

SOLVING PROBLEMS OF THREE RESERVOIRS INTERCONNECTED BY PIPES USING A CODE IN PYTHON LANGUAGE

Ahlem KHERROUBI¹, Ali BEDJAOU², Yamina BENKRIMA³

¹ Ph.D Student, Univ. of Biskra, Depart. of civil Engineering and Hydraulics, Sciences and technology Fac., Algeria, a.kherroubi@univ-biskra.dz

² Associate Professor, Univ. of Biskra, Depart. of civil Engineering and Hydraulics, Sciences and technology Fac., Algeria, a.bedjaoui@univ-biskra.dz

³ Associate Professor, Department of Exact Sciences, ENS Ouargla, 30000, Algeria
b-amina1@hotmail.fr

Abstract

This work consists of a contribution to the resolution of the problems related to the problems of three reservoirs which includes three different cases, namely: the calculation of flow rates, pipe sizes, and pressure head. Researchers proposed different solutions that start by imposing initial values and work through it to a balanced state using methods like predefined graphs or equations like Darcy-Weisbach and Colebrook-White. Despite their accuracy, users suggest fixed values of friction factor to solve the problem. The proposed python-based application solves all three cases utilizing three different methods Hager, Hazen-Williams, and RMM. The flow rate case is solved by a linear least squares regression to determine the exact values of pressure head and flow rates that verify the condition $\sum Q_i = 0$; the pipe commercial diameters are selected from a set of predefined tables for the case of pipe sizing calculation and finally, the pressure head in the last case is considered as the average of heads computed for each pipe. The analysis of the study case presented in the paper illustrated that the differences between the original and the model's results were significant (up to 30 l/s in terms of flow rates).

Keywords: Dimeter, Flow rate, Hager, Hazen-Williams, RMM, Python, Three reservoirs problems.

1. Introduction

In hydraulics, the problem of the three reservoirs interconnected by pipes is part of the complex analysis of the water system, and we are faced with three problems to solve, namely, i) computing the flows (and therefore the direction of flow) in the pipes after having fixed the diameters and proposing an initial value for the pressure head at the junction. The sum of the resulting flows must be zero or approach it as closely as possible; ii) computing the diameters of the pipes connecting the three reservoirs, knowing their dimensions, the piezometric head at the junction, and the flow rates supplying or issuing from each reservoir; and iii) computing the piezometric level at the junction point, knowing the diameters and the flow rates.

Despite the fact that there are many methods to solve each problem among those mentioned above, the solutions of the three cases mainly depend on the use of Bernoulli, Hazen-Williams, or Darcy-Weisbach equations.

The most encountered problem in practice is the first case where the flows are the main variables to be sought. However, it depends on identifying the unknown pressure head at the junction to subsequently satisfy the law of conservation of mass, that is, $(\sum Q_i = 0)$. Generally, this case can be solved by two iterative approaches: Proposing a flow rate (Q) in one of the pipes or proposing a

piezometric height or a pressure head at the junction (PH_j) and then determining the desired flow rates, either by:

i) Assume the flow rate in one of the pipes Q_1 and define the pressure head at the junction PH_j accordingly, then proceed with calculating the other flow rates (Q_2, Q_3, \dots) and verifying the continuity equation. The first flow rate can be corrected directly as part of the iterative process and continue the loop of computing PH_j, \dots [1], or it could be corrected by a correction coefficient computed based on the initial pipe's resistance value; in this case, it is preferable that the assumed flow rate is one a lower pipe's. [2]. Alternatively, it can be solved by assuming a velocity value in pipe $N^\circ 1$ after simplifying Bernoulli's equation to write the velocity equations in pipes $N^\circ 2$ and 3 as a function of velocity in pipe $N^\circ 1$, which makes it the targeted variable of the iteration process [3]. ii) Proposing an initial value of the pressure head at the junction PH_j and proceeding with one of the known relationships mentioned above to calculate flows, then PH_j must be corrected (by increasing or decreasing it according to the sign of $\sum Q_i$) until equality is ensured: $\sum Q_i = 0$. It should be noted that to use Bernoulli, Darcy-Weisbach, and Colebrook-White, many researchers make assumptions such as choosing a known friction factor (usually the same factors for all three pipes) or use Moody's diagram; other researchers choose the Hazen-Williams relationship and suggest a constant C coefficient to avoid using the Colebrook-White and Darcy-Weisbach relationships. [3], [4], [5]. iii) Rewriting the energy equation in terms of PH_j and using the false position method to iterate the pressure heads until convergence is reached [3]. iv) Using a diagram containing three columns that represent values of the hydraulic gradients j , diameters (D) and flows (Q), starting with a proposed PH_j and defining the correspondent hydraulic gradient then projecting a straight line on the columns j, D , and Q with repeating the process until $\sum Q_i = 0$ is verified [7]. v) Performing an interpolation between conducted values of PH_j until converging toward an admissible value (Haidra and al., 2018). vi) Plot the graph $\sum Q_i = f(PH_j)$ using 3 arbitrary values of PH_j and find the point that satisfies $\sum Q_i = 0$ [2].

The other two types of problems related to the three tanks are the computation of the diameters of each pipe and the hydraulic gradient; the solutions are less complicated, but they call for the use of the friction factor f , which is impossible to determine because the diameter is unknown likewise for the Reynolds number, which also depends on the diameter; some authors propose values of f to facilitate the calculation and evaluation of the diameter from Darcy's relation. This method is doubtful since the friction factors are unique values that depend on several parameters (the relative roughness ε/D , the Reynolds number R and the flow rate Q); therefore, an assumed value of f could generate errors after calculating the diameter.

For the calculation of the hydraulic gradient J , whose purpose is to evaluate the piezometric head at the junction generally, the computation is simple, and the relation of Darcy (despite the explicit aspect of the Colebrook-White relation required in the relation), or the Hazen-Williams relationship (if there is a perfect match or equivalence with the absolute roughness of the pipe and coefficient C), is sufficient.

Each of the solutions cited above has a deficiency; for the first category, during the calculation of Q , the drawback associated with this type of method lies in the extent of the iterative process if it were carried out manually. For the rest, precision is the problem in regard to f .

