Response of Fractional Ordered Magneto-Micropolar Thermoviscoelastic Half-Space Due To Hall Current And Rotation Under Ramp-Type Heating

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Abstract

Purpose-Present work is concerned with the response of magneto- micropolar viscoelastic half space due to the combined effects of hall current and rotation whose surface exposed to a ramp-type heating in the context of fractional order theory of thermoelasticity.

Design/methodology/approach- The fractional generalization of Lord-Shulman theory has been used to investigate the problem. Analytical-numerical technique based upon Laplace and Fourier transform has been employed to solve the resulting non–dimensional coupled field equations.

Findings-Expressions for stresses, displacement and temperature are obtained in the physical domain using numerical inversion technique. Numerical computed results are depicted graphically to illustrate the effects of hall current, rotation, ramp parameter and fractional parameterand comparisons of physical quantities are shown in the presence and absence of viscous nature of the half space.Some particular cases of interest are also deduced from the present study

Originality/Value-

Keywords-Micropolar thermoelasticity, Fractional order derivative, Hall Current, Ramp-Type Heating, Integral transforms.

1. Introduction

Classical theory of elasticity is inadequate to represent the behavior of materials possessing microstructure behavior,for example polycrystalline materials and materials with fibrous or coarse grain structure fall in this category. This theory successfully explains the behavior of material such as aluminium and steel where stresses remains within elastic limits, however discrepancies in classical theory and experimental results pointed out that microstructure might be important in describing the mechanical behavior of a material.Voigt [1]tried to remove these shortcomings of the classical theory by introducing additional couple vector to describe the interaction between two particles in a body, which introduced the concept of couple stresses in elasticity. Cosserat theory of elasticity given by Cosserat and Cosserat[2] was the first to introduced the theory of asymmetric elasticity. Eringen[3] and Eringen and Suhubi[4]used Cosserat medium equations and gave well known Micropolar theory and intended to be applied to materials for which the ordinary classical theory of elasticity fails owing to the microstructure of the material. In this theory, a load across surface element is transmitted not only by a force stress vector but also by a couple stress vector and the motion is characterized by six degrees of freedom, three of translation and three of microrotation.

Mechanical and thermal intercations are of practical importance due to their vast applications in various fields like aeronautics, high energy particle accelerators and nuclear reactors.Eringen [5]and Nowacki[6] incorporatedthermal effects in linear theory of micropolar. Tauchert, Claus Jr and Ariman[7] developed the linear theory of micropolar thermoelasticity and formulate the constitutive equations.Boschi and Ieşan [8] proposed a generalized theory of linear micropolar thermoelasticity that admits the possibility of second sound effect. Ciarletta[9] established the finite speed of thermal waves by using theory of micropolar thermoelasticity without energy dissipation.Sherief, Hamza and EI-Sayed [10] derived the generalized equation for the linear theory of micropolar thermoelasticity.

The magnetic effects in the theory of elasticity was developed with the possibility of extensive practical applications in diverse fields such as geophysics, optics, acoustics, plasma physics etc. It was assumed that both mechanical and electromagnetic fields interactions occur by means of Lorentz forces appearing in the equations of motions and a term appearing in Ohm's law describing the electric field generated by motion of a particle moving in the magnetic field.Kaliski and Nowacki [11]discussed about the wave type of equations inmagneto micropolar thermoelasticity. Othman et al. [12] studied the effects of rotation and modified Ohm's law on magneto-thermoelastic micropolar material withmicrotemperatures.

There are many materials which exhibit viscous effects when subjected to dynamic loading. The mechanics of such materials can be described by linear viscoelasticity theory. The study of generation and propagation of waves in micropolar viscoelastic materials is well- known and have practical applications in the various fields of science and technology such as, seismology, acoustics, aerospace and submarine structures. Eringen [13]constructed linear viscoelasticity theory for micropolar solids and derived the constitutive equations for rate dependent stress, strain, micro rotation and couple stress. McCarthy and Eringen[14] derived the propagation conditions and growth equations which govern the wave propagation in micropolarviscoelastic solid. Cicco and Nappa[15]discussed Saint-Venant's principle for micropolar viscoelastic solid and established a spatial decay estimate of Toupin type in the dynamic linear theory of micropolar viscoelastic material. Kumar and Sharma [16] investigated the propagation of waves in micropolar viscoelastic generalized thermoelastic solid having interficial imperfections Deswal et al. [17] recently studied the interaction in a fractional ordered micropolar thermo-viscoelastic half space with diffusion due to mechanical loading

