UtilitasMathematica
ISSN 0315-3681 Volume 121, 2024

A new sequence of numbers called Chintaginjala numbers, Chintaginjala polynomials and their properties.

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Abstract
Euler, Bernoulli and Gnocchi numbers are sequences of signed rational numbers, defined by various exponential generating functions. In this paper, I am introducing a new number sequence of signed rational numbers, polynomials called Chintaginjala numbers and Chintaginjala polynomials by using different generating function. I used McLaurin's infinite series for this number system. In this theory we find the relation between Chitaginjala numbers and Bernoulli's numbers. Chintaginjala numbers appear in twelve infinite series expansions. I derive 0 to 20 Chintaginjala numbers and 0 to 10 Chintagijala polynomials. In this study we derive the derivative and integration properties of Chintaginjala numbers, chintaginjala polynomials also we derive some forward difference(Delta) properties of Chitaginjala polynomials to calculate sum of n powers of n consecutive natural numbers. We also draw the graphs of first five Chintaginjala polynomials.

Keywords:Infiniteseries;Sequence;polynomials;bernoulli'snumbers;Recurrence relation .

1.Introduction:

Bernoulli , Euler and Gnocchi numbers are the most important invention in number theory. Bernoulli numbers B_n are sequence of rational numbers.
Bernoulli numbers[1]:

$$\frac{x}{e^x-1} = \sum_{n=0}^{\infty} B_n \frac{x^n}{n!}, (x \in R, |x| < 2\pi) \tag{1}$$

The recursive definition of the Bernoulli numbers $B_0 = 1$

$$B_n = \frac{-1}{n+1} \sum_{k=0}^{n-1} \binom{n+1}{k} B_k, (x \in R, |x| < 2\pi) \tag{2}$$

Equation (2) can be represented symbolically in "the umbral calculus"
$(B+1)^{n+1} = B_{n+1}$ left hand side is expanded by Binomial theorem exponents of B are form subscripts.

$$B_0 = 1, B_1 = \frac{-1}{2}, B_2 = \frac{1}{6}, B_3 = 0 B_4 = \frac{-1}{30}$$

Definition of Bernoulli polynomials

$$B_0(x), B_1(x) B_2(x) \ldots \qquad x \in R$$

by the generating function

$$\frac{te^{xt}}{e^t - 1} = \sum_{n=0}^{\infty} B_n(x) \frac{t^n}{n!}, (x, t \in R: |t| < 2\pi) \tag{3}$$

where

$$B_n(x) = \sum_{k=0}^{n} \binom{n}{k} B_k x^{n-k}, (x \in R, n \in I^+) \tag{4}$$

2 Chintaginjala's numbers

The Definition of the Chintaginjala's numbers are the
sequence of rational numbers $C_0, C_1, C_2 \ldots$
defined by the generating function

$$\frac{2t}{e^t - e^{-t}} = \sum_{n=0}^{\infty} C_n \frac{t^n}{n!}, (t \neq 0, \quad t \in R \quad |t| < 2\pi, n \in I^+) \tag{5}$$

$$C_0 = 1 C_n = 0, \quad for \quad n = 1,3,5, \ldots$$

Recursive definition of the Chintaginjala's even index numbers

$$C_n = \frac{-1}{n+1} \sum_{k=0}^{n-1} \binom{n+1}{k} C_k, \qquad for \quad n = 2,4,6, \ldots \tag{6}$$

equation (6) can be also expressed as an Umbrol calculus

$$(C+1)^{n+1} = C_{n+1} \quad forn = 2,4,6, \ldots \tag{7}$$

In the left hand side is expanded by Binomial Theorem , take exponents on C used as subscripts of C.

$C_0 = 1$, $C_1 = 0 C_2 = \frac{-1}{3}$, $C_3 = 0$, $C_4 = \frac{7}{15}$, $C_5 = 0$

$C_6 = \frac{-31}{21}$, $C_7 = 0$, $C_8 = \frac{127}{15}$, $C_9 = 0$, $C_{10} = \frac{-2555}{33}$, $C_{11} = 0$

$C_{12} = \frac{1414477}{1365}$, $C_{13} = 0$, $C_{14} = \frac{-57337}{3}$, $C_{15} = 0$, $C_{16} = \frac{118518239}{255}$, $C_{17} = 0$

$C_{18} = \frac{-1441025453}{100}$, $C_{19} = 0$, $C_{20} = \frac{1109651847}{2}$

Relation between Chintaginjala's and Bernoulli's numbers

$$C_n = (2 - 2^n) B_n, \quad \forall \quad n \in I^+ \tag{8}$$

2.1 Some basic properties of Chintaginjala's numbers

i) Every Chintaginjala number is a rational number.
ii) C_0 is only non zero integers.
iii) $C_{2n-1} = 0, \forall n \geq 1$ all other numbers are non zero
iv) Every chintaginjala number with index multiple of 4 is a positive rational number,
v) from C_2 all Chintaginjala numbers alternate in sign.
vi) The absolute value of even indexed Chintaginjala numbers has a minimum value of $\frac{1}{3}$when the index is 2.