The main objective of this work is to solve the problems related to the three reservoirs with perfect precision by avoiding any illogical simplification mentioned beforehand and by proposing a computerized program in Python based on the relations of the reference rough model method, on

the work of Hager and at the end on the relation of Hazen-Williams, it is up to the user to choose between these methods.

1. Methodology

1.1. Analytical methods

Throughout history, pipe-flow problems have been approached through different equations; some were simple and direct, and some were mathematical-based formulas. Three major equations were used in the built-in model to ensure the diversity of solutions.

a) Hager (1987), (2010): Willi. H. Hager developed two sets of equations to solve two pipe-flow problems, identification of flow rate and diameter [7]. Relying on *Colebrook-White* (C-W) and *Darcy-Weisbach* (D-W), the equations were meant to give direct solutions avoiding the iterative process. It is worth noting that the formulas are highly accurate and valid for all types of turbulent flow regimes, from smooth to fully rough [8].

b) Hazen-Williams (1906): The Hazen-Williams formula is an empirical equation used to estimate the head loss (or pressure loss) in a pipe. It is used in hydraulic engineering to model the behaviour of fluid flow in pipelines, particularly in water distribution systems. The formula takes into account the diameter of the pipe, the flow rate of the fluid, and the roughness coefficient of the pipe material. The equation is provided with a constant coefficient equivalent of pipe roughness for each pipe material [9].

Although many researchers object using it outside its flow rate and pipe size ranges, it remains widely used due to its simplicity, and it is still present in all water distribution system modelling software.

c) The Rough Model Method (RMM) (2007): The rough reference model method (RRM) is a theoretical approach developed at the Subterranean and Surface Hydraulics Research Laboratory (LARHYSS) of the University of Biskra, Algeria [10] which aims to solve the problems of the flows in the pipes, these problems are: either the determination of a flow carried by a pipe of diameter D or the evaluation of a diameter D (linear dimension) or the evaluation of Hydraulic gradient J . The parameters considered in this approach are (volume flow Q , linear dimension D , roughness of the pipe's inner wall ϵ , unit gradient J , kinematic viscosity of the liquid carried ν and a parameter of form $\alpha = h/D = 1$ in the case of pressurized flow [11].

Each unknown parameter will be determined from the other parameters, namely: $D = f(Q, J, \epsilon, \alpha)$; $Q = f(D, J, \epsilon, \alpha)$; and $J = f(Q, D, \epsilon, \alpha)$. The formulas derived from this method are valid for: $2300 \leq R \leq 10^8$ and $0 \leq \epsilon/D \leq 0,5$, covering the entire range of commercial pipes.

The (RMM) helps hydraulic specialists to solve the three categories of problems encountered in practice, also in the calculation of the friction factor f since the formula recommended by (*Achour et al, 2002*) gives a more precise result than that obtained by the *Colebrook-White* formula [12], [13].

Table 1 summarizes all the equations listed in the model for each category.

Table 1. Explicit equations to solve pipe-flow problems

<i>Variable</i>	<i>Methods</i>	<i>Equation</i>	<i>Equation</i>
-----------------	----------------	-----------------	-----------------

			<i>number</i>
Flow rate	Hazen-Williams	$Q_i = \left(\frac{J_i C_i^{1.852} D_i^{4.87}}{10.67} \right)^{1/1.852}$	(3)
	W.H Hager	$N_i = \sqrt{g J_i D_i^3 / \nu}$	(4)
		$q_i = -\frac{\pi}{\sqrt{2}} \log \left(\frac{\varepsilon / D_i}{3.7} + \frac{2.51}{N_i \sqrt{2}} \right)$	(5)
		$Q_i = q_i \sqrt{g J_i D_i^5}$	(6)
RMM	$\bar{R}_i = 4\sqrt{2} \frac{\sqrt{g J_i D_i^3}}{\nu}$	(7)	
	$Q_i = -4\sqrt{2} g A_i \sqrt{R_{Hi} J_i} \log \left(\frac{\varepsilon / D_i}{3.7} + \frac{10.04}{\bar{R}_i} \right)$ R _{Hi} : hydraulic radius A _i : area of the cross-section of pipe	(8)	
Diameter	Hazen-Williams	$D_i = \left(\frac{10.67 Q_i^{1.852}}{J_i C_i^{1.852}} \right)^{1/4.87}$	(9)
	W.H Hager	$D_{0i} = \left[\frac{Q_i^2}{g J_i} \right]^{1/5}$	(10)
		$v_i^* = \nu D_{0i} / Q_i$	(11)
		$\varepsilon_i^* = \varepsilon / D_{0i}$	(12)
		$D_i = D_i^* D_{0i}$	(13)
		➤ For smooth turbulent flow $\varepsilon_i^* \rightarrow 0$: $D_i^* = \frac{2}{5} \log \left[-\frac{54.64}{\log v_i^*} \right]$	(14)
Applicable for the ranges: $10^{-9} < v^* < 10^{-3}$ and $10^4 < R < 4.10^9$ ➤ For rough turbulent flow $v_i^* \rightarrow 0$: $D_i^* = \frac{\varepsilon_i^{*0.03}}{1.853} ; \quad \text{for } 10^{-8} < \varepsilon / D < 7.10^{-4}$	(15)		
$D_i^* = \frac{\varepsilon_i^{*1/16}}{1.422} ; \quad \text{for } 7.10^{-4} < \varepsilon / D < 7.10^{-2}$	(16)		
RMM	$\bar{D} = (2\pi^2)^{-1/5} \left[\frac{Q^2}{gJ} \right]^{1/5}$	(17)	
	$\bar{R} = \frac{4Q}{\pi v \bar{D}}$	(18)	
		(19)	

		$\Psi = 1.35 \left[-\log \left(\frac{\varepsilon/\bar{D}}{4.75} + \frac{8.5}{\bar{R}} \right) \right]^{-2/5}$ $D = \Psi \cdot \bar{D}$	(20)
Hydraulic gradient	Hazen-Williams	$J_i = \frac{10.67 Q_i^{1.852}}{C_i^{1.852} D_i^{4.87}}$	(21)
	RMM	$R = \frac{4Q}{\pi Dv}$	(22)
		$\bar{R} = 2R \left[-\log \left(\frac{\varepsilon/D}{3.7} + \frac{5.5}{R^{0.9}} \right) \right]^{-1}$	(23)
		$f = \left[-2\log \left(\frac{\varepsilon/D}{3.7} + \frac{10.04}{\bar{R}} \right) \right]^{-2}$	(24)
		$J_i = \frac{8 f_i Q_i^2}{g \pi^2 D_i^5}$	(25)

For all the cases, the index *i* represents the element's ID (pipe or reservoir), which varies from 1 to *n*. Additionally, the hydraulic gradients used in the flow rate and pipe sizing categories are obtained through equations (1) and (2) for all three methods.