Fractional calculus has recently been used for modeling severalphysical processes particularly in the field of heat conduction, diffusion, viscoelasticity, control theory and mechanics of solids etc. Fractional derivatives gaining popularity due to its non local properties and global dependency behavior.Abel [18] was the first to use fractional calculus in solving an integral equation that arises in the formulation of the tautochrone problem. The applications of fractional calculus to viscoelasticity was established by Caputo and Mainardi[19] andBagley and Torvik [20]. The mathematical aspect of theory of fractional calculus and its applications may be found in the books by Oldham and Spanier [21],Miller and Ross [22] and Podlubny [23]. The review article ofRossikhin and Shitikova [24] discussed various applications of fractional calculus to problems connected with vibrations and waves in solid with hereditarily elastic properties.Thermoelasticity is another field where the fractional calculus has been applied to explain various cases of heat conduction. Povstenko[25]constructed the nonlocal generalization of the Fourier law and heat conduction by using time and space fractional derivatives.Sherief et al. [26] introduced a new model of thermoelasticity using fractional calculus and proved a uniqueness theorem where fractional parameter ranges between 0 and 1.Ezzat [27] constructed a new model of the magneto-thermoelasticity theory in the context of new consideration of heat conduction with fractional order derivative.Youssef [28]developed another model of the generalized theory of thermoelasticity using fractional calculus in which different cases of conductivity were defined corresponding to different values of the fractional parameter.Shaw and Mukhopadhyay [29] studied the generalized theory of micropolar thermoelasticity with two temperatures using fractional calculus.Kumar and Gupta[30]applied fractional order derivatives to study reflection and transmission of plane waves at the interface of an elastic half space and a micropolar thermoelastic half space. Kumar et al. [31]discussedthe plane deformation due to thermal source in a fractional order thermoelastic media. Deswal et. al[32] studied the effect fractional parameter and diffusion in a micropolar themroelastic half space under ramp type mechanical loading. Sharma and Khator [33] examined some problems of power generation due to renewable sources.

Hall effect is one of the important tool to understand the electronic properties of materials and it is the basis of devices such as position detectors and magnetic field measurements. This effect is due to the interaction of electric and magnetic fields acting on a conducting materials. There is a development of additional potential difference between the opposite surfaces of a conductor due to the hall effect, which induced a current perpendicular to both electric and magnetic field. This current is termed as hall current. Cowling [33]established that if the strength of magnetic field is very strong then hall current cannot be neglected and Ohm's law need to be modified to include hall current. Zakaria[34] discussed the effect of hall current and rotation on magneto micropolar generalized thermoelasticity due to ramp type heating. Kumar et. al [35]discussed the combined effects of hall current and rotation in fractional order magneto micropolar thermoelastic half space under ramp type heating conditions. Othman and Abd-Elaziz [36]recently studied the effects of hall current and rotation on a magneto-thermoelastic half space with microtemperature and void.Kumar and Devi[35] analyzed interaction due to hall current and rotation in modified couple stress elastic half-space subjected to ramp-type loading.Kaushal et al [50] investigated boundary value problem in frequency domain by considering modified Green- Lindsay thermoelastic medium.

Viscoelastic materials are shock absorber used for isolating vibrations, dampening noise and have many other applications. With special emphasis on viscous behavior, present research work studied theeffects of hall current, rotation and fractional order parameter on a micropolar thermo-viscoelastic half spaceunder a strong magnetic field.The transformed components of displacement, stress and temperature distribution are obtainedby using Laplace and Fourier transforms. Viscous behavior is significantly depends upon temperature so a ramp type heating application is employed to get the solution in the complete form.The resulting quantities are computed numerically and depicted graphically. Comparisons are made in the presence and absence of various effects.

