Response Of Fractional Order Theories Of Thermoelasticity In a Micropolar Viscothermoelastic Solid

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Abstract
The present work deals with the study of thermoelastic interaction in a micropolar viscoelastic solid usingfractional order theories of thermoelasticityderived by Sherief et.al (2010),Ezzat (2010)and Youssef (2010).A mathematical model has been developed to find a common solution corresponding to different theories under consideration. Laplace and Fourier transform techniques are used to find the general solution in the transform domain. The general solution then applied to a specific problem of a micropolar viscothermoelastic half space, which is subjected to a ramp-type increase in temperature with zero stress, and ramp-type increase in normal load with zero temperature change physical field quantities(displacement,stresses,temperature field) are obtained in transformed domain.These quantities are recovered in the physical domain by using numerical inversion technique and displayed in the form of graphs. Some particular case of interest are also deduced from the present study

Keywords: Micropolar viscothermoelastic,Fractional order derivative, Ramp-type load,integral transformations.

1 Introduction:

The well-known micropolar theory of elasticity was established by Eringen[1] in which the granular character of the medium were taken into consideration .This theory has been used extensively by many researchersin analyzing the problems of high frequency and short wavelength vibrations where, otherwise the classical theory unable to produce accurate results. The deformations in the medium are described by microrotation and micro translation.Eringen[2] and Nowacki [3]introduced the theory of micropolarthermoelasticity by including the thermal effects in the micropolar theory of elasticity.
In the theory of thermoelasticity the classical coupled and uncoupled theory of thermoelasticity presents a paradox of impossible phenomena of infinite velocity of thermal signals and to remove this paradox, the generalized theory of thermoelasticitywas proposed. The generalized theories areable to explain the many problems which involve high heat fluxes, low temperature and instantaneous heat inputs. Many generalization are available to coupled theory of elasticity, one can refer to Ignaczak[4], Chandrasekharaiah[5] and Hetnarski and Ignaczak[6] for extensive review of generalized theories.From these generalized theories, the theory given by Lord and Shulman [7] and Green and Lindsay [8] have been used in large number of investigations.
During recent years, Fractional calculus is being used to develop several interesting modals to study the physical processes particularly in the field of heat conduction, diffusion, viscoelasticity and mechanics

of solids etc. Caputo and Mainardi [9],[10]applied fractional derivatives in the linear theory of viscoelasticity and verified the results experimentally for some metals while Bagley and Torvik [11]established a theoretical basis for the application of fractional calculus to viscoelasticity.Rossikhin andShitikova[12] discussed about the application of fractional calculus to various problems of mechanics of solids. Fractional calculus and its application as well as the historical development may be found in the books by Oldham and Spanier[13] , Miller and Ross [14] and in Podlubny[15]

Fractional calculus hasfound its applications in the various field but investigations in the theory of fractional order thermoelasticity have started quite recently.Povstenko[16] investigated the nonlocal generalization of the Fourier law and heat conduction by using time and space fractional derivatives. Sherief et al. [17]proposed a new model of thermoelasticity using fractional calculus and proved a uniqueness theorem where fractional parameter αlies between 0 and 1 and heat conduction equation is of the form

Youssef[18] introduced a new modal of thermoelasticityby using fractional calculus within fractional parameter range $(0 < \alpha \leq 2)$ covering different cases of conductivity,The interval $(0 < \alpha < 1)$ represented a weak conductivity, $\alpha = 1$a normal conductivity and $(1 < \alpha \leq 2)$ corresponds to strong conductivity.Another new theory of fractional order generalized thermoelasticity using the new Taylor series expansion of time fractional order has been developed by Ezzat [19]where fractional order parameter α lies in interval $0 < \alpha \leq 1$.

Several researches have solved different problem using different fractional order theories of thermoelasticity.Kumar and Gupta [20] studied the reflection and transmission of plane waves at the interface of an elastic half space and a micropolar thermoelastic half space with fractional order derivative by using heat conduction equation derived by Ezzat [19].Kumar et.al [21] studied the plane deformation due to thermal source in a fractional order thermoelastic media.Shaw and Mukhopadhyay [22] discussed the generalized theory of micropolar thermoelasticity with two temperatures using fractional calculus.Deswal and Kalkal[22]discussed the plane waves in a fractional order micropolar magneto- thermoelastic half space by using Sherief et. al [17].Hussein[23]uses Ezzat theory to investigated the fractional order thermoelastic problem for an infinitely long solid circular cylinder by using cylindrical polar coordinates.Sharma and Khator [24] examined some problems of power generation due to renewable sources.Kaushal et al [31]investigated boundary value problem in frequency domain by considering modified Green- Lindsay thermoelastic medium.

