1-SOLITON SOLUTION OF KADOMTSEV-PETVIASHVILI EQUATION WITH GENERALIZED EVOLUTION USING SIGMOID FUNCTION

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Abstract
This study calculatesthe solution of (3+1)-dimensional generalized Kadomtsev–Petviashvili(KP) equation by using sigmoid function. KP equations is the 2D analog of the Korteweg-de Vries (KdV) equation that models the flow of shallow water waves. The solitary wave ansatz method is applied to accomplish the integration of such equations. The parameter domain is the key factor to recognizethese types of equations. Numerical simulations and graphs also produced in this study.

Keywords: Generalized KP equation; solitons; integrality;simulations

1. Introduction :

Solitary waves become incredible move along the past 30 years. In this area many researchers has been studied extensively and produce the good results [1-11].The main phases in this area has been completely understood is the integrability of the solitary waves (1+1)D, (1+2)D and (1+3)D equations.Now a days a lot of study is going on with the integrality aspects in (1+N)D equations. Previously many people using the nonlinear analog of Fourier Transform method, which is Inverse Scattering Transform (IST), this gives the nonlinear evolution equations. There are plenty of modern methods of integrability available to solve these Nonlinear evolution equations namely exponential method, $\frac{G'}{G}$method,Adomain decomposition method,F-expansion method ,Riccati's equation method etc., We need to be cautious of implementing these type of modern integrability methods.

In this article I introduced one new method of integrabilityapplied to carryout the integration of KadomtsevPetviashvilli (KP) equation. This KP equation is a 2D analog of Korteweg –de Vries (KdV) equation is the mathematical models of shallow water waves.If wave propagation along x-axis, with the Equilibrium of non-linearity and dispersion is also in the x-axis direction. A proper dependence on the y-variable also has a contribution that occurs at the same order as non-linearity and dispersion.In this paper. I discussed three generalized variants of KP equation. This new technique will be adopted to integrate such equations is the solitary wave ansatz method. My numerical simulations and graphs give the better results than other methods.

2. Mathematical analysis using sigmoid function

Three variants of the generalized KadomtsevPetviashvilli (KP) equation given by [8]

Varaint I

$$\{(q^l)_t + a(q^n)_x + b(q^n)_{xxx}\}_x + cq_{yy} = 0 \quad (2.1)$$

Variant II

$$\{(q^l)_t + a(q^{n+1})_x + b[q(q^n)_{xx}]_x\}_x + cq_{yy} = 0 \quad (2.2)$$

and

Variant III
$\{(q^l)_t + a(q^{2n})_x + b[q^n(q^n)_{xx}]_x\}_x + cq_{yy} = 0$ ________________ (2.3)

In (2.1) to (2.3) the first term indicates generalized evolution. The special case $l = 1$
Is regular evolution term. Coefficients a are nonlinear terms, the coefficients b are the nonlinear dispersion terms along x-axis and the coefficients c represents the linear dispersion in the y-axis. Wazwaz[8] already solved these equations with $l = 1$in the year 2005.In this paper I also calculated
Compactons and periodic solutions.
2.1 Varian I
To solve the generalized KP equation I choose the hypothesis
as Sigmoidfunction $q(x,y,t) = \frac{A}{(1+e^{-\tau})^p}$ _____________ (2.4)
Where$\tau = b_1x + b_2y - vt$, $p > 0$ _____________ (2.5)
for solitons to exist.
A denotes the amplitude of the soliton whileb_1, b_2represents the inverse widths along thex,yaxes respectively and v is the velocity. The exponent $p$is unknown at this point and will be evaluated during the on the derivation of the solitons (2.1) to (2.3).
let$S = 1 + e^{-\tau}$

$$(q^l)_t = \frac{-A^l vlp e^{-\tau}}{S^{pl+1}}$$

$$\Rightarrow (q^l)_t = -A^l vlp\left(\frac{1}{S^{pl}} - \frac{1}{S^{pl+1}}\right)$$

$$(q^l)_{tx} = -A^l vlpb_1\left[lp\left(\frac{1}{S^{pl}} - \frac{1}{S^{pl+1}}\right) - (lp+1)\left(\frac{1}{S^{pl+1}} - \frac{1}{S^{pl+2}}\right)\right]$$

$$(q^l)_{tx} = -A^l vlpb_1\left[\frac{lp}{S^{pl}} - \frac{(2lp+1)}{S^{pl+1}} + \frac{(lp+1)}{S^{pl+2}}\right] \text{___________ (2.6)}$$

$$(q^n)_x = -A^n b_1 np\left(\frac{1}{S^{np}} - \frac{1}{S^{np+1}}\right)$$

$$(q^n)_{xx} = -A^n {b_1}^2 np\left(\frac{np e^{-\tau}}{S^{np+1}} - \frac{(np+1)e^{-\tau}}{S^{np+2}}\right)$$

$$(q^n)_{xx} = -A^n {b_1}^2 np\left(np\left(\frac{1}{S^{np}} - \frac{1}{S^{np+1}}\right) - (np+1)\left(\frac{1}{S^{np+1}} - \frac{1}{S^{np+2}}\right)\right)$$