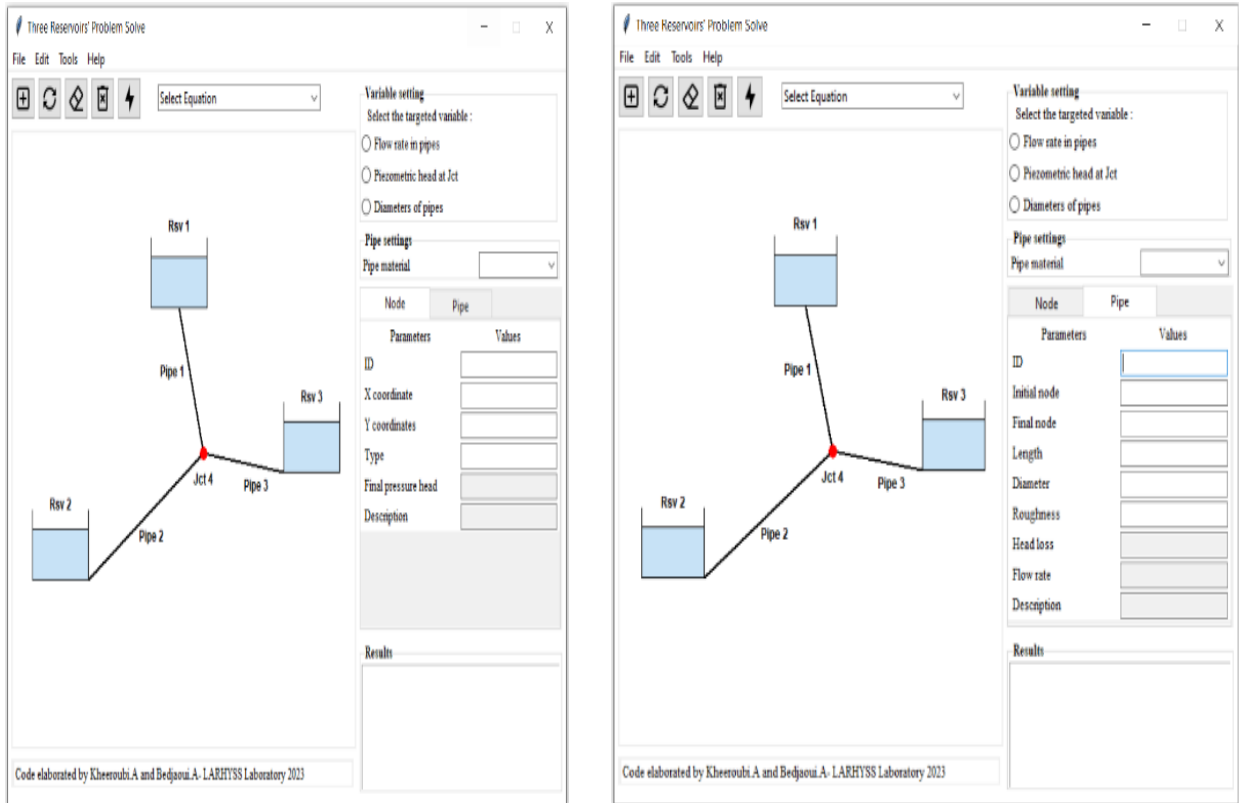
$$\Delta H_i = PH_i - PH_j \quad (1)$$

$$j_i = \frac{\Delta H_i}{L_i} \quad (2)$$

1.2. Simulation model

The presented application was developed using Python as a programming language, while the database was created in MySQL workbench 8.0, presented by the MySQL team at Oracle. This is a Public License version 2.

The home interface is divided into two main parts, a Canvas that displays the schemas of three reservoirs connected with a junction via three pipes. The second part contains the widgets for inserting data and displaying results. Figure 1 represents an overall look at the main interface.



a) Nods

b) Pipes

Figure (1):The main interface of the Three Reservoirs Problem solver “TRP solver, a)Nods and b)Pipes.

The principal features in the application interface can be described as follows:

1- Buttons:

- ⚡: Run button to launch the simulation.
- ⊕: Add button to insert elements into the database.
- ↻: Refresh button to update any changes on elements.
- ✖: Delete button to erase element from the database.
- ☒: Clear button to erase all data from the database.

2- Drop-down list:

This combo-box permits the user to select one of three methods listed in the model: Hager, Hazen-Williams, and RMM.

3- Option buttons:

This part is to select the targeted variables to be determined.

a. **Flow rate in pipes:** checking this radio button launches the part of the solver that computes the flow rates in the pipes, following the steps in Figure 2.a.

b. **Piezometric head at Jct:** this radio button launches the part of the solver that computes the piezometric head (the pressure head) at the junction, following the steps in Figure 2.b.

c. **Diameters of pipes:** this radio button launches the part of the solver that computes the diameters and selects the commercial correspondent diameters, then recalculate the pressure head at the junction that corresponds to the selected pipe diameters following the steps in Figure 2.c.

4- Pipe settings:

This option enables the user to select the pipe material to be used especially for the third case of calculating and selecting pipe diameters. There are five types included in the database (PVC, HDPE, Steel, Cast iron, Asbestos cement).