In the present article a common analytical expressions for displacement, stress and temperature distribution for three fractional order theories presented by Sherief, Ezzat and Youssef have been derived in micropolar viscothermalelastic half-space.The problem has been solved by using Laplace and Fourier transform techniques and the transformed components of displacement, stress and temperature change are obtained.Theresulting quantitiesare computed numerically for two different boundary conditions. The results obtained are depicted graphically and a comparison is made between the three theories under consideration.Application of this problem are found in the field of geomechanics where interest is in various phenomenon occurring in earthquakes, oil industries and measurements of stresses and temperature distribution due to certain sources

2. Governing Equations

Following Eringen[24] the constitutive relations and equations of motion in a homogeneous, isotropic micropolar viscothermoelastic solid are given by

$$t_{ij} = \lambda^* u_{r,r}\delta_{ij} + \mu^*\left(u_{i,j} + u_{j,i}\right) + k^*\left(u_{j,i} - \epsilon_{ijr}\phi_r\right) - \nu T\delta_{ij} \quad (1)$$

$$m_{ij} = \alpha^*\phi_{r,r}\delta_{ij} + \beta^*\phi_{i,j} + \gamma^*\phi_{j,i} \quad (2)$$

$$(\mu^* + K^*)\nabla^2\vec{u} + (\lambda^* + \mu^*)\nabla(\nabla.\vec{u}) + k^*(\nabla \times \overrightarrow{\phi}) - \nu\nabla \mathrm{T} = \rho\left(\frac{\partial^2\vec{u}}{\partial t^2}\right) \tag{3}$$

$$(\alpha^* + \beta^* + \gamma^*)\,\nabla(\nabla.\vec{\phi}) - \gamma^*\nabla \times (\nabla \times \vec{\phi}) + k^*(\nabla \times \vec{u}) - 2k^*\vec{\phi} = \rho j\left(\frac{\partial^2\vec{\phi}}{\partial t^2}\right) \tag{4}$$

where

$\lambda, \mu, k, \alpha, \beta, \gamma, \lambda^*, \mu^*, K^*, \alpha^*, \beta^*, \gamma^*$arematerial constants and ρ is the density, j is the microinertia $\vec{\phi}$ is the microrotation vector .where $t_{ij}$is the force stress tensor,u_i is the component of the displacement vector,ϕ_i is the component of the mirorotation vector,m_{ij} is the couple stress tensor,ρ is the density,j is the micro inertia, $\in_{ijr}$ is the permutation symbol

Following Sherief [17] , Ezzat [19] , Youssef [18], unified equation of heat conduction is

$$K^*\nabla^2 T = \rho C^*\left(\frac{\partial^{p_1}}{\partial t^{p_1}} + \frac{\tau_0^{p_2}}{p_2!}\frac{\partial^{\alpha_0+1}}{\partial t^{\alpha_0+1}}\right)T + \nu T_0\left(\frac{\partial^{p_1}}{\partial t^{p_1}} + \frac{\tau_0^{p_2}}{p_2!}\frac{\partial^{\alpha_0+1}}{\partial t^{\alpha_0+1}}\right)\nabla.\vec{u} \tag{5}$$

where

$p_1 = 1, p_2 = 1$ for Sherief theory

$p_1 = 1, p_2 = \alpha_0$ for Ezzat theory

$p_1 = \alpha_0, p_2 = 1$ for Youssef theory

$p_1 = 1, p_2 = 1, \alpha_0 = 1$ For Lord and Shulman theory

and K^* is coefficient of thermal conductivity, $\nu = (3\lambda + 2\,\mu + k)\alpha_t$,$\alpha_t$ is the coefficient of thermal linear expansion, $C^*$is the specific heat at constant strain,T is change in temperature of the medium at any time,T_0 is the reference temperature of the body, τ_0 represents the thermal relaxation times.
where fractional order derivative of a functions is taken as per following definition

$$\frac{\partial^{\alpha_0}}{\partial t^{\alpha_0}}F(x,t) = \begin{cases} F(x,t) - F(x,0)\alpha_0 \to 0 \\ I^{1-\alpha_0}\dfrac{\partial}{\partial t}F(x,t) \quad 0 < \alpha_0 < 1 \\ \dfrac{\partial F(x,t)}{\partial t}\alpha_0 = 1 \end{cases}$$

Here,I^{α_0}represntsthe Riemann-Liouville integral operator of fractional order $\alpha_0$given as follows

$$I^{\alpha_0}F(t) = \frac{1}{\Gamma(\alpha_0)}\int_0^t (t-s)^{\alpha_0-1}F(s)ds$$

3. Formulation of the Problem

We consider a homogeneous, isotropic, micropolar thermoelastic half space with fractional order derivative in an undisturbed state at uniform temperature T_0 .The origin of rectangular Cartesian coordinate system (x_1, x_2, x_3) is taken at any point on the plane surface having the surface of half space as the plane $x_3 = 0$ and x_3-axis points vertically downwards into the medium. For the two dimensional problem we assume the components of the displacement $\boldsymbol{u}$ and microrotation vector $\vec{\phi}$ of the form

$$\vec{u} = (u_1, 0, u_3)\ ,\ \vec{\phi} = (0, \phi_2, 0) \tag{6}$$

To facilitate the solution, the following dimensionless quantities are introduces

$x_i^{'} = \frac{\omega^*}{c_1}x_i,\ \ u_i^{'} = \frac{\rho\omega^* c_1}{\nu T_0}u_i\ ,\ \ \phi_2^{'} = \frac{\rho c_1^2}{\nu T_0}\phi_2\ , m_{ij}^{'} = \frac{\omega^*}{c_1\nu T_0}m_{ij}\ , t^{'} = \omega^* t\ ,\ \ \ T^{'} = \frac{T}{T_0}$