2. Basic equations

The governing differential equations for homogeneous, isotropic micropolar thermoelastic viscoelastic solid, when the hall current and rotation effect are taken into accountcan be expressed as :

The equations of motion taking into account the Lorentz forces :

$$(\mu^*+k^*)\nabla^2\vec{u}+(\lambda^*+\mu^*)\nabla(\nabla.\vec{u})+k^*(\nabla\times\vec{\phi})-\nu^*\nabla T+\vec{F} = \rho\left(\frac{\partial^2\vec{u}}{\partial t^2}+\vec{\Omega}\times(\vec{\Omega}\times\vec{u})+2\left(\vec{\Omega}\times\frac{\partial\vec{u}}{\partial t}\right)\right) \quad (1)$$

$$(\alpha^*+\beta^*+\gamma^*)\,\nabla(\nabla.\vec{\phi})-\gamma^*\nabla\times(\nabla\times\vec{\phi})+k^*(\nabla\times\vec{u})-2k^*\vec{\phi} = \rho j\left(\frac{\partial^2\vec{\phi}}{\partial t^2}+\left(\vec{\Omega}\times\frac{\partial\vec{\phi}}{\partial t}\right)\right) \quad (2)$$

FollowingSherief et al. [37], the heat conduction equation:

$$K^*\nabla^2T=\rho C^*\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^{\theta+1}}{\partial t^{\theta+1}}\right)T+\nu^*T_0\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^{\theta+1}}{\partial t^{\theta+1}}\right)\nabla.\vec{u} \quad (3)$$

In the above heat conduction equation, we have taken into consideration the following definition of fractional order derivative

$$\frac{\partial^\theta}{\partial t^\theta}f(x,t)=\begin{cases} f(x,t)-f(x,0)\theta\to 0 \\ I^{1-\theta}\frac{\partial}{\partial t}f(x,t) \quad 0<\theta<1 \\ \frac{\partial f(x,t)}{\partial t}\theta=1 \end{cases}$$

whereI^θ is the Riemann-Liouville fractional integral operator of order θdefined as

$$I^\theta f(t)=\frac{1}{\Gamma(\theta)}\int_0^t(t-s)^{\theta-1}f(s)ds$$

The constitutive relations:

$$\sigma_{ij}=\lambda^*u_{r,r}\delta_{ij}+\mu^*(u_{i,j}+u_{j,i})+k^*(u_{j,i}-\epsilon_{ijr}\phi_r)-\nu^*T\delta_{ij} \quad (4)$$

$$m_{ij}=\alpha^*\phi_{r,r}\delta_{ij}+\beta^*\phi_{i,j}+\gamma^*\phi_{j,i} \quad (5)$$

where

$$\lambda^*=\lambda\left(1+\alpha_1\frac{\partial}{\partial t}\right),\ \mu^*=\mu\left(+\alpha_2\frac{\partial}{\partial t}\right),\ k^*=k\left(1+\alpha_3\frac{\partial}{\partial t}\right),\ \alpha^*=\alpha\left(1+\alpha_4\frac{\partial}{\partial t}\right),\ \beta^*=\beta(1+\alpha_5\frac{\partial}{\partial t}),\gamma^*=\gamma\left(1+\alpha_6\frac{\partial}{\partial t}\right),\nu^*=\nu\left(1+\beta_1\frac{\partial}{\partial t}\right) \quad (6)$$

where $\sigma_{ij}$is the force stress tensor,u_i is the component of the displacement vector,ϕ_i is the component of the mirorotation vector,m_{ij} is the couple stress tensor,ρ is the density,j is the micro inertia, ϵ_{ijr} is the permutation symbol. $\lambda,\mu,k,\alpha,\beta,\gamma,\lambda^*,\mu^*,k^*,\alpha^*,\beta^*,\gamma^*,\nu^*$ arematerial constants. $\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5,\alpha_6,\beta_1$ are viscoelastic relaxation time. K^* is the coefficient of thermal conductivity, $\nu=(3\lambda+2\mu+k)\alpha_t$,$C^*$ is the specific heat at constant strain,α_t is the coefficient of thermal linear expansion, θdenotes thefractional order parameter,T is the change in temperature of the medium at any time,T_0 is the reference temperature of the body, τ_0 is the thermal relaxation times.For the Lord Shulman (L-S) theory $\theta=1$ and $\theta=0$ corresponds to coupled theory of thermoelasticity.