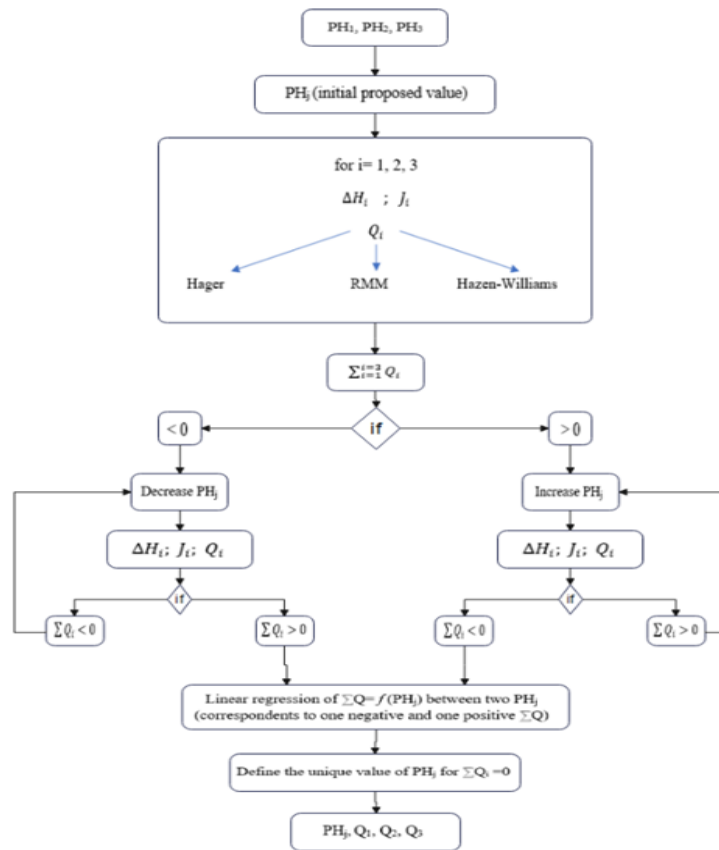
5- Tabs:

- Node: holds the entry fields characterizing the reservoirs and the junction
- Pipe: holds the entry fields characterizing the pipes

6- **Text box:** This part displays the results and suggestions based on the selected category.

The procedure of using the model can be described in the following five key steps:

- Selecting the computation method.
- Selecting the targeted variable to be identified.
- Selecting the pipe material
- Inserting the necessary data of nodes and links depending on the selected case.
- Launching the simulation process, where the solver will follow one of three processes illustrated in Figure 2.



(a)

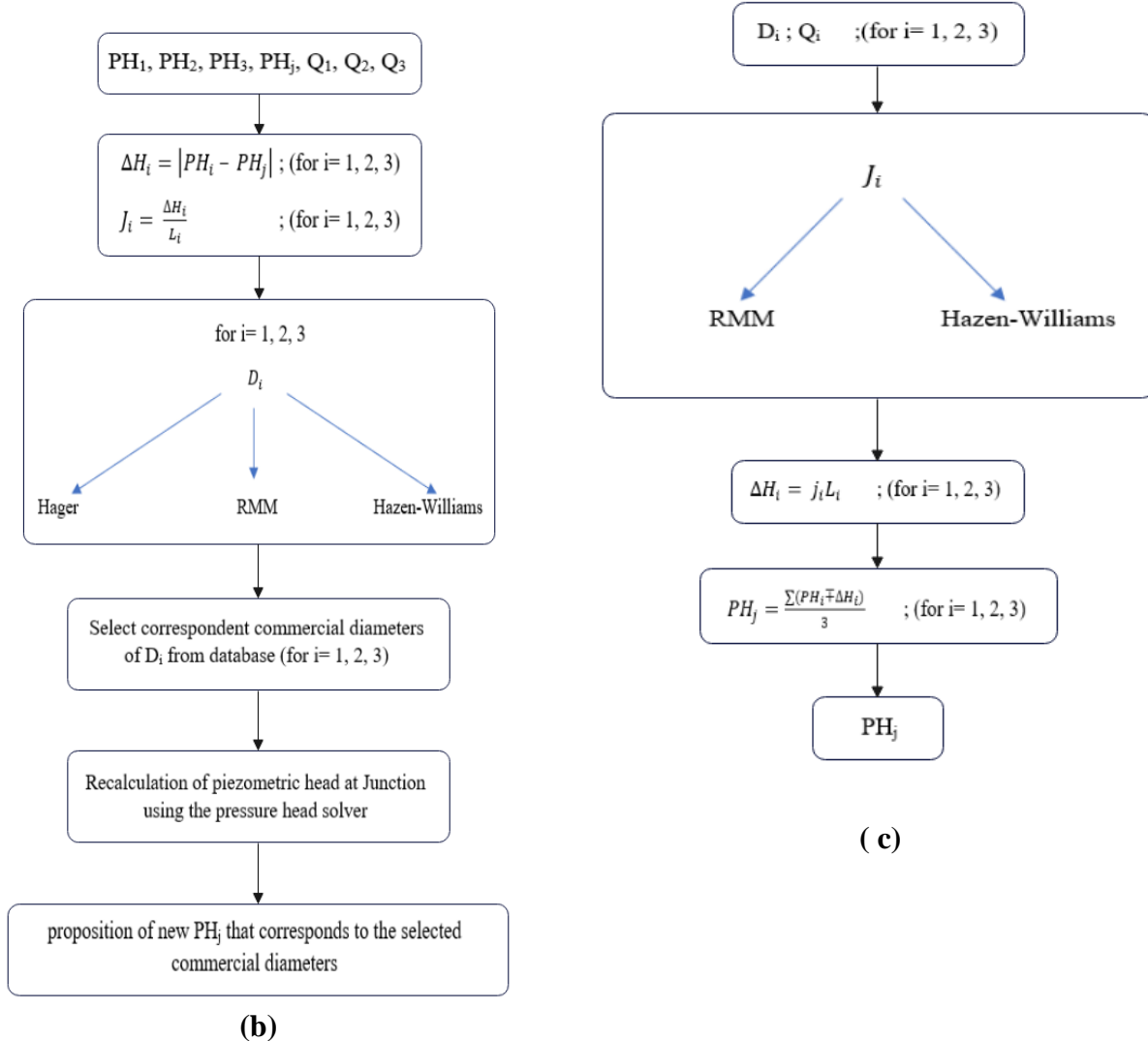


Figure (2):Flow charts of the three reservoirs problem solver;
a)-Flow rate calculation process,
b)-Pipe size calculation process,
c)- Pressure head calculation process)