$\tau_0^{'} = \omega^*\tau_0\ , t_{ij}^{'} = \frac{t_{ij}}{\nu T_0}\ ,\ \ F_1^{'} = \frac{F_1}{\nu T_0}\ , T_1^{'} = \frac{T_1}{T_0}\ ,$

where$\omega^* = \frac{\rho c^* c_1^2}{K^*},\ c_1^2 = \frac{\lambda+2\mu+K}{\rho}$, (7)

and $\lambda^* = \lambda\left(1 + Q_1\frac{\partial}{\partial t}\right), \mu^* = \mu\left(1 + Q_2\frac{\partial}{\partial t}\right),$

$k^* = k\left(1 + Q_3\frac{\partial}{\partial t}\right), \alpha^* = \alpha\left(1 + Q_4\frac{\partial}{\partial t}\right),$

$\beta^* = \beta\left(1 + Q_5\frac{\partial}{\partial t}\right),\ \gamma^* = \gamma\left(1 + Q_6\frac{\partial}{\partial t}\right),$

where

$Q_i = \omega^*\left(\frac{\lambda_v}{\lambda}, \frac{\mu_v}{\mu}, \frac{k_v}{k}, \frac{\alpha_v}{\alpha}, \frac{\beta_v}{\beta}, \frac{\gamma_v}{\gamma}\right),\ \ (i = 1, \ldots 6)$

The displacement components are related to the potential functions Φ & ψ as

$$u_1 = \frac{\partial\Phi}{\partial x_1} - \frac{\partial\psi}{\partial x_3}\ \ ,\ \ u_3 = \frac{\partial\Phi}{\partial x_3} + \frac{\partial\psi}{\partial x_1} \tag{8}$$

Making use of (8) in the equations (3)-(5), with the aid of (7) and after suppressing the primes we get

$$\left[\left(1 + \delta_0\frac{\partial}{\partial t}\right)\nabla^2 - \frac{\partial^2}{\partial t^2}\right]\Phi - \left(1 + Q_7\frac{\partial}{\partial t}\right)T = 0 \tag{9}$$

$$\left[a_1\left(1 + Q_2\frac{\partial}{\partial t}\right) + a_2\left(1 + Q_3\frac{\partial}{\partial t}\right)\right]\nabla^2\psi + a_2\left(1 + Q_3\frac{\partial}{\partial t}\right)\phi_2 - \frac{\partial^2\psi}{\partial t^2} = 0 \tag{10}$$

$$\left[\left(1 + Q_6\frac{\partial}{\partial t}\right)\nabla^2 - 2a_3\left(1 + Q_3\frac{\partial}{\partial t}\right) - a_4\frac{\partial^2}{\partial t^2}\right]\phi_2 - a_3\left(1 + Q_3\frac{\partial}{\partial t}\right)\nabla^2\psi = 0 \tag{11}$$

$$\nabla^2 T - (\omega^*)^{p_1-1}\left(\frac{\partial^{p_1}}{\partial t^{p_1}} + \frac{\tau_0^{p_2}}{p_2!}(\omega^*)^{\alpha_0+1-(p_1+p_2)}\frac{\partial^{\alpha_0+1}}{\partial t^{\alpha_0+1}}\right)T$$
$$- a_5\left(\frac{\partial^{p_1}}{\partial t^{p_1}} + \frac{\tau_0^{p_2}}{p_2!}(\omega^*)^{\alpha_0+1-(p_1+p_2)}\frac{\partial^{\alpha_0+1}}{\partial t^{\alpha_0+1}}\right)\nabla^2\Phi = 0 \tag{12}$$

where$a_1 = \frac{\mu}{\rho c_1^2}$,$a_2 = \frac{k}{\rho c_1^2}$, $a_3 = \frac{k c_1^2}{\gamma(\omega^*)^2}$,$a_4 = \frac{\rho j\ c_1^2}{\gamma}$, $a_5 = \frac{\nu^2 T_0(\omega^*)^{(p_1-2)}}{\rho K^*}$, $\delta_0 = \frac{\lambda Q_1 + 2\mu Q_2 + k Q_3}{\rho c_1^2}$,
$Q_7 = \frac{(3\lambda Q_1 + 2\mu Q_2 + k Q_3)\alpha_t}{\nu}$

To solve the problem, we define the Laplace and Fourier transform as follows

$\overline{f}(x_1\ , x_3, s) = \int_0^\infty f(x_1\ , x_3, t)e^{-st}dt$ (13)

$\overline{\overline{f}}(\xi\ , x_3, s) = \int_{-\infty}^\infty \overline{f}(x_1\ , x_3\ , s)e^{i\xi x_1}dx_1$ (14)

Applying the Laplace transform define by (13) on (9)-(12), then applying Fourier transform define by (14) on the resulting equations and after some simplification, we obtained the following system of ordinary differential equations

$(D^4 + b_1 D^2 + b_2)\overline{\overline{\Phi}} = 0$(15)