$$\Rightarrow (q^n)_{xx} = -A^n {b_1}^2 np\left[\frac{np}{S^{np}} - \frac{(2np+1)}{S^{np+1}} + \frac{(np+1)}{S^{np+2}}\right] \text{_______________(2.7)}$$

$$(q^n)_{xxx} = -A^n {b_1}^3 np\left[\frac{(np)^2 e^{-\tau}}{S^{np+1}} - \frac{(np+1)(2np+1)e^{-\tau}}{S^{np+2}} + \frac{(np+1)(np+2)e^{-\tau}}{S^{np+3}}\right]$$

$$\Rightarrow (q^n)_{xxx} = -A^n {b_1}^3 np\left[\frac{(np)^2}{S^{np}} - \frac{(3n^2p^2+3np+1)}{S^{np+1}} + \frac{(3n^2p^2+5np+3)}{S^{np+2}} - \frac{(n^2p^2+3np+2)}{S^{np+3}}\right] \text{______ (2.8)}$$

$$(q^n)_{xxxx} = -A^n {b_1}^4 np\left[(np)^2 np\left(\frac{1}{S^{np}} - \frac{1}{S^{np+1}}\right) - (3n^2p^2 + 3np + 1)(np + 1)\left(\frac{1}{S^{np+1}} - \frac{1}{S^{np+2}}\right) + (3n^2p^2 + 5np + 3)(np + 2)\left(\frac{1}{S^{np+2}} - \frac{1}{S^{np+3}}\right) - (n^2p^2 + 3np + 2)(np + 3)\left(\frac{1}{S^{np+3}} - \frac{1}{S^{np+4}}\right)\right]$$

$$\Rightarrow (q^n)_{xxxx} = -A^n {b_1}^4 np\left[\frac{n^3p^3}{S^{np}} - \frac{(4n^3p^3+6n^2p^2+4np+1)}{S^{np+1}} + \frac{(6n^3p^3+17n^2p^2+17np+7)}{S^{np+2}} - \frac{(4n^3p^3+17n^2p^2+24np+12)}{S^{np+3}} + \frac{(n^3p^3+6n^2p^2+11np+6)}{S^{np+4}}\right] \qquad (2.9)$$

$$\Rightarrow q_{yy} = -A^n {b_2}^2 p\left[\frac{p}{S^p} - \frac{(2p+1)}{S^{p+1}} + \frac{(p+1)}{S^{p+2}}\right] \qquad (2.10)$$

Substitute (2.6) to (2.10) in the equation (2,1)

$$\frac{-l^2p^2A^lvb_1}{S^{pl}} + \frac{A^lvb_1(2lp+1)}{S^{pl+1}} - \frac{A^lvb_1lp(lp+1)}{S^{pl+2}} + \frac{aA^n{b_1}^2n^2p^2}{S^{np}} - \frac{aA^n{b_1}^2np(2np+1)}{S^{np+1}} + \frac{aA^n{b_1}^2np(np+1)}{S^{np+2}} + \frac{bA^n{b_1}^4n^4p^4}{S^{np}} - \frac{bA^n{b_1}^4np(4n^3p^3+6n^2p^2+4np+1)}{S^{np+1}} + \frac{bA^n{b_1}^4np(6n^3p^3+17n^2p^2+7np+7)}{S^{np+2}} - \frac{bA^n{b_1}^4np(4n^3p^3+17n^2p^2+24np+12)}{S^{np+3}} + \frac{bA^n{b_1}^4np(n^3p^3+6n^2p^2+11np+6)}{S^{np+4}} + \frac{Ac{b_2}^2p^2}{S^p} - \frac{Ac{b_2}^2p(2p+1)}{S^{p+1}} + \frac{Ac{b_2}^2p(p+1)}{S^{p+2}} = 0 \qquad (2.11)$$

From (2.11) equating the exponents of p and $np + 2$ gives

$$np + 2 = p \Rightarrow p = \frac{2}{1-n}$$

$$-vb_1n^2p^2 + a{b_1}^2n^2p^2 + b{b_1}^4n^4p^4 = 0$$

$$\Rightarrow -v + ab_1 + b{b_1}^3n^2p^2 = 0$$

$$\Rightarrow v = ab_1 + b{b_1}^3n^2p^2$$

$$\Rightarrow v = ab_1 + \frac{4bb_1{}^3n^2}{(1-n)^2} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_} \quad (2.12)$$

$$-np(np+1)vb_1A^n + aA^n np(np+1){b_1}^2 + bA^n {b_1}^4 np(6n^3p^3 + 17n^2p^2 + 17np + 7) + Ac{b_2}^2 p(p+1) = 0$$

divide by $np(np+1)b_1A^n$

$$-v + ab_1 + \frac{b{b_1}^3 np(6n^3p^3 + 17n^2p^2 + 17np + 7)}{(np+1)} + \frac{A^{1-n}c{b_2}^2(p+1)}{b_1 n(np+1)} = 0 \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} (2.13)$$