Following Zakaria (2012), the generalized Ohm's law including Hall current

$$\vec{J}=\frac{\sigma_0}{1+m^2}\left(\vec{E}+\mu_0\left(\frac{\partial\vec{u}}{\partial t}\times\vec{H}\right)-\frac{\mu_0}{e\,n_e}\vec{J}\times\vec{H}\right) \quad (6)$$

where$\vec{E}$ is the intensity vector of the electric field, $m=\omega_e t_e$ is the Hall parameter, t_e is the electron collision time,$\omega_e=\frac{e\,B_0}{m_e}$ is the electron frequency,e is the charge of an electron,B_0 is the magnetic induction,m_e mass of an electron,$\sigma_0=\frac{e^2n_et_e}{m_e}$is the electric conductivity and n_e is the electron number density,μ_0 is the magnetic permeability and $\vec{J}$is the conduction current density vector.

3.Formulation of the problem

Consider a homogeneous, isotropic, magneto-micropolar thermoviscoelastic half space with fractional order derivative in an undisturbed state at uniform temperatureT_0.The origin of rectangular cartesian coordinate system (x_1,x_2,x_3) is taken at any point on the plane surface having the surface of half space as the plane $x_3=0$ and x_3-axis points vertically downwards into the medium.

For the two dimensional problem we assume the components of the displacement $\vec{u}$ and microrotation vector $\vec{\phi}$ of the form

$\vec{u} = (u_1, 0, u_3)$, $\vec{\phi} = (0, \phi_2, 0)$ and $\vec{\Omega} = (0, \Omega_0, 0)$

A magnetic field with constant intensity, namely $\vec{H} = (0, H_0, 0)$ acts in the direction of x_2- axis.We also assume that$\vec{E} = \mathbf{0}$, the generalized Ohm's gives $J_2 = 0$ everywhere in the medium where the current density vector components J_1 and J_3 are given by

$$J_1 = \frac{\sigma_0 \mu_0 H_0}{1+m^2}\left(m\frac{\partial u_1}{\partial t} - \frac{\partial u_3}{\partial t}\right)$$
$$J_3 = \frac{\sigma_0 \mu_0 H_0}{1+m^2}\left(\frac{\partial u_1}{\partial t} + m\frac{\partial u_3}{\partial t}\right) \tag{7}$$

To facilitate the solution, the following dimensionless quantities are introduced:

$$x_i^{'} = \frac{\omega^*}{c_1}x_i, \quad u_i^{'} = \frac{\rho\omega^* c_1}{\nu T_0}u_i, \quad \phi_2^{'} = \frac{\rho c_1^2}{\nu T_0}\phi_2, (t^{'}, \tau_0^{'}, \alpha^{'}_1, \alpha^{'}_2, \alpha^{'}_3, \alpha^{'}_4, \alpha^{'}_5, \alpha^{'}_6, \beta^{'}_1) = \omega^*(t, \tau_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \beta_1), \; m_{ij}^{'} = \frac{\omega^*}{c_1 \nu T_0}m_{ij}, \; T^{'} = \frac{T}{T_0} \tag{8}$$
$$t_{ij}^{'} = \frac{t_{ij}}{\nu T_0}, \quad \Omega^{'} = \frac{\Omega}{\omega^*}, \; M = \frac{\sigma_0 \mu_0^2 H_0^2}{\rho\omega^*}$$

where$\omega^* = \frac{\rho c^* c_1^2}{K^*}$, $c_1^2 = \frac{\lambda+2\mu+k}{\rho}$ and M is the Hartmann number or magnetic parameter

The displacement components are related to the potential functionsΦ andψas

$$u_1 = \frac{\partial\Phi}{\partial x_1} - \frac{\partial\psi}{\partial x_3}, \quad u_3 = \frac{\partial\Phi}{\partial x_3} + \frac{\partial\psi}{\partial x_1} \tag{9}$$

Making use of (9) in the equations (1)-(3), with the aid of (8) and after suppressing the primes, we obtain

$$\left((1+\delta_0\frac{\partial}{\partial t})\nabla^2 + \Omega_0^2 - \frac{M}{1+m^2}\frac{\partial}{\partial t} - \frac{\partial^2}{\partial t^2}\right)\Phi - \left(2\Omega_0 + \frac{M}{1+m^2}\right)\frac{\partial\psi}{\partial t} - (1+\beta_1\frac{\partial}{\partial t})T = 0 \tag{10}$$