Chintaginjala numbers appear in 12 Infinite series expansions.

$$1.\ \tan x = \sum_{n=1}^{\infty} 2^{2n} \frac{(2^{2n}-1)}{(2n)!(2^{2n}-2)} |C_{2n}| x^{2n-1}, (|x| < \frac{\pi}{2})$$

$$2.\ \cot x = \frac{1}{x} - \sum_{n=1}^{\infty} \frac{1}{(2n)!(2^{2n}-2)} |C_{2n}| x^{2n-1}, (0 < |x| < \pi)$$

$$3.\ \csc x = \frac{1}{x} + \sum_{n=1}^{\infty} \frac{1}{(2n)!} |C_{2n}| x^{2n-1}, (0 < |x| < \pi)$$

$$4.\ \tanh x = -\sum_{n=1}^{\infty} 2^{2n} \frac{(2^{2n}-1)}{(2n)!(2^{2n}-2)} C_{2n} x^{2n-1}, (|x| < \frac{\pi}{2})$$

$$5.\ \coth x = \frac{1}{x} - \sum_{n=1}^{\infty} \frac{1}{(2n)!(2^{2n}-2)} C_{2n} x^{2n-1}, (0 < |x| < \pi)$$

$$6.\ \mathrm{csc}hx = \frac{1}{x} + \sum_{n=1}^{\infty} \frac{1}{(2n)!} C_{2n} x^{2n-1}, (0 < |x| < \pi)$$

$$7.\ \log|\sin x| = -\log|\csc x|$$
$$= \log|x| - \sum_{n=1}^{\infty} \frac{1}{2n(2n)!} |C_{2n}| x^{2n}, (0 < |x| < \pi)$$

$$8.\ \log\cos x = -\log\sec x$$
$$= -\sum_{n=1}^{\infty} \frac{2^{2n-1}(2^{2n}-1)}{2n(2n)!(2^{2n}-2)} |C_{2n}| x^{2n}, (|x| < \frac{\pi}{2})$$

$$9.\ \log|\tan x| = -\log|\cot x|$$
$$= \log|x| + \sum_{n=1}^{\infty} \frac{2^{2n-1}}{n(2n)!} |C_{2n}| x^{2n}, (0 < |x| < \frac{\pi}{2})$$

$$10. \log|\sinh x| = -\log|\operatorname{csch} x| \\ = \log|x| - \sum_{n=1}^{\infty} \frac{2^{2n-1}}{n(2n)!(2^{2n}-2)} C_{2n} x^{2n}, (0 < |x| < \pi)$$

$$11. \log\cosh x = -\log\operatorname{sech} x \\ = \log|x| - \sum_{n=1}^{\infty} \frac{2^{2n-1}(2^{2n}-1)}{n(2n)!(2^{2n}-2)} C_{2n} x^{2n}, (|x| < \frac{\pi}{2})$$

$$12. \log|\tanh x| = -\log|\coth x| \\ = \log|x| + \sum_{n=1}^{\infty} \frac{2^{2n-1}}{n(2n)!} C_{2n} x^{2n}, (0 < |x| < \frac{\pi}{2})$$

Even indexed Chintaginjala number in to the Zeta function

$$C_n = (-1)^{\frac{1}{2}(n+4)} \frac{(2^n-2)2n!}{(2\pi)^n} \zeta(n), \quad for \quad n = 2,4,6,\ldots \\ = (-1)^{\frac{1}{2}(n+4)} \frac{(2^n-2)2n!}{(2\pi)^n} \sum_{k=1}^{\infty} \frac{1}{k^n}, \quad for \quad n = 2,4,6,\ldots$$

Conversely Zeta of an even positive integer n into Chintaginjala number

$$\zeta(n) = \sum_{k=1}^{\infty} \frac{1}{k^n} \\ = \frac{(2\pi)^n}{(2^n-2)2n!} |C_n|, \quad for \quad n = 2,4,6,\ldots$$

this gives

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots = \frac{\pi^2}{6} \\ 1 + \frac{1}{2^4} + \frac{1}{3^4} + \ldots = \frac{\pi^4}{90} \\ 1 + \frac{1}{2^6} + \frac{1}{3^6} + \ldots = \frac{\pi^6}{945}$$

$$1+\frac{1}{2^8}+\frac{1}{3^8}+\ldots=\frac{\pi^8}{9450}$$
$$1+\frac{1}{2^{10}}+\frac{1}{3^{10}}+\ldots=\frac{\pi^{10}}{93555}$$
$$1+\frac{1}{2^{12}}+\frac{1}{3^{12}}+\ldots=\frac{691\pi^{12}}{638512875}$$