The model solver is divided into three distinguished parts, each one to resolve a specific case of the above-mentioned problems through different mechanisms like loops or direct approaches. Firstly, the flow rate solver (Figure 2.a) gets initiated by proposing a value for the pressure head at the junction PH_j , identifying the flow direction and calculating the flow rate and $\sum Q_i$ by one of the three proposed methods (Hager, RMM and Hazen-Williams). The sign of the flow summation $\sum Q_i$ determine the loop to follow:

1) If the value is positive, PH_j will be increased by 0,1, and the loop of calculating the parameters ΔH_i , J_i , Q_i , and $\sum Q_i$ will continue until obtaining a negative value of $\sum Q_i$; the solver then sets the last values of PH_j that resulted one positive and one negative $\sum Q_i$ and their correspondent outcomes into two arrays of X and Y respectively. Scipy. The stats module permits

to identify the slope and the intercept of the equation $\sum Q_i = f(PH_j)$ using `scipy.stats.linregress` which computes the linear least squares regression for the two arrays X and Y, and the process ends by calculating PH_j which verifies the equation $\sum Q_i = 0$ and consequently identifies the final values of flow rates.

2) If the value is negative, PH_j will be decreased by 0,1 and proceed with the loop mentioned earlier until obtaining a positive value of $\sum Q_i$, after that the same steps will be carried out to compute the final values of PH_j and flow rates.

Next in order, the diameter solver (Figure 2.b) starts off by computing the diameters through one of the methods and the solver selects the closest commercial diameters from a set of diameter tables listed in the database according to the pipe material chosen by the user. Afterwards, the solver recalculates and proposes the new pressure head at the junction via the selected commercial diameters.

Finally, the pressure head solver computes the head loss between the three reservoirs through one of two methods (RMM and Hazen-Williams) and defines the final pressure head at the junction by the average of pressure heads calculated via each pipe.

2.3 Numerical application

An example selected from the literature ‘Hydraulique generale’ [A.Lencastre 1996], page 161, section 4.32 ‘cas de plusieurs réservoirs reliés entre eux’ (case of multiple interconnected reservoirs) is considered in the comparative analysis. Figure 3 represents the schema and Tables 2 and 3 summarize the data.

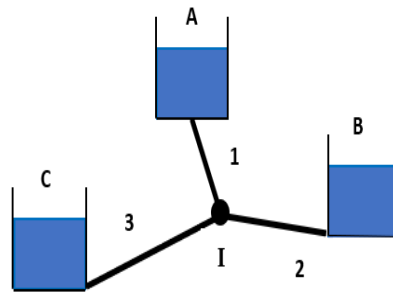


Figure 3. Reservoirs network schema

Table 2. Reservoir’s piezometric heads.

Parameter	Reservoir A	Reservoir B	Reservoir C
Pressure head (m)	60	20	08

Table 3. Pipes characteristics.

Parameter	Pipe 1	Pipe 2	Pipe 3
Length (m)	600	500	1800
Diameter (mm)	300	200	400
Roughness (mm)	0.025	0.025	0.025
Hazen-Williams’s coefficient	140	140	140

Note that Hazen Williams' coefficients were not presented in the original example; the values were selected from [14] and [15] for the pipe material originally presented in the example (Asbestos Cement).

According to the original example, the first proposed pressure head at the junction was 30 m, and using the chart given by Lencastre (1996) the process took three iterations to consider 27 m as the final pressure head at the junction with the following results: $Q_1= 0.395 \text{ m}^3/\text{s}$, $Q_2= 0.061 \text{ m}^3/\text{s}$, $Q_3= 0.330 \text{ m}^3/\text{s}$ and $Q_1-Q_2-Q_3=0,004 \text{ m}^3/\text{s}$.

Without any explanation, the proposed values become: $Q_1= 0.393 \text{ m}^3/\text{s}$, $Q_2= 0.061 \text{ m}^3/\text{s}$, and $Q_3= 0.332 \text{ m}^3/\text{s}$

2. Results and Discussion

To validate the performance of the model and highlight the variability between the different methods integrated in the solver, the results were classified according to the considered cases.

Case 1: Flow rate computation

For all three methods, the initial proposed value of the pressure head at the junction was the one presented in the original example. The simulation model then variates the values by increasing or decreasing by a step of 0.1 to ensure accuracy. Figure 4 exemplifies the function $\sum Q_i = f(\text{PH}_j)$ of one of the methods.

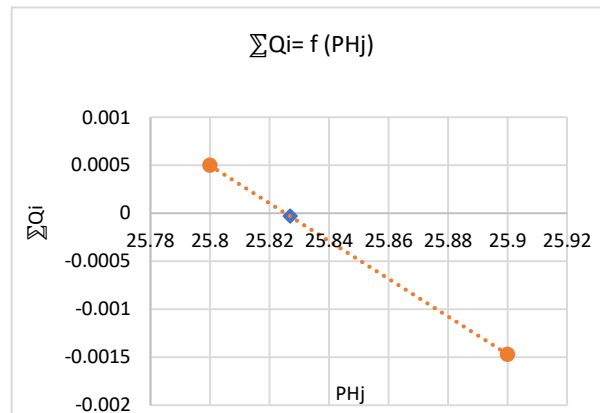


Figure. 4. Representation of summation of flow rates as a function of junction pressure head

From the chart, it can be seen that a narrow range of the two considered PH_j (one for negative and one for positive $\sum Q_i$) would enhance the calculation of the slope and intercept, which affects the precision by reducing the margin of error. The details are more explicit in Table 4.