$$\left\{\left[a_1\left(1+\alpha_2\frac{\partial}{\partial t}\right) + a_2\left(1+\alpha_3\frac{\partial}{\partial t}\right)\right]\nabla^2 + \Omega_0^2 - \frac{M}{1+m^2}\frac{\partial}{\partial t} - \frac{\partial^2}{\partial t^2}\right\}\psi + \left(2\Omega_0 + \frac{M}{1+m^2}\right)\frac{\partial\Phi}{\partial t} + a_2\left(1+\alpha_3\frac{\partial}{\partial t}\right)\phi_2 = 0 \tag{11}$$

$$\left[\left(1+\alpha_6\frac{\partial}{\partial t}\right)\nabla^2 - 2a_3\left(1+\alpha_3\frac{\partial}{\partial t}\right) - a_4\frac{\partial^2}{\partial t^2}\right]\phi_2 - a_3\left(1+\alpha_3\frac{\partial}{\partial t}\right)\nabla^2\psi = 0 \tag{12}$$

$$\left(\nabla^2 - \frac{\partial}{\partial t} + \tau_0(w^*)^{\theta-1}\frac{\partial^{\theta+1}}{\partial t^{\theta+1}}\right)T - a_5\left(\frac{\partial}{\partial t} + \tau_0(w^*)^{\theta-1}\frac{\partial^{\theta+1}}{\partial t^{\theta+1}}\right)\left(1+\beta_1\frac{\partial}{\partial t}\right)\nabla^2\Phi = 0 \tag{13}$$

where $\delta_0 = \frac{\lambda\alpha_1+2\mu\alpha_2+k\alpha_3}{\rho c_1^2}$, $\beta_1 = \frac{(3\lambda\alpha_1+2\mu\alpha_2+k\alpha_3)\alpha_t}{\nu}$, $a_1 = \frac{\mu}{\rho c_1^2}$, $a_2 = \frac{k}{\rho c_1^2}$, $a_3 = \frac{k c_1^2}{\gamma w^{*2}}$, $a_4 = \frac{\rho j c_1^2}{\gamma}$, $a_5 = \frac{\nu^2 T_0}{\rho K^* w^*}$,

To solve the problem, we define the Laplace and Fourier transform as follows

$$\overline{f}(x_1, x_3, s) = \int_0^\infty f(x_1, x_3, t)e^{-st}dt \tag{14}$$

$$\tilde{\overline{f}}(\xi, x_3, s) = \int_{-\infty}^\infty \overline{f}(x_1, x_3, s)e^{i\xi x_1}dx_1 \tag{15}$$

Applying the Laplace transform defined by (14) to both sides of equations (10)-(13), then applying Fourier transform defined by (15) to both sides of the resulting equations and after some simplification, we obtain

$$\{D^8 + AD^6 + BD^4 + CD^2 + F\}(\tilde{\overline{\phi}}, \tilde{\overline{\psi}}, \tilde{\overline{T}}, \tilde{\overline{\phi}}_2) = 0 \tag{16}$$

where

$$A = R_1 + R_3 - R_6 - R_7 - R_8 + a_3 a_5$$

B=$(R_1 - R_7 - R_8)(R_3 - R_6 - a_3 a_5) + R_2 R_4 - R_3 R_6 - R_1 R_7 - a_3 R_5 \xi^2 + R_8 \xi^2$

C=$(R_8\xi^2 - R_1 R_7)(R_3 - R_6 - a_3 a_5) - (R_1 - R_7 - R_8)(R_3 R_6 + a_3 R_5 \xi^2) - R_2 R_4 (R_6 + R_7)$

$D = \frac{d}{dx_3}$, F=$-(R_8\xi^2 - R_1 R_7)(R_3 - R_6 - a_3 a_5) + R_2 R_4 R_6 R_7$

$R_1 = \frac{\Omega_0^2}{1+\delta_0 s} - \xi^2 - \frac{s^2}{1+\delta_0 s} - \frac{Ms}{(1+m^2)(1+\delta_0 s)}$, $R_2 = \frac{(2\Omega_0 + \frac{M}{1+m^2})s}{1+\delta_0 s}$,$R_3 = \frac{1+\beta_1 s}{1+\delta_0 s}$

$R_4 = \frac{1}{a_1(1+\alpha_2 s)+a_2(1+\alpha_3 s)}\left(\Omega_0^2 - s^2 - \frac{Ms}{1+m^2}\right) - \xi^2$, $R_5 = \frac{(2\Omega_0 + \frac{M}{1+m^2})s}{a_1(1+\alpha_2 s)+a_2(1+\alpha_3 s)}$