3 Chintaginjala's Polynomials

$$C_0(x), C_1(x), C_2(x)\ldots (x\in R)$$

By generating function

$$\frac{2te^{xt}}{(e^t-e^{-t})}=\sum_{n=0}^{\infty} C_n(x)\frac{t^n}{n!}, (x,t\in R, t\neq 0: |x|<2\pi) \quad (9)$$

$$C_n(x)=\sum_{k=0}^{n}\binom{n}{k}C_k x^{n-k} \quad (10)$$

Second order Bernoulli's Polynomial 0 to 10

$$C_0(x)=1$$
$$C_1(x)=x$$
$$C_2(x)=x^2-\frac{1}{3}$$
$$C_3(x)=x^3-x$$
$$C_4(x)=x^4-2x^2+\frac{7}{15}$$
$$C_5(x)=x^5-\frac{10}{3}x^3+\frac{7}{3}x$$
$$C_6(x)=x^6-5x^4+7x^2-\frac{31}{21}$$
$$C_7(x)=x^7-7x^5+\frac{49}{3}x^3-\frac{31}{3}x$$
$$C_8(x)=x^8-\frac{98}{3}x^4+\frac{124}{3}x^2-\frac{127}{15}$$
$$C_9(x)=x^9-12x^7+\frac{294}{5}x^5-124x^3+\frac{381}{5}x$$
$$C_{10}(x)=x^{10}-15x^8+98x^6-310x^4+381x^2-\frac{2555}{33}$$

3.1 Some basic properties of Chintaginjala's polynomials.

i) All the coefficients are rational numbers.
ii)The coefficients are alternate in sign.
iii)The degree of $C_n(x)$ is n.
iv) $C_n(x+1)-C_n(x-1)=2n(x)^{n-1}$.
v) $|C_n(x)|\leq C_n \quad \forall x\in(0,1), \quad n\geq 2$
vi) $\frac{d}{dx}(C_n(x))=nC_{n-1}(x) \quad$ n$\geq$ 1

vii) $\int_0^x C_n(t)dt = \frac{1}{n+1}[C_{n+1}(x) - C_{n+1}]$
viii) $\int_{x-1}^{x+1} C_n(t)dt = 2x^n$
ix) $\int_0^2 C_n(t)dt = 2, \quad \forall n \in I^+$

3.2 Some Difference properties of Chintaginjala's polynomials.

from the definition of $C_n(x)$ we can observe that

$$\Delta\left\{\frac{C_n(x+1)+C_n(x)}{2n}\right\} = (x+1)^{n-1}, \quad for \quad n \geq 1 \tag{11}$$

$$Proof: \Delta\left\{\frac{C_n(x+1)+C_n(x)}{2n}\right\} = \frac{\Delta C_n(x+1)+\Delta C_n(x)}{2n}$$
$$= \frac{1}{2n}[C_n(x+2) - C_n(x+1) + C_n(x+1) - C_n(x)]$$
$$= \frac{1}{2n}[C_n(x+2) - C_n(x)]$$
$$= (x+1)^{n-1}$$

For example $n = 3 \quad then$

$$\Delta\left\{\frac{C_3(x+1) + C_3(x)}{6}\right\} = \frac{1}{6}[C_3(x+2) - C_3(x)]$$
$$= \frac{1}{6}[(x+2)^3 - (x+2) - x^3 + x]$$
$$= (x+1)^2$$

from (11) , applying inverse difference operator on both sides we get

$$\left\{\frac{C_n(x+1)+C_n(x)}{2n}\right\} = \Delta^{-1}(x+1)^{n-1} \tag{12}$$

suppose x takes integer values from the equation (12) we can easily observe that

$$\sum_{x=0}^{m-1} (x+1)^{n-1} = \frac{1}{2n}[C_n(x+1) + C_n(x)]_{x=0}^{m}$$

For example $n = 3 \sum_{x=0}^{m-1} (x+1)^2 = \frac{1}{6}[C_3(x+1) + C_3(x)]_{x=0}^{m}$

$$= \frac{1}{6}[C_3(m+1) + C_3(m) - C_3(1) - C_3(0)]$$
$$= \frac{1}{6}[(m+1)^3 - (m+1) + m^3 - m]$$
$$\Rightarrow 1^2 + 2^2 + 3^2 + \ldots + m^2 = \frac{1}{6}m(m+1)(2m+1)$$

4 Conclusion

It is easily to understand that these chintaginjala’s numbers are expressed in 12 infinite series.Chintaginjala’s polynomials are expressed in terms of sum of powers of first n consecutive natural numbers. We can use these Chintaginjala’s numbers in various engineering problems.

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