Table. 4. The imposed pressures at the junction with the resulting flows from the simulation model
For proposed $PH_i=30$ m

Methods	Calculated PH_j (m)	Q_1 (m^3/s)	Q_2 (m^3/s)	Q_3 (m^3/s)	$\square Q_i$ (m^3/s) $\times 10^{-5}$
RMM (Eq.07)	25.83	0.3626	0.0542	0.3085	-3
Hager (Eq.06)	25.83	0.3628	0.0542	0.3086	-2
Hazen-Williams (Eq.03)	26.3	0.3474	0.0534	0.2941	-5

As can be seen, the number of iterations is high (39 and 43) because the solver uses a step of 0.1, but for Hazen-Williams (39), the number is lower than that of RMM and Hager (43 iterations). It is also noted that the flow rates, the sum of the flow rates and the calculated pressure heads resulting from using Hager's relations and the RMM are the same. However, the Hazen-Williams' relationship presented a higher calculated pressure head (26.296 m) at the junction and smaller discharge values than those obtained by the Hager and RMM relationships. Although $\sum Q$ for H-W is -5.10^{-5} , but when using the pressure head obtained according to the relation of H-W in either of the other two methods, the sum of the flow rates at the junction node becomes $-0,009 m^3/s$, which is considered a high-value. This difference in the results is mainly due to the choice of coefficient C. Figure 5 displays the results obtained for this case study using Hager's method, where the results mentioned in table 4 above are grouped together.

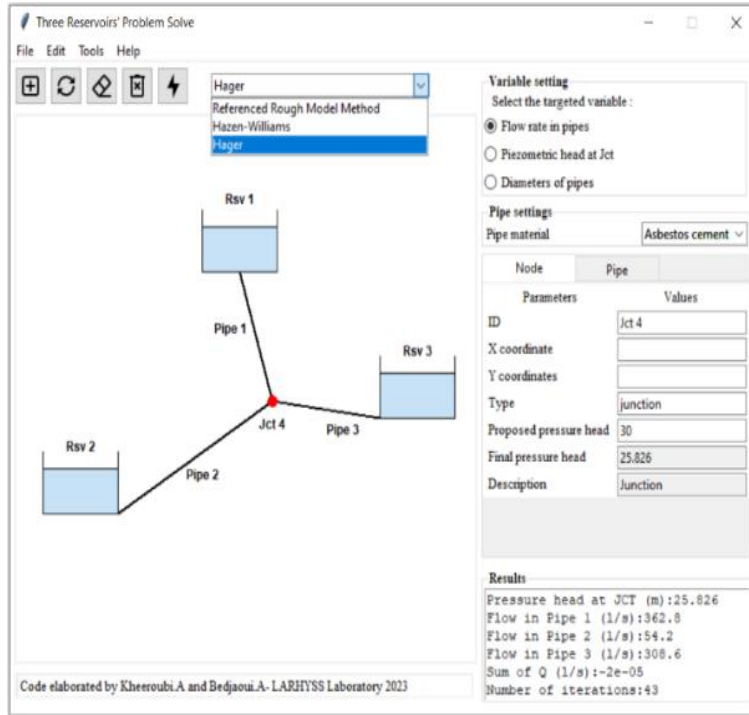


Figure (5). Flow rate calculation results

Case 2: Pipe size calculation

For this part, the flow rates were considered as known (from the previous case respectively) and the diameters were computed for the calculated head losses in Table 4, then the model selects the commercial diameters from the database according to the selected pipe material. Table 5 represents the calculated and the corresponding commercial diameters and as shown in Figure 6.

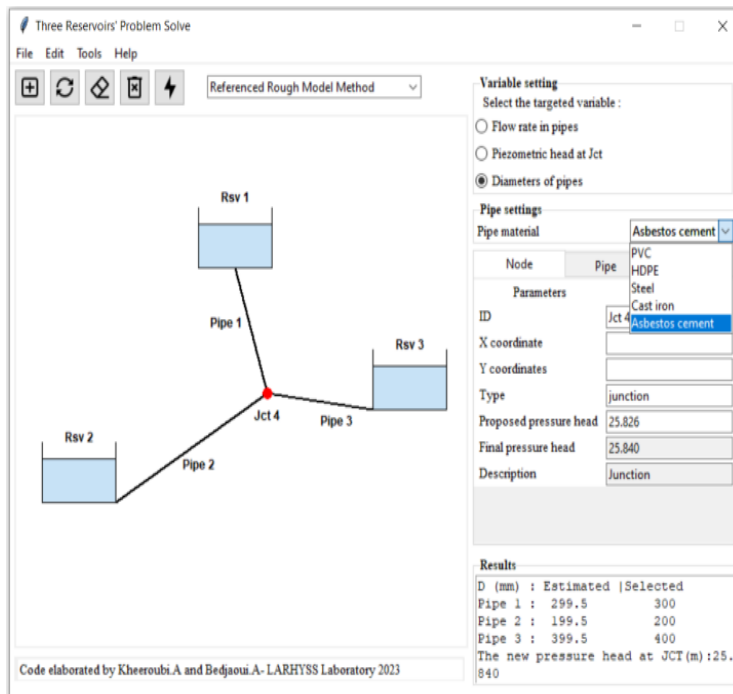


Figure (6):Diameter calculation results

After the selection of pipe sizes, the simulation model recalculates and proposes a new pressure head at the junction that corresponds to the newly selected diameters, as displayed in Table 6.

Table 5. Calculated and selected commercial diameters

Method	Pipe1		Pipe2		Pipe3	
	D calculated	D selected	D calculated	D selected	D calculated	D selected
RMM (Eq.20)	0.299	0.300	0.199	0.200	0.399	0.400
Hager (Eq.15)	0.296	0.300	0.182	0.200	0.391	0.400
Hazen-Williams(Eq.09)	0.299	0.300	0.199	0.200	0.406	0.400

As illustrated above, all three methods delivered accurate results, yet RMM was the most consistent method compared to *Hager* for pipe 2 and (*H-W*) for pipe 3. Nonetheless, the pipe size selection process by the model covers the slight differences where all the selected diameters are the same.