$R_6 = \frac{a_2(1+\alpha_3 s)}{a_1(1+\alpha_2 s)+a_2(1+\alpha_3 s)}$,$R_7 = \frac{2a_3(1+\alpha_3 s)+a_4 s^2}{1+\alpha_6} + \xi^2$, $R_8 = \frac{a_3(1+\alpha_3 s)}{1+\alpha_6 s}$

$R_9 = \xi^2 + s + \tau_0(\omega^*)^{\theta-1}s^{\theta+1}$, $R_{10} = a_5(s + \tau_0(\omega^*)^{\theta-1}s^{\theta+1})(1+\beta_1 s)$

The solution of equation (16) satisfying the radiation conditions that $\tilde{\overline{\Phi}}, \tilde{\overline{\psi}}, \tilde{\overline{\phi}}_2$ and $\tilde{\overline{T}} \to 0$ as $x_3 \to \infty$ can be written as

$$\begin{cases} \tilde{\tilde{\Phi}} = A_1 e^{-m_1 x_3} + A_2 e^{-m_2 x_3} + A_3 e^{-m_3 x_3} + A_4 e^{-m_4 x_3} \\ \tilde{\tilde{\psi}} = r_1 A_1 e^{-m_1 x_3} + r_2 A_2 e^{-m_2 x_3} + r_3 A_3 e^{-m_3 x_3} + r_4 A_4 e^{-m_4 x_3} \\ \tilde{\tilde{\phi}}_2 = s_1 A_1 e^{-m_1 x_3} + s_2 A_2 e^{-m_2 x_3} + s_3 A_3 e^{-m_3 x_3} + s_4 A_4 e^{-m_4 x_3} \\ \tilde{\tilde{T}} = p_1 A_1 e^{-m_1 x_3} + p_2 A_2 e^{-m_2 x_3} + p_3 A_3 e^{-m_3 x_3} + p_4 A_4 e^{-m_4 x_3} \end{cases} \tag{17}$$

where,

$$r_i = \frac{m_i^4 + (R_1 - R_9 - R_3 R_{10}) m_i^2 + R_3 R_{10} \xi^2 - R_1 R_9}{R_2 m_i^2 - R_2 R_9} \tag{18}$$

$$q_i = -\left(\frac{R_5}{R_6} + \frac{(m_i^2 + R_3)(m_i^4 + (R_1 - R_9 - R_3 R_{10}) m_i^2 + R_3 R_{10} \xi^2 - R_1 R_9)}{R_2 m_i^2 - R_2 R_9} \right) \tag{19}$$

$$p_i = \frac{R_{10}(m_i^2 - \xi^2)}{m_i^2 - R_9} \tag{20}$$

$A_i (i = 1,2,3,4)$are unknown parameter depending s and ξ. These parameters are to be determined from the boundary conditions. m_i $(i = 1,2,3,4)$are the roots of the characteristic equation

$$n^8 + A n^6 + B n^4 + C n^2 + F = 0 \tag{21}$$

With the help of equations (9) and (17) we obtained the displacement component $\tilde{\tilde{u}}_1$ and $\tilde{\tilde{u}}_3$

$$\begin{aligned} \tilde{\tilde{u}}_1 = {} & (-\iota\xi + m_1 r_1) A_1 e^{-m_1 x_3} + (-\iota\xi + m_2 r_2) A_2 e^{-m_2 x_3} + (-\iota\xi + m_3 r_3) A_3 e^{-m_3 x_3} \\ & + (-\iota\xi + m_4 r_4) A_4 e^{-m_4 x_3} \end{aligned} \tag{22}$$

$$\begin{aligned} \tilde{\tilde{u}}_3 = {} & (-m_1 - \iota\xi r_1) A_1 e^{-m_1 x_3} + (-m_2 - \iota\xi r_2) A_2 e^{-m_2 x_3} + (-m_3 - \iota\xi r_3) A_3 e^{-m_3 x_3} \\ & + (-m_4 - \iota\xi r_4) A_4 e^{-m_4 x_3} \end{aligned} \tag{23}$$

4.Applications

To determine the response of half space, following boundary conditions has been applied on free surface $x_3 = 0$