$np = \frac{2n}{1-n}$, $np + 1 = \frac{2n}{1-n} + 1 = \frac{1+n}{1-n}$, $p + 1 = \frac{3-n}{1-n}$

$$\frac{(6n^3p^3 + 17n^2p^2 + 17np + 7)}{(np+1)} = \frac{7n^3 + 21n^2 + 13n + 7}{n^3 - n^2 - n + 1}$$

and $\frac{(p+1)}{n(np+1)} = \frac{3-n}{n^2+n}$

then from (2.13)

$$v = ab_1 + \frac{b{b_1}^3(7n^3+21n^2+13n+7)}{n^3-n^2-n+1} + \frac{A^{1-n}c{b_2}^2(3-n)}{b_1(n^2+n)} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_} (2.14)$$

$$\Rightarrow A^{1-n} = \left\{\left[\frac{4b{b_1}^3n^2}{(1-n)^2} - \frac{b{b_1}^3(7n^3 + 21n^2 + 13n + 7)}{n^3 - n^2 - n + 1}\right]\frac{b_1(n^2+n)}{c{b_2}^2(3-n)}\right\}$$

$$\Rightarrow A = \left\{\left[\frac{4b{b_1}^3n^2}{(1-n)^2} - \frac{b{b_1}^3(7n^3+21n^2+13n+7)}{n^3-n^2-n+1}\right]\frac{b_1(n^2+n)}{c{b_2}^2(3-n)}\right\}^{\frac{1}{1-n}} \quad \text{\_\_\_\_\_\_\_\_\_\_\_} (2.15)$$

So 1-soliton solution of variant-I is given by

$$q = \frac{A}{\left(1-e^{-(b_1x+b_2y-vt)}\right)^{\frac{2}{1-n}}} ,$$

Where $n = \frac{1}{2}$, v and A values can be chosen from the equations (2.12) and (2.15) respectively.

Figure 1 i) Profile of variant –III KP solution at $t = 0$
ii)Profile of variant –III KP solution at $t = 3$
iii) Profile of variant –III KP solution at $t = 5$
the parameter values are chosen are $a = b = c = 0.5$, $b_1 = 0.5$, $b_2 = -0.5$

2.2 Variant – II :

$(q^l)_{tx} = -A^l vlp\mathrm{e}^{-\tau} b_1 \left[\frac{lp}{S^{pl}} - \frac{(2lp+1)}{S^{pl+1}} + \frac{(lp+1)}{S^{pl+2}}\right]$ ___________ (2.6)

$$(q^{n+1})_x = A^{n+1} b_1 (n+1)p \left(\frac{1}{S^{(n+1)p}} - \frac{1}{S^{(n+1)p+1}}\right)$$

$$(q^n)_{xx} = A^{n+1} {b_1}^2 (n+1)p \left(\frac{(n+1)p\mathrm{e}^{-\tau}}{S^{(n+1)p+1}} - \frac{\big((n+1)p+1\big)\mathrm{e}^{-\tau}}{S^{(n+1)p+2}}\right)$$

$$(q^{n+1})_{xx} = A^{n+1} {b_1}^2 (n+1)p \Bigg((n+1)p \left(\frac{1}{S^{(n+1)p}} - \frac{1}{S^{(n+1)p+1}}\right) - \big((n+1)p+1\big)\left(\frac{1}{S^{(n+1)p+1}} - \frac{1}{S^{(n+1)p+2}}\right)\Bigg)$$

$(q^{n+1})_{xx} = A^{n+1} {b_1}^2 (n+1)p \left[\frac{(n+1)p}{S^{(n+1)p}} - \frac{(n+1)p+(n+1)^2p^2}{S^{(n+1)p+1}} + \frac{(n+1)p+1}{S^{(n+1)p+2}}\right]$ _____(2.16)

$(q^n)_{xx} = A^n {b_1}^2 np \left[\frac{np}{S^{np}} - \frac{(2np+1)}{S^{np+1}} + \frac{(np+1)}{S^{np+2}}\right]$ ______________ (2.7)

$$q(q^n)_{xx} = A^{n+1} {b_1}^2 np \left[\frac{np}{S^{(n+1)p}} - \frac{(2np+1)}{S^{(n+1)p+1}} + \frac{(np+1)}{S^{(n+1)p+2}}\right]$$

$$[q(q^n)_{xx}]_x = A^{n+1} {b_1}^3 np \Bigg[np(n+1)p\left(\frac{1}{S^{(n+1)p}} - \frac{1}{S^{(n+1)p+1}}\right) - \big((n+1)p+1\big)(2np+1)\left(\frac{1}{S^{(n+1)p+1}} - \frac{1}{S^{(n+1)p+2}}\right) + \big((n+1)p+2\big)(np+1)\left(\frac{1}{S^{(n+1)p+2}} - \frac{1}{S^{(n+1)p+3}}\right)\Bigg]$$

$$\Rightarrow [q(q^n)_{xx}]_x = A^{n+1} {b_1}^3 np \Bigg[\frac{(n^2+n)p^2}{S^{(n+1)p}} - \frac{\big((3n^2+3n)p^2+(3n+1)p+1\big)}{S^{(n+1)p+1}} + \frac{\big((3n^2+3n)p^2+(6n+2)p+3\big)}{S^{(n+1)p+2}} - \frac{\big((n^2+n)p^2+(3n+1)p+2\big)}{S^{(n+1)p+3}}\Bigg]$$