$(D^4 + b_3 D^2 + b_4)\overline{\overline{\psi}} = 0$ (16)

where

$b_1 = -(a_5 l_1 l_4 + l_2 + l_3)$
$b_2 = l_2\, l_3 + a_5 l_1 l_4 \xi^2$
$b_3 = -(l_7 + l_9 - l_8 l_{10})$
$b_4 = l_7 l_9 - l_8 l_{10}\xi^2$
$l_1 = s^{p_1} + \frac{\tau_0^{p_2}}{p_2!}(\omega^*)^{\alpha+1-(p_1+p_2)} s^{\alpha+1}$
$l_2 = \xi^2 + (\omega^*)^{(p_1-1)} l_1$
$l_3 = \xi^2 + s^2/(1+\delta_0 s\,)$
$l_4 = \frac{1+Q_7 s}{1+\delta_0 s}$
$l_5 = a_1(1+\mathrm{Q}_2\mathrm{s}) + \mathrm{a}_2(1+\mathrm{Q}_3\mathrm{s})$
$l_6 = \mathrm{a}_2(1+\mathrm{Q}_3\mathrm{s})$
$l_7 = \xi^2 + s^2/l_5$
$l_8 = l_6/l_5\,,\ l_9 = \xi^2 + \frac{2\mathrm{a}_3(1+\mathrm{Q}_1\mathrm{s})+\mathrm{a}_4\mathrm{s}^2}{1+\mathrm{Q}_6\mathrm{s}}\,,\ l_{10} = \frac{\mathrm{a}_3(1+\mathrm{Q}_3\mathrm{s})}{1+\mathrm{Q}_6\mathrm{s}}$
$D = \frac{d}{dx_3}$(17)

The solution of equations (15) and (16) satisfying the radiation conditions that $\tilde{\tilde{\Phi}}, \tilde{\tilde{\psi}}, \tilde{\tilde{\phi}}_2$ and $\tilde{\tilde{T}} \to 0$ as $x_3 \to \infty$ gives
$\left\{\tilde{\tilde{\Phi}}, \tilde{\tilde{T}}\right\} = \sum_{i=1}^{2}\{1, r_i\} A_i e^{-m_i x_3}$(18)
$\left\{\tilde{\tilde{\psi}}, \tilde{\tilde{\phi}}_2\right\} = \sum_{j=3}^{4}\{1, S_j\} A_j e^{-m_j x_3}$(19)
where
$r_i = (m_i - l_3)/l_4\,,\ \ i = 1,2$
$\mathrm{S}_j = \frac{1}{l_8}(l_7 - m_j^2)\ \ j = 3,4$
where m_1 , m_2 are the roots of the equation (15) and m_3 , $m_4$are the roots of the equation (16)
With the help of equations (18)-(19) and (8) we obtained the displacement component $\tilde{\tilde{u}}_1$ and $\tilde{\tilde{u}}_3$

$$\tilde{\tilde{u}}_1 = -\mathrm{i}\,\xi \mathrm{A}_1 e^{-m_1 x_3} - i\xi A_2 e^{-m_2 x_3} + A_3 m_3 e^{-m_3 x_3} + A_4 m_4 e^{-m_4 x_3} \quad (20)$$

$$\tilde{\tilde{u}}_3 = -\mathrm{m}_1\mathrm{A}_1 e^{-m_1 x_3} - m_2 A_2 e^{-m_2 x_3} - \mathrm{i}\,\xi A_3 e^{-m_3 x_3} - \mathrm{i}\,\xi A_4 e^{-m_4 x_3} \quad (21)$$

4. Boundary Conditions

The surface of the half space is subjected to two different cases of boundary conditions

Case-i

The surface of the half space is assumed to be stress-free and subjected to a ramp-type increase in heating,which depends on the coordinate x_1 and time t, so we have

$$t_{33}(x_1,0,t) = t_{31}(x_1,0,t) = m_{32} = 0\ ,\mathrm{T}(x_1\,,0\,,t\,) = T_1 G(t) F(x_1) \quad (22)$$

where G(t)as defined by Mishra et.al [25]is given by G(t) =

$$\left\{\begin{matrix} 0 & t \le 0 \\ \frac{t}{t_0} & 0 < t \le t_0 \\ 1 & t > t_0 \end{matrix}\right\} \quad (23)$$

where T_1 is a constant temperature and $t_0 \ge 0$ is a fixed moment of time to raise the temperature. This means that the boundary of the half-space which is initially at temperature T_0 is suddenly raised to a temperature equal to the function $T_1 G(t) F(x_1)$and maintained at this temperature then on. $F(x_1)$ is an arbitrary function of x_1 and is consider as

$F(x_1) = H(b - |x_1|)$ (24)

Here b is a constant and H is a Heaviside unit step function.

From the condition (22)we can say that the thermal source acts on a band of width 2b which is centered around the x_1-axis on the surface of the half space $(x_3 = 0)$ and is zero everywhere else.