(i) Thermal boundary condition

The boundary of the half space is subjected to different types of thermal sources, which depends on the coordinate x_1 and the time t of the form

$$T(x_1, 0, t) = F(x_1)\delta(t) \tag{24}$$

Applying Laplace and Fourier transform on (24) we get

$$\tilde{\tilde{T}}(\xi, 0, s) = \tilde{\tilde{F}}(\xi) \tag{25}$$

(a) Concentrated thermal point source

In this case $F(x_1) = \delta(x_1)$ with $\tilde{\tilde{F}}(\xi)$=1

(b) Uniformly distributed thermal source

$$F(x_1) = \begin{cases} 1 \; if \; |x| \le a \\ 0 \; if \; |x| > a \end{cases}$$

with $\tilde{\tilde{F}}(\xi) = \frac{2\sin(\xi a)}{\xi}$, $\xi \neq 0$

(b) Linearly distributed thermal source

$$F(x_1) = \begin{cases} 1 - \dfrac{|x|}{a} \; if \; |x| \le a \\ 0 \; if \; |x| > a \end{cases}$$

with $\tilde{\tilde{F}}(\xi) = \frac{2[1 - \cos(\xi a)]}{\xi^2 a}$, $\xi \neq 0$

(ii) Mechanical boundary conditions.

Under the assumption that the surface of the half space is traction free, we have

$$\sigma_{33}(x_1, 0, t) = \sigma_{31}(x_1, 0, t) = m_{32} = 0 \tag{26}$$

Applying Laplace and Fourier transform on (26), we obtain

$$\tilde{\tilde{\sigma}}_{33}(\xi, 0, s) = 0, \quad \tilde{\tilde{\sigma}}_{31}(\xi, 0, s) = 0 \quad , \tilde{\tilde{m}}_{32} = 0 \tag{27}$$

Using equations (4)-(5), equation (8) and equations (14)-(15), the nonvanishing stress components in transform domain can be written as

$$\tilde{\tilde{\sigma}}_{33} = -a_6 i\xi(1 + \alpha_1 s)\tilde{\tilde{u}}_1 + (1 + \delta_0 s) D\tilde{\tilde{u}}_3 - \tilde{\tilde{T}} \tag{28}$$

$$\tilde{\tilde{\sigma}}_{31} = -a_1(1 + \alpha_2 s) i\xi\tilde{\tilde{u}}_3 + [a_1(1 + \alpha_2 s) + a_2(1 + \alpha_3 s)] D\tilde{\tilde{u}}_3 - a_2(1 + \alpha_3 s)\tilde{\tilde{\phi}}_2 \tag{29}$$

$$\tilde{\tilde{\Box}}_{32} = \Box_7(1 + \Box_6\Box)\Box\tilde{\tilde{\Box}}_2 \tag{30}$$

where$\Box_6 = \frac{\Box}{\Box\Box_1^2}$, $\Box_7 = \frac{\Box\Box^{*2}}{\Box\Box_1^4}$

Substitute the values of $\tilde{\tilde{\Box}}_1, \tilde{\tilde{\Box}}_3, \tilde{\tilde{\Box}}, \tilde{\tilde{\Box}}_2$ from (17)in the boundary condition (25), (27) and using (28)–(30),provide us a system of four non homogeneous equations in four unknown and after simplification we obtained the components of displacement, stresses and temperature field:

$$\tilde{\tilde{\square}}_{1} = \frac{1}{\Delta}[(-\iota\xi + m_{1}\square_{1})\Delta_{1}\square^{-\square_{1}\square_{3}} + (-\iota\xi + m_{2}\square_{2})\Delta_{2}\square^{-\square_{2}\square_{3}} + (-\iota\xi + m_{3}\square_{3})\Delta_{3}\square^{-\square_{3}\square_{3}} + (-\iota\xi + m_{4}\square_{4})\Delta_{4}\square^{-\square_{4}\square_{3}} \quad (31)$$

$$\tilde{\tilde{\square}}_{3} = \frac{1}{\Delta}[(-m_{1} - \iota\xi\square_{1})\Delta_{1}\square^{-\square_{1}\square_{3}} + (-m_{2} - \iota\xi\square_{2})\Delta_{2}\square^{-\square_{2}\square_{3}} + (-m_{3} - \iota\xi\square_{3})\Delta_{3}\square^{-\square_{3}\square_{3}} + (-m_{4} - \iota\xi\square_{4})\Delta_{4}\square^{-\square_{4}\square_{3}} \quad (32)$$