$[q(q^n)_{xx}]_{xx} =$

$$A^{n+1}{b_1}^4 np\Bigg[(n^2+n)p^2(n+1)p\left(\frac{1}{S^{(n+1)p}}-\frac{1}{S^{(n+1)p+1}}\right) - \big((n+1)p+1\big)\big((3n^2+3n)p^2+(3n+1)p+1\big)\left(\frac{1}{S^{(n+1)p+1}}-\frac{1}{S^{(n+1)p+2}}\right) + \big((n+1)p+2\big)\big((3n^2+3n)p^2+(6n+2)p+3\big)\left(\frac{1}{S^{(n+1)p+2}}-\frac{1}{S^{(n+1)p+3}}\right) + \big((n+1)p+3\big)\big((n^2+n)p^2+(3n+1)p+2\big)\left(\frac{1}{S^{(n+1)p+3}}-\frac{1}{S^{(n+1)p+4}}\right)\Bigg]$$

$$\Rightarrow [q(q^n)_{xx}]_{xx} = A^{n+1}{b_1}^4 np\Bigg[\frac{n(n+1)^2p^3}{S^{(n+1)p}} - \frac{\big((4n^3+8n^2+4n)p^3+(6n^2+7n+1)p^2+(4n+2)p+1\big)}{S^{(n+1)p+1}} + \frac{(6n(n+1)^2p^3+(18n^2+21n+3)p^2+(19n+9)p+7)}{S^{(n+1)p+2}} - \frac{(4n(n+1)^2p^3+(18n^2+21n+3)p^2+(26n+12)p+12)}{S^{(n+1)p+3}} + \frac{(n(n+1)^2p^3+(6n^2+7n+1)p^2+(11n+5)p+6)}{S^{(n+1)p+4}}\Bigg]$$

____________________ (2.17)

$$q_{yy} = A^n {b_2}^2 p\left[\frac{p}{S^p}-\frac{(2p+1)}{S^{p+1}}+\frac{(p+1)}{S^{p+2}}\right] \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (2.10)}$$

Substitute (2.6) , (2.10) ,(2.16) and (2.17) in the equation (2,2)

$$-A^l vlpb_1\left[\frac{lp}{S^{pl}}-\frac{(2lp+1)}{S^{pl+1}}+\frac{(lp+1)}{S^{pl+2}}\right] + aA^{n+1}{b_1}^2(n+1)p\left[\frac{(n+1)p}{S^{(n+1)p}}-\frac{(n+1)p+(n+1)^2p^2}{S^{(n+1)p+1}}+\frac{(n+1)p+1}{S^{(n+1)p+2}}\right] + bA^{n+1}{b_1}^4 np\Bigg[\frac{n(n+1)^2p^3}{S^{(n+1)p}} - \frac{\big((4n^3+8n^2+4n)p^3+(6n^2+7n+1)p^2+(4n+2)p+1\big)}{S^{(n+1)p+1}} + \frac{(6n(n+1)^2p^3+(18n^2+21n+3)p^2+(19n+9)p+7)}{S^{(n+1)p+2}} - \frac{(4n(n+1)^2p^3+(18n^2+21n+3)p^2+(26n+12)p+12)}{S^{(n+1)p+3}} + \frac{(n(n+1)^2p^3+(6n^2+7n+1)p^2+11p+5)}{S^{(n+1)p+4}}\Bigg] + cA^n{b_2}^2p\left[\frac{p}{S^p}-\frac{(2p+1)}{S^{p+1}}+\frac{(p+1)}{S^{p+2}}\right]=0$$

____________________ (2.18)

From (2.18) equating the exponents of p and $(n+1)p+2$ gives
$(n+1)p+2=p \Rightarrow p=\frac{-2}{n}$.
Also equating the exponents of lp and $(n+1)p$ gives

$$(n+1)p=lp \Rightarrow l=(n+1)$$

From (2.18) , setting the coefficients of $\frac{1}{S^{(n+1)p+j}}$ for $j = 0,4$ to zero we get

$$-A^l v b_1 l^2 p^2 + aA^{n+1} {b_1}^2 (n+1)^2 p^2 + bA^{n+1} {b_1}^4 n^2 (n+1)^2 p^4 = 0$$

But $l = (n+1)$

$$\Rightarrow -A^{n+1} v b_1 (n+1)^2 p^2 + aA^{n+1} {b_1}^2 (n+1)^2 p^2 + bA^{n+1} {b_1}^4 n^2 (n+1)^2 p^4 = 0$$

$$\Rightarrow -v + ab_1 + b{b_1}^3 n^2 p^2 = 0$$

$$\Rightarrow v = ab_1 + 4b{b_1}^3 \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (2.19)}$$

and

$$bA^{n+1} {b_1}^4 np(n(n+1)^2 p^3 + (6n^2 + 7n + 1)p^2 + (11n+5)p + 6) = 0$$

Which is not possible.
so , there is no possibility of 1-soliton solution for this variant –II.

