, 1_\Phi\}$. Let $\psi : F \to \Delta$ and g : Δ ⟶ Φ be identity mappings. Then ψ and ξ are f.ma.e-pc maps ξ ∘ ψ : *F* ⟶ Φ is not f.ma.e-pc, since for the FMAe-O set τ2 in Φ, ψ−1(τ2) = τ2 which is not Fe-PO set in *F*.

Theorem 3.50 If ψ : *F* → Δ is f.e-p.i and ξ : Δ → Φ is f.ma.e-pc, hen ξ ∘ ψ : *F* → Φ is a f.ma.e-pc.

Proof. Let γ be a FMAe-O set in Φ. Since ξ is f.ma.e-pc, ξ−1(γ) is a Fe-PO set in Δ. Again since ψ is f.e-p.i, ψ−1(ξ−1(γ)) = (ξ ∘ ψ)−1(γ) is a Fe-PO set in *F*. Hence ξ ∘ ψ is f.ma.e-pc.

Theorem 3.51If $\psi : F \to \Delta$ is f.e-pc and $\xi : \Delta \to \Phi$ is f.ma.e-pc, then $\xi \circ \psi : F \to \Phi$ is a f.ma.e-c.
Proof. Let γ be a FMAe-O set in Φ. Since ξ is f.ma.e-pc, $\xi-1(\gamma)$ is a Fe-PO set in Δ. Again since ψ is f.e-pc, $\psi-1(\xi-1(\gamma)) = (\xi \circ \psi)-1(\gamma)$ is a Fe-O set in F. Hence $\xi \circ \psi$ is f.ma.e-c.
Theorem 3.52 If $\psi : F \to \Delta$ is f.e-p.i and $\xi : \Delta \to \Phi$ is $*$-f.e-pc, then $\xi \circ \psi : F \to \Phi$ is a f.ma.e-pc.
Proof. Let γ be a FMAe-O set in Φ. Since every FMAe-O set is a Fe-O set, γ is a Fe-O set in Φ. Since ξ is $*$-f.e-pc, $\xi-1(\gamma)$ is a Fe-PO set in Δ. Again since ψ is f.e-p.i, $\psi-1(\xi-1(\gamma)) = (\xi \circ \psi)-1(\gamma)$ is a Fe-PO set in F. Hence $\xi \circ \psi$ is f.ma.e-pc.

IV. CONCLUSION

The notion of fuzzy e-open sets is remarkable one. By means of this, fuzzy e-paraopen set introduced and studied. Also various fuzzy mappings and comparisons with appropriate examples investigated.

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