Note that the pressure head of Hager’s method in this case is calculated using RMM equations (22 - 25) since Hager did not present any equations to calculate the hydraulic gradient.

Table 6. New proposed pressure heads at the junction corresponding to selected diameters

Methods	RMM	Hager	Hazen-Williams
Node pressure head (m)	25.84	25.84	26.84

The calculated heads at the junction using RMM are not far from the values stated in Table 4 (a difference of 0.01m). The variation is evident between *Hazen-Williams* and the rest of the methods (a difference of 1 m).

Case 3: Pressure head calculation

In this category, only RMM and Hazen-Williams are considered. Utilizing the flow rates from Table 4 and the diameters presented in the example (Figure 7), the pressure heads obtained are summarized in Table 7.

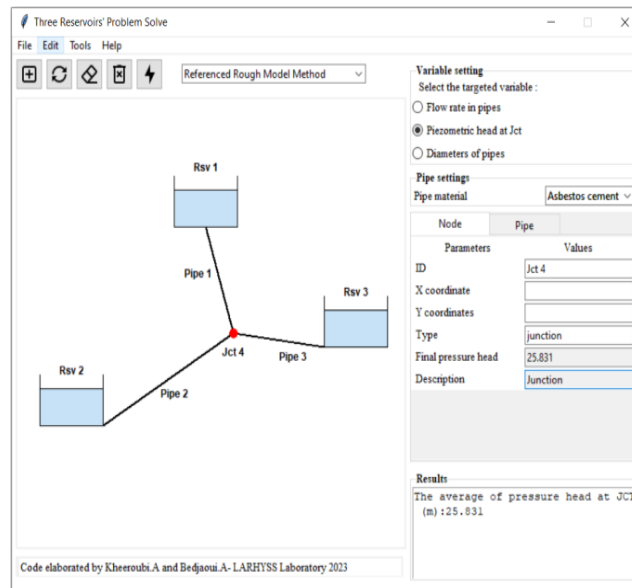


Figure (7): Calculated pressure head at the junction by using the RMM equation.

Table 7. Calculated pressure head at the junction

Methods	RMM (Eq.25)	Hazen-Williams (Eq.21)
Pressure head at the junction (m)	25.83	26.30

The difference between the pressure heads obtained in this case and these in Table 4 is almost insignificant (0.004m and 0.001m respectively).

The results indicate that the model generated different results from the original example where the iterations were ceased at $PH_j = 27$ m, while in the model, the value decreased to 25.827 m for (*RMM and Hager*) and 26.3 m for *Hazen Williams*, while the value 27 m generated $\sum Q = -0.02267$ m³/s according to the model as highlighted in appendix A1 which displays all the pressure heads and $\sum Q_i$ of the flows at the junction. The difference between the flows is significant (differences up to 30 l/s and more).

The difference in outcomes between *Hager* and *RMM* is insignificant since both methods are analytical. However, that was not the case with *Hazen-Williams*; the difference between H-W and the two other methods was up to 0.5 m and at times 1m in the calculated pressure head. This discrepancy may be due to the selected coefficient *C* since it was selected based on the pipe material without considering the roughness or any other parameter; therefore, the selection of *C* is crucial in this process and impacts the results significantly.

Note that many researchers have proposed alternative methods for calculating *C* that include some or all of the flow parameters to avoid the uncertainty associated with the choice of *Hazen-Williams's* coefficients. One of these methods is the approach of B.Achour and L. Amara[16], , who introduced a diagram to evaluate *C* from no-dimensional number *C** expressed graphically (Fig. 8.) as a function of two no-dimensional numbers: the relative roughness and ϵ/D and the *Reynolds* number *R*.

$$R = \frac{1}{2} \bar{R} \log \left[\frac{\epsilon/D}{3.7} + \frac{10.04}{\bar{R}} \right]^{-1} \quad (26)$$

After identifying *C** using Figure 5, the coefficient *C* is estimated using equation (27).

$$C^* = \frac{C_{hw}}{g^{0.54} D^{-0.01} v^{-0.08}} \quad (27)$$

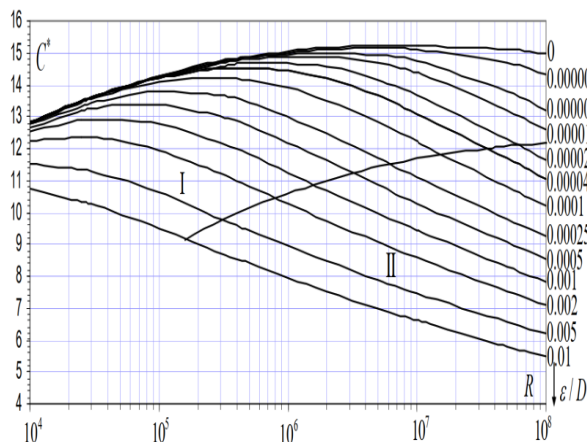


Figure (8):Dimensionless diagram for determining the Hazen-Williams coefficient in a pipe-flow.

I : transition zone,
II : rough zone.(Achour and Amara, 2020)

3. Conclusion

The main objective of this work is to present a precise alternative solution to solve three cases of the problems of the three reservoirs, namely the calculation of the flow conveyed by each pipe, the evaluation of the dimensions of the pipes from the three reservoirs and the calculation of the pressure head at the junction node. The proposed code in Python language consists of three parts, a flow rate solver that uses a loop to ensure convergence toward the exact value of the pressure head at the junction node while ensuring, through iterations, that ($\sum Q_i=0$) by using the Python library `scipy.stats` to calculate linear regression based on the least squares method for the PH_j and $\sum Q$ datasets. In addition, the pipe sizing solver allows us to compute and select commercial diameters closest to the calculated ones and proposes an adequate pressure head at the junction with the adopted diameters. Finally, the pressure head solver calculates the head losses to evaluate the head at the junction.

The model based on the Python language uses Hager relations, RMM and Hazen-Williams relation for each solver to solve the three problems.