2.3 Variant – III :

$$(q^l)_{tx} = -A^l v l p \mathrm{e}^{-\tau} b_1 \left[\frac{lp}{S^{pl}} - \frac{(2lp+1)}{S^{pl+1}} + \frac{(lp+1)}{S^{pl+2}}\right] \text{\_\_\_\_\_\_\_\_\_\_\_\_ (2.6)}$$

$$q^{2n} = \frac{A^{2n}}{S^{2np}}$$

$$(q^{2n})_x = A^{2n} b_1 2np \left(\frac{1}{S^{2np}} - \frac{1}{S^{2np+1}}\right)$$

$$(q^{2n})_{xx} = A^{n+1} {b_1}^2 2np \left(\frac{2np\mathrm{e}^{-\tau}}{S^{2np+1}} - \frac{(2np+1)\mathrm{e}^{-\tau}}{S^{2np+2}}\right)$$

$$(q^{2n})_{xx} = A^{2n} {b_1}^2 2np \left(2np\left(\frac{1}{S^{2np}} - \frac{1}{S^{2np+1}}\right) - (2np+1)\left(\frac{1}{S^{2np+1}} - \frac{1}{S^{2np+2}}\right)\right)$$

$$(q^{2n})_{xx} = A^{2n} {b_1}^2 2np \left[\frac{2np}{S^{2np}} - \frac{(4np+1)}{S^{2np+1}} + \frac{(2np+1)}{S^{2np+2}}\right] \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (2.20)}$$

We already know

$$(q^n)_{xx} = A^n {b_1}^2 np \left[\frac{np}{S^{np}} - \frac{(2np+1)}{S^{np+1}} + \frac{(np+1)}{S^{np+2}}\right] \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (2.7)}$$

$$q^n (q^n)_{xx} = A^{2n} {b_1}^2 np \left[\frac{np}{S^{2np}} - \frac{(2np+1)}{S^{2np+1}} + \frac{(np+1)}{S^{2np+2}}\right]$$

$$[q^n (q^n)_{xx}]_x = A^{2n} {b_1}^3 np \left[2n^2p^2\left(\frac{1}{S^{2np}} - \frac{1}{S^{2np+1}}\right) - (2np+1)^2\left(\frac{1}{S^{2np+1}} - \frac{1}{S^{2np+2}}\right) + (2np+1)(np+1)\left(\frac{1}{S^{2np+2}} - \frac{1}{S^{2np+3}}\right)\right]$$

$$\Rightarrow [q^n(q^n)_{xx}]_x = A^{2n}{b_1}^3 np\left[\frac{2n^2p^2}{S^{2np}} - \frac{(6n^2p^2+4np+1)}{S^{2np+1}} + \frac{(6n^2p^2+8np+3)}{S^{2np+2}} - \frac{(2n^2p^2+4np+2)}{S^{2np+3}}\right]$$

$$[q^n(q^n)_{xx}]_{xx} = A^{2n}{b_1}^4 np\left[\frac{4n^3p^3}{S^{2np}} - \frac{(16n^3\,p^3+14n^2p^2+6np+1)}{S^{2np+1}} + \frac{(24n^3\,p^3+42n^2p^2+28np+7)}{S^{2np+2}} - \frac{(16n^3\,p^3+42n^2p^2+38np+12)}{S^{2np+3}} + \frac{(4n^3\,p^3+14n^2p^2+16np+6)}{S^{2np+4}}\right]$$

________________ (2.21)

$q_{yy} = A^n{b_2}^2 p\left[\frac{p}{S^p} - \frac{(2p+1)}{S^{p+1}} + \frac{(p+1)}{S^{p+2}}\right]$ ______________ (2.10)

Substitute (2.6) , (2.10) ,(2.20) and (2.21) in the equation (2.3)

$$-A^l vlpb_1\left[\frac{lp}{S^{pl}} - \frac{(2lp+1)}{S^{pl+1}} + \frac{(lp+1)}{S^{pl+2}}\right] + aA^{2n}{b_1}^2 2np\left[\frac{2np}{S^{2np}} - \frac{(4np+1)}{S^{2np+1}} + \frac{(2np+1)}{S^{2np+2}}\right] + \mathrm{b}A^{2n}{b_1}^4 np\left[\frac{4n^3p^3}{S^{2np}} - \frac{(16n^3\,p^3+14n^2p^2+6np+1)}{S^{2np+1}} + \frac{(24n^3\,p^3+42n^2p^2+28np+7)}{S^{2np+2}} - \frac{(16n^3\,p^3+42n^2p^2+38np+12)}{S^{2np+3}} + \frac{(4n^3\,p^3+14n^2p^2+16np+6)}{S^{2np+4}}\right] + cA^n{b_2}^2 p\left[\frac{p}{S^p} - \frac{(2p+1)}{S^{p+1}} + \frac{(p+1)}{S^{p+2}}\right] = 0$$ ________________(2.22)

From (2.18) equating the exponents of p and $2np+2$ gives

$2np+2 = p \Rightarrow p = \frac{2}{1-2n}$.