Applying Laplace and Fourier transform on (22) we get

$\tilde{\bar{t}}_{33}(\xi, 0, s) = 0, \tilde{\bar{t}}_{31}(\xi, 0, s) = 0, \tilde{\bar{m}}_{32} = 0, \tilde{\bar{T}}(\xi, 0, s) = T_1 \bar{G}(s)\hat{F}(\xi)$ (25)

where $\bar{G}(s) = \frac{(1-e^{-st_0})}{t_0 s^2}$ and $\hat{F}(\xi) = \frac{2\sin(b\xi)}{\xi}$

Applying Laplace and Fourier transform on (1)-(2) and with the aid of (7), we get

$$\tilde{\bar{t}}_{33} = -a_6 i\xi\tilde{\bar{u}}_1 + D\tilde{\bar{u}}_3 - \tilde{\bar{T}} \quad (28)$$

$$\tilde{\bar{t}}_{31} = -a_7 i\xi\tilde{\bar{u}}_3 + a_1 D\tilde{\bar{u}}_3 - a_2\tilde{\bar{\phi}}_2 \quad (29)$$

$$\tilde{\bar{m}}_{32} = a_8 D\tilde{\bar{\phi}}_2 \quad (30)$$

and$a_6 = \frac{\lambda}{\rho c_1^2}$, $a_7 = \frac{\mu}{\rho c_1^2}$, $a_8 = \frac{\gamma w^{*2}}{\rho c_1^4}$

Substitute the values of $\tilde{\bar{u}}_1$,$\tilde{\bar{u}}_3$,$\tilde{\bar{T}}$,$\tilde{\bar{\phi}}_2$ from (18)-(21) in the boundary condition (25), 27) and using (28)-(30) we obtained a system of four non homogeneous equations in four unknown and after some simplification, we obtained the components of stresses and temperature change for as

Case-i

$$\tilde{\bar{t}}_{33} = T_1\frac{\bar{G}(s)\hat{F}(\xi)}{\Delta}(d_{11}\Delta_1 e^{-m_1x_3} + d_{12}\Delta_2 e^{-m_2x_3} + d_{13}\Delta_3 e^{-m_3x_3} + d_{14}\Delta_4 e^{-m_4x_3}) \quad (31)$$

$$\tilde{\bar{t}}_{31} = T_1\frac{\bar{G}(s)\hat{F}(\xi)}{\Delta}(d_{21}\Delta_1 e^{-m_1x_3} + d_{22}\Delta_2 e^{-m_2x_3} + d_{23}\Delta_3 e^{-m_3x_3} + d_{24}\Delta_4 e^{-m_4x_3}) \quad (32)$$

$$\tilde{\bar{m}}_{32} = -a_8 T_1\frac{\bar{G}(s)\hat{F}(\xi)}{\Delta}(d_{33}\Delta_3 e^{-m_3x_3} + d_{34}\Delta_4 e^{-m_4x_3}) \quad (33)$$

$$\tilde{\bar{T}} = T_1\frac{\bar{G}(s)\hat{F}(\xi)}{\Delta}(r_1\Delta_1 e^{-m_1x_3} + r_2\Delta_2 e^{-m_2x_3}) \quad (34)$$

where

$\Delta = \begin{vmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ d_{21} & d_{22} & d_{23} & d_{24} \\ d_{31} & d_{32} & d_{33} & d_{34} \\ d_{41} & d_{42} & d_{43} & d_{44} \end{vmatrix}$, where Δ_i , $i = 1,2,3,4$ areobtained from Δ by interchanging i^{th}column by the column $[0, 0,0,1]^T$

Case-ii

$$\tilde{\bar{t}}_{33} = -F_1\frac{\bar{G}(s)\hat{F}(\xi)}{\Delta}\left(d_{11}\Delta_1' e^{-m_1x_3} + d_{12}\Delta_2' e^{-m_2x_3} + d_{13}\Delta_3' e^{-m_3x_3} + d_{14}\Delta_4' e^{-m_4x_3}\right) \quad (35)$$

$$\tilde{\bar{t}}_{31} = -F_1\frac{\bar{G}(s)\hat{F}(\xi)}{\Delta}(d_{21}\Delta_1' e^{-m_1x_3} + d_{22}\Delta_2' e^{-m_2x_3} + d_{23}\Delta_3' e^{-m_3x_3} + d_{24}\Delta_4' e^{-m_4x_3}) \quad (36)$$

$$\tilde{\bar{m}}_{32} = a_8 F_1\frac{\bar{G}(s)\hat{F}(\xi)}{\Delta}(d_{33}\Delta_3' e^{-m_3x_3} + d_{34}\Delta_4' e^{-m_4x_3}) \quad (37)$$

$$\tilde{\bar{T}} = -F_1\frac{\bar{G}(s)\hat{F}(\xi)}{\Delta}(r_1\Delta_1' e^{-m_1x_3} + r_2\Delta_2' e^{-m_2x_3}) \quad (38)$$

where Δ_i', $i = 1,2,3,4$ are obtained from Δ by interchanging i^{th}column by the column $[1, 0,0,0]^T$

and

$$d_{1i} = \ -a_6\xi^2 + m_i^2 - r_i i = 1,2$$
$$d_{1j} = i\xi m_j(-a_6 + 1), \quad j = 3,4$$
$$d_{2p} = \ i\xi m_p(a_7 + a_1), \quad\ p = 1,2$$
$$d_{2q} = \ -\left(a_7\xi^2 + a_1 m_q + a_2 s_q\right), \quad q = 3,4$$
$$d_{31} = 0, d_{32} = 0\ , d_{3l} = s_l m_l, \quad l = 3,4$$
$$d_{4n} = r_n\ , d_{43} = 0\ , d_{44} = 0\ , \ \ n = 1,2$$