The use of the proposed code to solve the example treated in the book (Hydraulique générale, French edition) where the problem studied was the determination of the flow rates in the three pipes gave ($Q_1=0.3626 \text{ m}^3/\text{s}$, $Q_2=0.0542 \text{ m}^3/\text{s}$ and $Q_3=0.3085 \text{ m}^3/\text{s}$) which shows a significant difference compared to the values of the flows mentioned in the book ($Q_1= 0.393 \text{ m}^3/\text{s}$, $Q_2= 0.061 \text{ m}^3/\text{s}$ $Q_3= 0.332 \text{ m}^3/\text{s}$) and the differences recorded are 30 l/s, 7 l/s and 22.67 l/s, respectively for pipes 1, 2 and 3.

Likewise, the pressure head at the junction node was fixed after 3 iterations at 27 m according to the book, while the code corrects it and proposes 25.83 m and ensures that the algebraic sum of the flow rates at the junction node is zero.

It should be observed that the most precise results (whether for flows or diameters or the elevation at the junction node) are obtained with the Hager and RMM relations. In contrast, the results obtained by applying the Hazen- Williams method differ from those obtained by the two other methods; this is due to the choice of the coefficient $C =140$, which is not in value equivalent to the absolute roughness $\epsilon =0.025 \text{ mm}$ of the pipe.

Appendix

Appendix A1. Example of pressure heads and their correspondent summation of flows

X	Y
30	-0.07798
29.9	-0.07619
29.8	-0.07439
29.7	-0.07259
29.6	-0.07078
29.5	-0.06898
29.4	-0.06717
29.3	-0.06535
29.2	-0.06353
29.1	-0.06172
29	-0.0599
28.9	-0.05806

X	Y
28.8	-0.05623
28.7	-0.0544
28.6	-0.05256
28.5	-0.05073
28.4	-0.04889
28.3	-0.04703
28.2	-0.04518
28.1	-0.04333
28	-0.04146
27.9	-0.03961
27.8	-0.03775
27.7	-0.03587

X	Y
27.6	-0.034
27.5	-0.03213
27.4	-0.03024
27.3	-0.02835
27.2	-0.02647
27.1	-0.02458
27	-0.02267
26.9	-0.02078
26.8	-0.01887
26.7	-0.01695
26.6	-0.01503
26.5	-0.01311

X	Y
26.4	-0.01119
26.3	-0.00925
26.2	-0.00731
26.1	-0.00528
26	-0.00342
25.9	-0.00147
25.8	0.0005

Conflicts of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

The authors would like to acknowledge the precious help of Professor Achour Bachir in the preparation of this paper.

References

- [1] M. C. Potter, D. C. Wiggert, B. M. Ramadan, *Mechanics of fluid*, Fourth Edition, Cengage learning, USA, 785 p, 2011
- [2] M. A. Haidara, E. M. Valentine, An effective alternate method for solving the classic three reservoirs and multi-reservoirs problem, *Civil Engineering Research Magazine*, Vol. 40, No. 3, 2018
- [3] J.F. Douglas, J. M. Gasiorek, J. A. Swaffield, L. B. Jack, *Fluid mechanics*, Fifth Edition, Pearson, Harlow, England, 941 p, 2005
- [4] R.V. Giles, B.J. Evett, C. Liou, *Mécanique des fluides et hydraulique. Série Schaum*, second edition, McGraw-Hill Inc, Paris, France, 1995, (In French)
- [5] Streeter V. L., Wylie E. B., Bedford K. W., *Fluid mechanics*, Ninth Edition, WCB/McGraw-Hill, USA, 555 p, 1998
- [6] L. Satiawati, Numerical solution of discharge calculations of the three reservoir problems, 2nd international conference on Earth Science, Mineral, and Energy, conference proceeding, 2020 DOI 10.1063/5.0010096
- [7] A. Lencastre, *Hydraulique générale*, Editions Eyrolles, Paris, France, 1996, (In French)
- [8] H. W. Hager, A. J. Schleiss constructions hydrauliques, Écoulements stationnaires - *Traité de génie civil*, vol. 15. Edition Presses Polytechniques Romandes, Suisse. 2009, (in French)
- [9] W.H. Hager, *Wastewater Hydraulics theory and practice*, Springer Heidelberg Dordrecht London New York, 2010. pp. 18-28.
- [10] N. F. A. Bombardelli and H. M. Garcia, M. ASCE., Hydraulic Design of Large-Diameter Pipes, *Journal of Hydraulic engineering ASCE* / November 2003, pp. 839-846. DOI: 10.1061/~ASCE!0733-9429~2003!129:11~839
- [11] Achour B., *Calculating conduits and channels, Pressurized conduits and channels, Volume 1, First Edition, Capital Edition, Algeria. 2007*, (In French).
- [12] B. Achour, A. Bedjaoui, Turbulent pipe flow computation using the Rough Model Method (RMM), *Journal of Civil Engineering and Science*. Vol.1, pp.36–41, 2012
- [13] B. Achour, A. Bedjaoui, M. Khattaoui, M. Debabeche, Contribution au calcul des écoulements uniformes à surface libre et en charge [Contribution for the computation of open channel and pressurized flow], *Larhyss Journal*, No 1, pp. 7-36, 2002, (In French)
- [14] G.S.C. Assuncao, D. Marcelein, J.C.V.H. Filho, D.J. Schiozer, M.S. De Castro, (2020), Friction factor equations accuracy for single and two-phase flows, *Proceedings of the ASME 39th International Conference on Ocean, Offshore and Arctic Engineering OMAE2020*, August 3-7, Virtual, Online, 2002

- [15] N. Jacomovic, M. Stamenic, P. Kolendic, D. Dordevic, B. Radanov ,A Novel Method for the Inclusion of pipe Roughness in the Hazen-Williams Equation, FME Transactions, Vol. 43, No. 1, pp. 35–39, 2015
- [16] G.S. Williams , A. Hazen, Hydraulic tables, JHON WILEY & SONS, New York, USA, 1905
- [17] B. Achour., L. Amara, Theoretical considerations on flow regime dependency of the Hazen-Williams coefficient, LARHYSS Journal, P-ISSN 1112-3680/E-ISSN 2521-9782, (42), pp.53-62, 2020