Also equating the exponents of lp and $2np$ gives

$$2np = lp \Rightarrow l = 2n$$

From (2.22) , setting the coefficients of $\frac{1}{S^{2np+j}}$ for $j = 0, 2, 4$ to zero we get

$$-A^l vb_1 l^2p^2 + 4aA^{2n}{b_1}^2 n^2p^2 + 4bA^{2n}{b_1}^4 n^4p^4 = 0$$

But $l = 2n$

$$\Rightarrow -4A^{2n}vb_1n^2p^2 + 4aA^{2n}{b_1}^2 n^2p^2 + 4bA^{2n}{b_1}^4 n^4p^4 = 0$$

$$\Rightarrow -v + ab_1 + b{b_1}^3 n^2p^2 = 0$$

Replace $p = \frac{2}{1-2n}$.

$$\Rightarrow v = ab_1 + 4b{b_1}^3\frac{n^2}{(1-2n)^2}$$ ______________ (2.23)

$$-A^l vlpb_1(lp+1) + aA^{2n}{b_1}^2 2np(2np+1) + \mathrm{b}A^{n+1}{b_1}^4 np(24n^3\,p^3+42n^2p^2+28np+7) + cA^n{b_2}^2p^2 = 0$$

Replace $l = 2n$

$$-A^{2n}v2npb_1(2np+1)+aA^{2n}{b_1}^2 2np(2np+1) + \mathrm{b}A^{2n}{b_1}^4 np(24n^3\,p^3+42n^2p^2+28np+7)+cA{b_2}^2p^2=0$$

$$-v+ab_1+\mathrm{b}{b_1}^3\frac{(24n^3\,p^3+42n^2p^2+28np+7)}{2(2np+1)}+\frac{cA^{1-2n}{b_2}^2p^2}{2n(2np+1)}=0$$

$$v=ab_1+\mathrm{b}{b_1}^3\frac{(24n^3\,p^3+42n^2p^2+28np+7)}{2(2np+1)}+\frac{cA^{1-2n}{b_2}^2p^2}{2n(2np+1)}$$

$$v=ab_1+\mathrm{b}{b_1}^3\frac{(24n^3\,p^3+42n^2p^2+28np+7)}{2(2np+1)}+\frac{cA^{1-2n}{b_2}^2p^2}{2n(2np+1)}$$

____________________ (2.24)

From (2.23) and (2.24)

$$4b{b_1}^3\frac{n^2}{(1-2n)^2}=\mathrm{b}{b_1}^3\frac{(24n^3\,p^3+42n^2p^2+28np+7)}{2(2np+1)}+\frac{cA^{1-2n}{b_2}^2p^2}{2n(2np+1)}$$

Replace $p=\frac{2}{1-2n}$

$$b{b_1}^3\left[\frac{4n^2}{(1-2n)^2}-\frac{(24n^3+28n^2+14n+7)}{2(2n+1)(1-2n)^2}\right]=\frac{cA^{1-2n}{b_2}^2}{n(2n+1)}$$

$$A^{1-2n}=\frac{-b{b_1}^3n(24n^3+28n^2+14n+7)}{2c{b_2}^2(1-2n)^2}$$

$$A=\left[\frac{-b{b_1}^3n(24n^3+28n^2+14n+7)}{2c{b_2}^2(1-2n)^2}\right]^{\frac{1}{1-2n}}$$

____________________ (2.25)

Finally, 1-soliton solution of variant-III is given by

$$q=\frac{A}{(1-e^{-(b_1x+b_2y-vt)})^{\frac{2}{1-2n}}}$$

Where v and A values can be chosen from the equations (2.23) and (2.25) respectively.
Figure 2 i) Profile of variant –III KP solution at $t = 0$
ii)Profile of variant –III KP solution at $t = 5$
iii) Profile of variant –III KP solution at $t = 10$
the parameter values are chosen are $a = b = c = 0.5$, $b_1 = 0.5$, $b_2 = -0.5$