5. Particular case:

(i)By putting $p_1 = 1\ , p_2 = 1$ in equations (20)-(21) and (31)-(38), we obtained the expressions for displacement, stresses and temperature for fractional order theory developed by Sherief [17]

(ii) By putting $p_1 = 1\ , p_2 = \alpha_0$ in equations (20)-(21) and (31)-(38), we obtained the expressions for displacement, stresses and temperature for fractional order theory developed byEzzat [19]

(iii) By putting $p_1 = \alpha_0\ , p_2 = 1$ in equations (20)-(21) and (31)-(38), we obtained the expressions for displacement, stresses and temperature for fractional order theory developed by Youssef [18]

(iv)By putting $\alpha_0 = 1$in equations (20)-(21) and (31)-(38), we obtained the expressions for displacement, stresses and temperature for Lord-Shulman theory

6. Inversion of the transforms

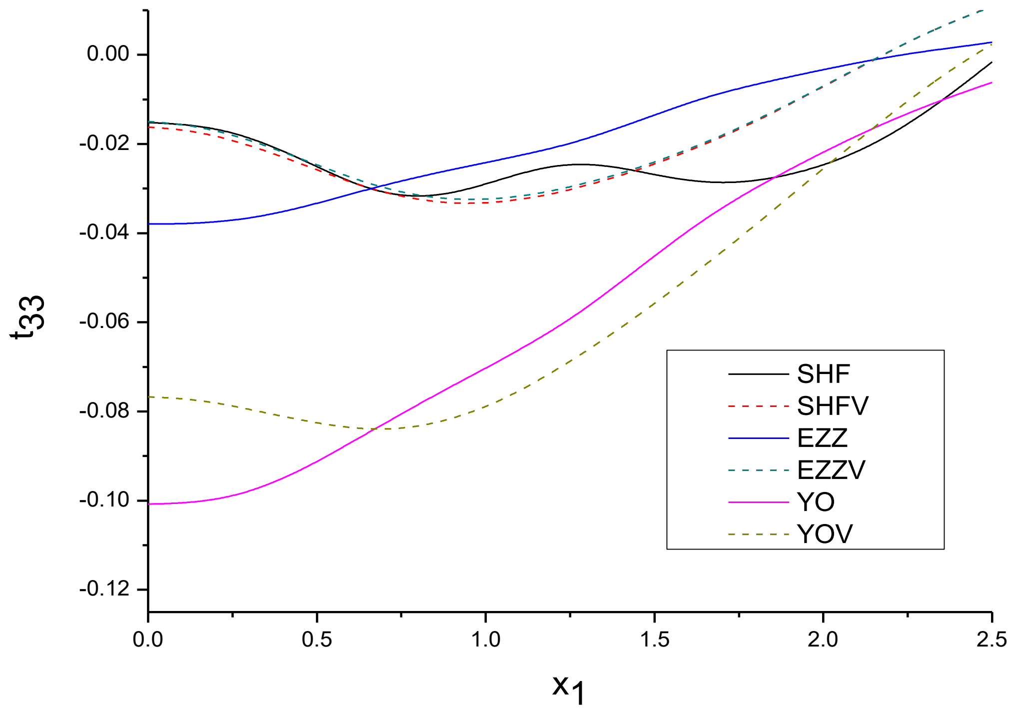
The transformed stresses and temperature distribution are the functions of x_3 and the parameter of Laplace and Fourier transform s and ξ respectively and hence of the form $f(\xi, x_3, s)$ .It is difficult to find the inverse Laplace and Fourier transform due to complicated solutions for displacement, temperature and stresses in Laplace –Fourier domain .So numerical computations have been carried out to find the solution in physical domain, we invert the Laplace and Fourier transforms by using the method described by Youssef [26]

7. Numerical results and discussion

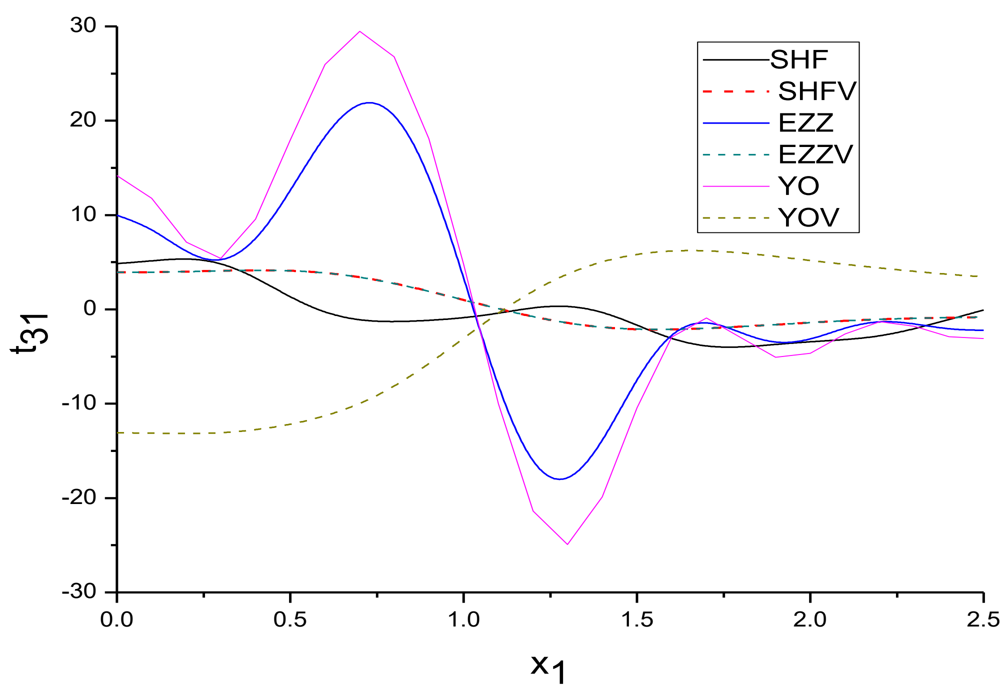
In order to illustrate the contribution of fractional parameter, effect of ramp type increase in load or temperature on the field variables a numerical analysis is carried out, Following Eringen[27], Deswal et.al [28]the physical datafor a magnesium crystal is given below