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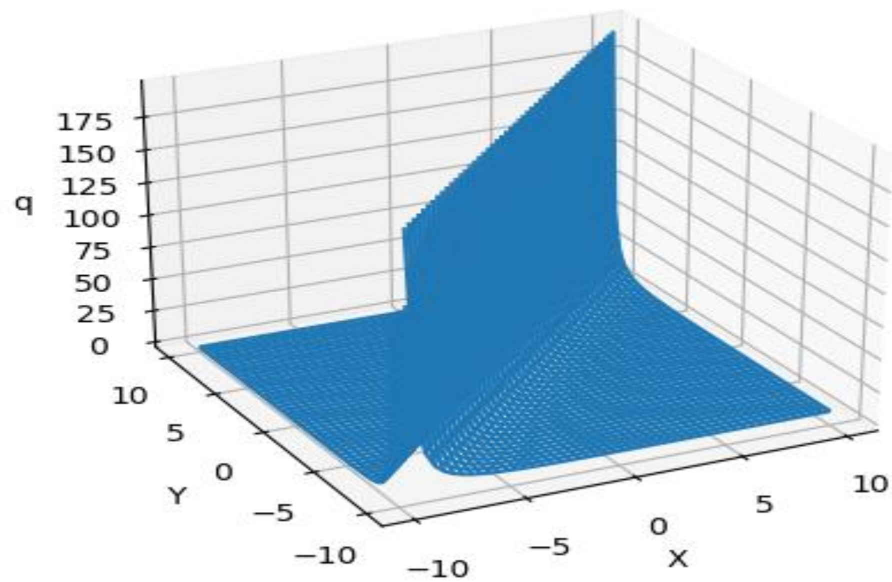
(i)



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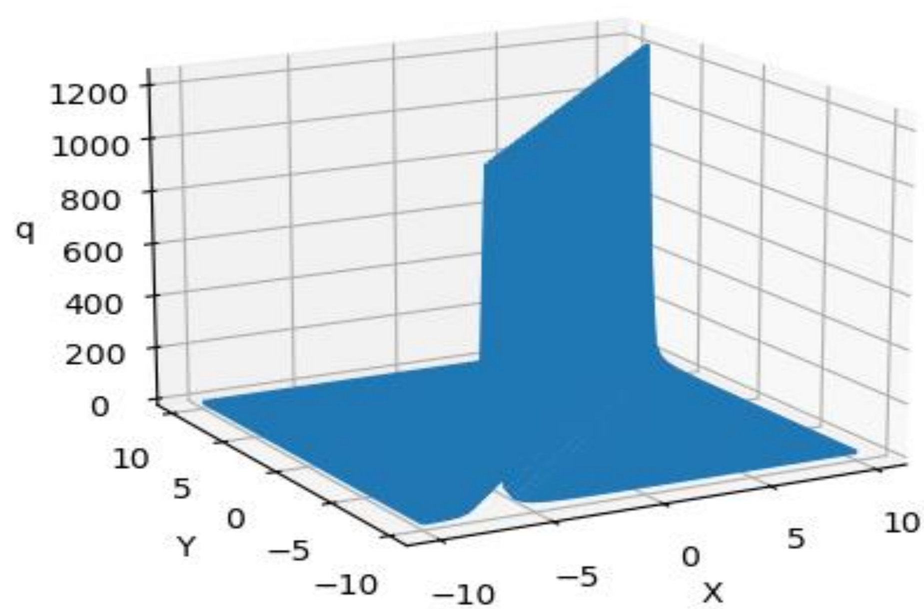

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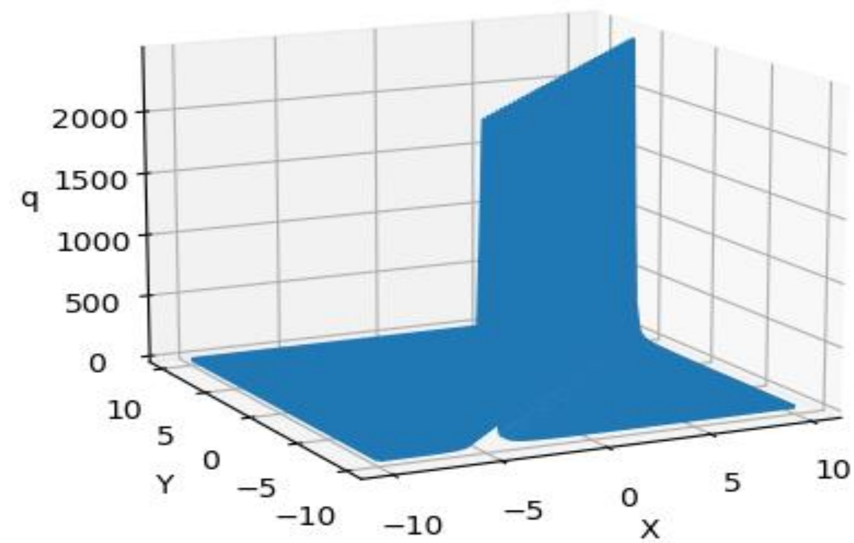
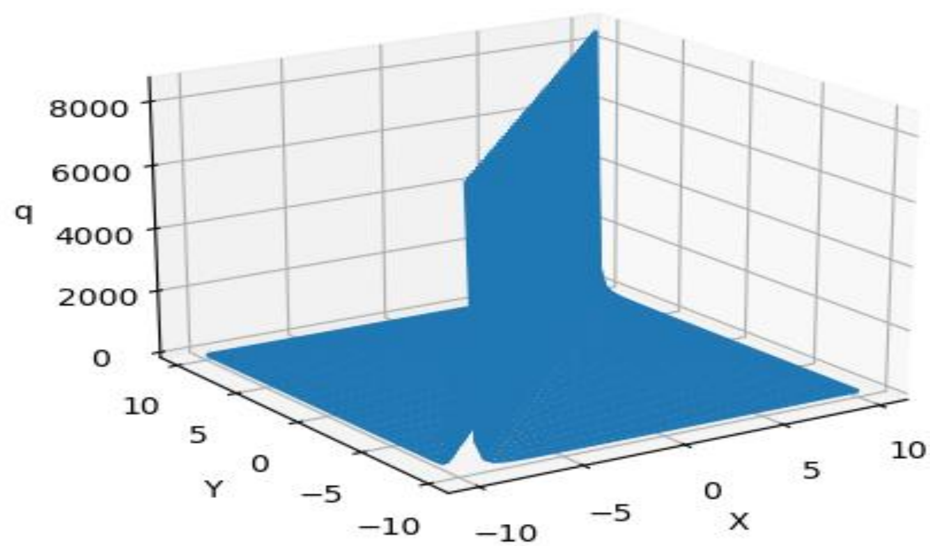


Figure 1 (i) Profile of variant-I KP soliton at $t = 0$, Iii) Profile of variant-I KP soliton at $t = 3$ (iii) Profile of variant-I KP soliton at$t = 5$

(i)



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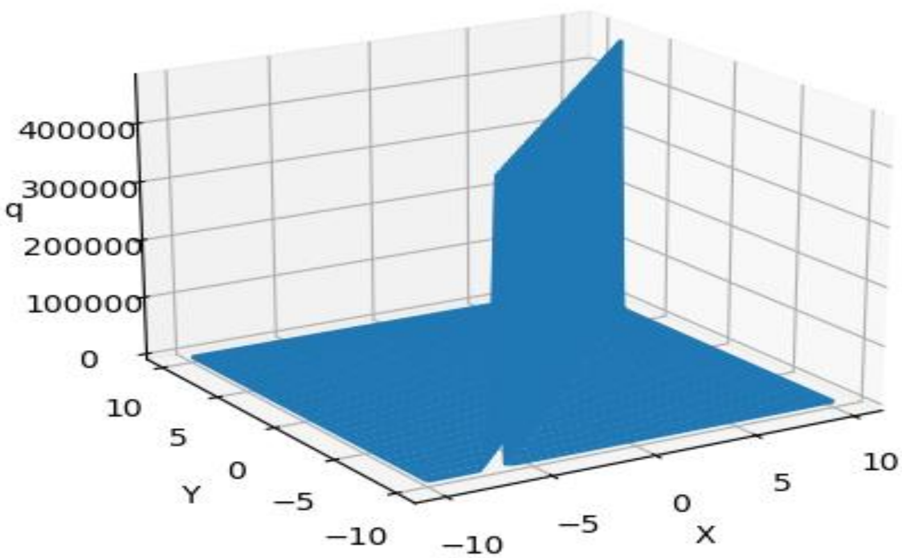

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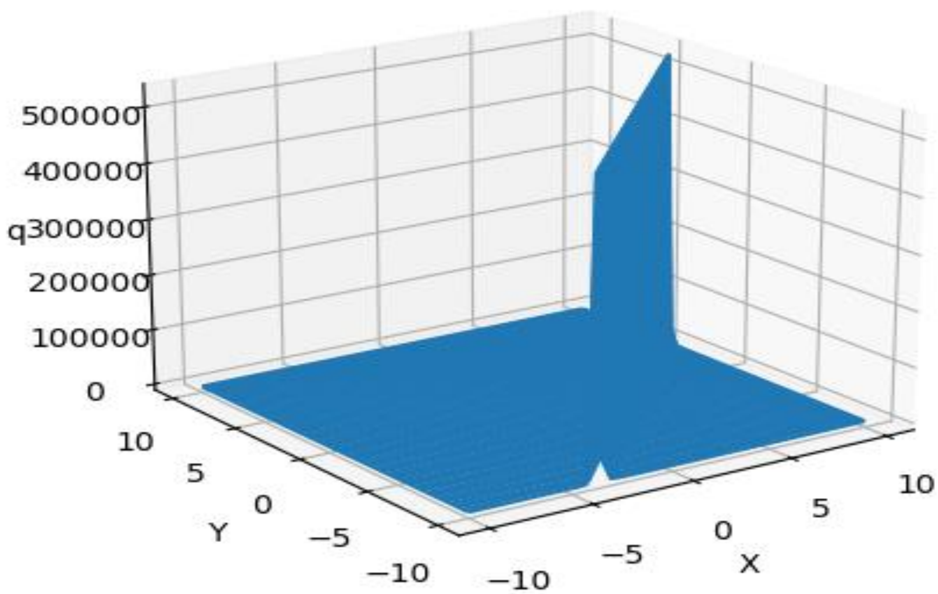

Figure 2 (i) Profile of variant-I KP soliton at $t = 0$, Iii) Profile of variant-I KP soliton at $t = 5$
(iii) Profile of variant-I KP soliton at $t = 10$