$\lambda = 9.4\ X10^{11} kgm^{-1}s^{-2}$, $\mu = 4.0X10^{11} kgm^{-1}s^{-2}$, $T_0 = 298K$, $K = 1.0X10^{11} kgm^{-1}s^{-2}$
$j = 0.2X10^{-19} m^2$, $\gamma = 0.779X10^{-9} kgms^{-2}$, $\rho = 1.74X10^3 kgm^{-3}$, $\alpha_t = 2.36X10^{-5} K^{-1}$
$C^* = 9.623X10^2 Jkg^{-1}K^{-1}$, $K^* = 2.510Wm^{-1}K^{-1}$, $\tau_0 = 0.02s$, b = 1

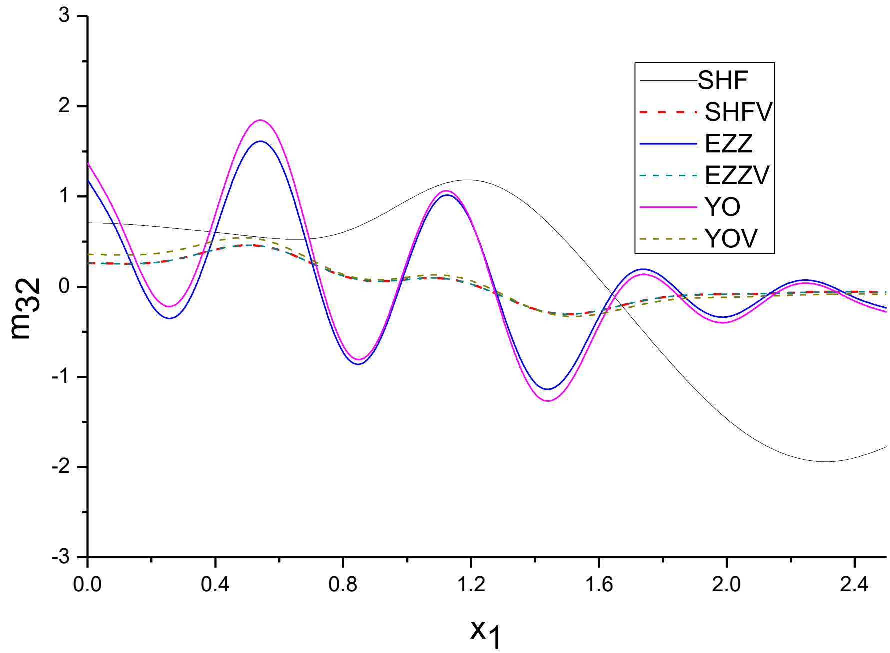
The computations are carried out for a single value of time t = 0.1 for fractional parameter value ($\alpha_0 = 0.75$) on the surface of the plane $x_3 = 1$ in the range$0 \leq x_1 \leq 2.5$.The numerical values of normal stress,tangential stress , tangential couple stress and temperaturechanges on the surface of the plane due to a ramp type heating.We have investigated how the stresses and temperature vary with distance x_1 according to different fractional order theories.



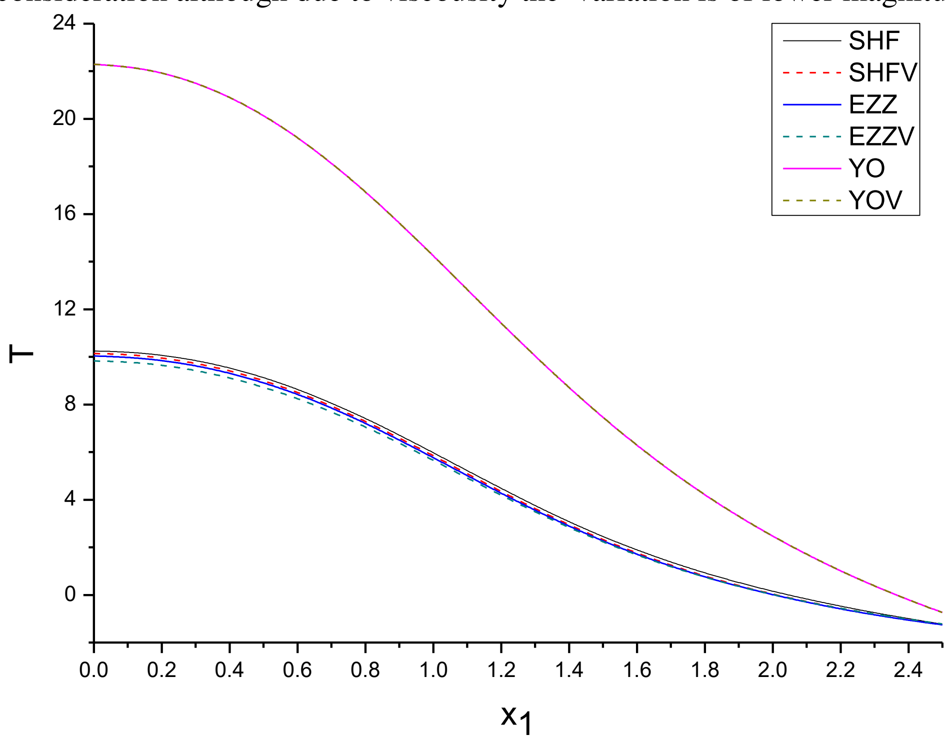
Magnitude of t_{33} remain higher due to viscosity for initial values of x_1 and then behavior and variations of t_{33} remain oscillatory as x_1 advances for all the considered cases.



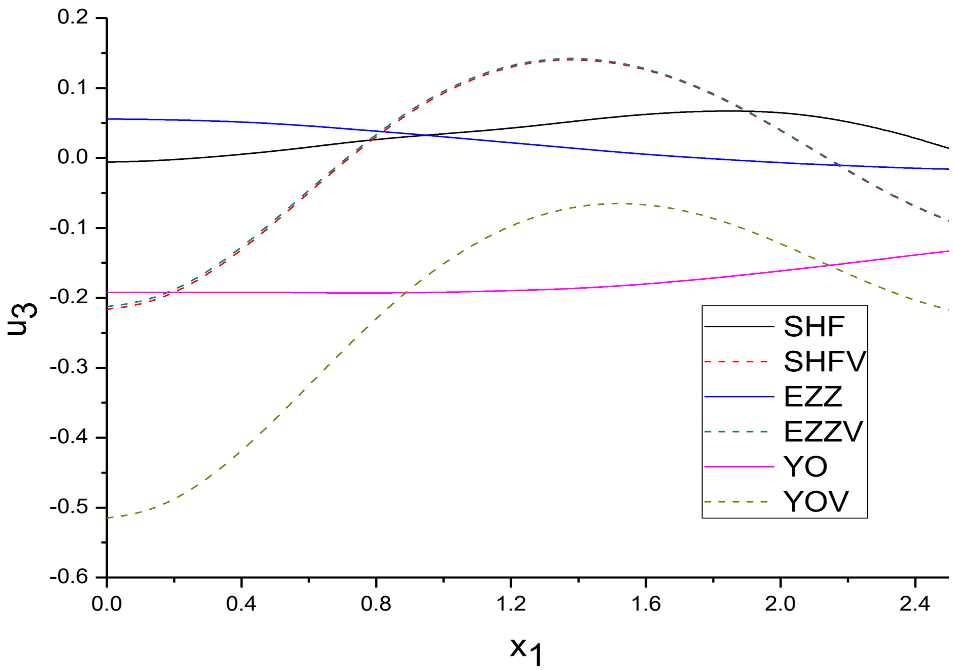
The values of t_{31} due to viscosity remain lower for bounded values of x_1 then fluctuated as x_1 increases further depicting the viscous impact.



Behaviour and variation of m_{32} remain oscillatory for all values of x_1 for all the theories under consideration although due to viscousity the variation is of lower magnitude



The values of T decreases monotonically for all the assumed model and converges faraway from the values of x_1.



Due to viscosity the values of u_3 remains lower for 0≤ x_1 ≤ 0.2.
Due to viscosity$u_3$depict Parabolic variation for SHFV,EZZV& YOV whereas withoutviscosity magnitude of u_3 advances with small variation.

Conclusion: In this paper two-dimensional problem is examined to explain the response of fractional order theories of thermoelasticity in a micropolar viscothermoelastic solid due to thermal source.The governing equations after converting in to two dimensional and then potential function are used to decouple the equations which will give new set of equations. The Laplace and Fourier transform technique is used for further simplification. The transform component of physical field quantities (displacement, stresses, temperature field) are obtained numerical inversion technique is used to explore the impact of viscosity on different fractional order theories of thermos elasticity. From the empirical study following observations are made. Viscosity impact is dominated for all the assumed model on normal stress and tangential stress. Viscosity impact decreases the oscillation of magnitude for tangential couple stress. Predominant impact is observed in Youssef model.

Finally when examining the present problem in relation to the actual behavior specific material properties and geometry of model it gains significance relevance.Geomechanics, seismic, shar dynamics and other field find practical application through the specialized theory of micropolar viscothermoelasticity.

Declaration

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Mamatha Kumari
Literature view, solved the research problem, writing the discussion of the numerical results
Rajneesh

Problem formulation, methodology, editing, software, data evaluation, conclusion, editing.
Rama
Conceptualization, formulation, methodology, supervision, visualization.

All the authors read and agreed to submit for publication

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