$$\tilde{\tilde{\square}}_{33} = \frac{\tilde{\tilde{\square}}(\square)}{\Delta}(\square_{11}\Delta_{1}\square^{-\square_{1}\square_{3}} + \square_{12}\Delta_{2}\square^{-\square_{2}\square_{3}} + \square_{13}\Delta_{3}\square^{-\square_{3}\square_{3}} + \square_{14}\Delta_{4}\square^{-\square_{4}\square_{3}}) \quad (33)$$

$$\tilde{\tilde{\square}}_{31} = \frac{\tilde{\tilde{\square}}(\square)}{\Delta}(\square_{21}\Delta_{1}\square^{-\square_{1}\square_{3}} + \square_{22}\Delta_{2}\square^{-\square_{2}\square_{3}} + \square_{23}\Delta_{3}\square^{-\square_{3}\square_{3}} + \square_{24}\Delta_{4}\square^{-\square_{4}\square_{3}}) \quad (34)$$

$$\tilde{\tilde{\square}}_{32} = \frac{\tilde{\tilde{\square}}(\square)}{\Delta}(\square_{31}\Delta_{1}\square^{-\square_{1}\square_{3}} + \square_{32}\Delta_{2}\square^{-\square_{2}\square_{3}} + \square_{33}\Delta_{3}\square^{-\square_{3}\square_{3}} + \square_{34}\Delta_{4}\square^{-\square_{4}\square_{3}}) \quad (35)$$

$$\tilde{\tilde{\square}} = \frac{\tilde{\tilde{\square}}(\square)}{\Delta}(\square_{41}\Delta_{1}\square^{-\square_{1}\square_{3}} + \square_{42}\Delta_{2}\square^{-\square_{2}\square_{3}} + \square_{43}\Delta_{3}\square^{-\square_{3}\square_{3}} + \square_{44}\Delta_{4}\square^{-\square_{4}\square_{3}}) \quad (36)$$

where

$$\square_{1\square} = -\square_{6}\square^{2}(1 + \square_{1}\square) - \iota\xi a_{6}(1 + \square_{1}\square)\square_{\square}\square_{\square} + (1 + \square_{0}\square)(\square_{\square}^{2} + \iota\xi\square_{\square}\square_{\square}) - \square_{\square}$$

$$\square_{2\square} = \square_{1}(\iota\xi\square_{\square} - \square^{2}\square_{\square})(1 + \square_{2}\square) - (\iota\xi\square_{\square} - \square_{\square}^{2}\square_{\square})[\square_{1}(1 + \square_{2}\square) + \square_{2}(1 + \square_{3}\square)] - \square_{2}(1 + \square_{3}\square)\square_{\square}$$

$$\square_{3\square} = -\square_{8}\square_{\square}\square_{\square}(1 + \square_{6}\square)$$

$\square_{4\square} = \square_{\square}$ where, $(\square = 1,2,3,4)$

and

$$\Delta = \begin{vmatrix} \square_{11} & \square_{12} & \square_{13} & \square_{14} \\ \square_{21} & \square_{22} & \square_{23} & \square_{24} \\ \square_{31} & \square_{32} & \square_{33} & \square_{34} \\ \square_{41} & \square_{42} & \square_{43} & \square_{44} \end{vmatrix}$$

and $\Delta_{\square}$, $\square = 1,2,3,4$ is obtain from Δ by interchanging $\square^{\square h}$column by the column $[0, 0, 0, 1]^{\square}$

5.Particular cases

(i) **Response without viscous effect:** Viscous effect can be removed by settingviscoelastic relaxation time, $\square_{\square} = 0, (\square = 1 - 6)$in the equation (6), which implies that $\square^{*} = \square$, $\square^{*} = \square$, $\square^{*} = \square$, $\square^{*} = \square$, $\square^{*} = \square$, $\square^{*} = \square$, $\square^{*} = \square$.Considering these modifications, we can obtained the expressions for components of displacement, stresses and temperature field in a fractional order magneto-micropolar thermoelastic half space with hall current and rotation effects. The expressions obtained are good in agreement with the previous results obtained by Kumar et al. [35]

(ii) **Response without fractional effect:**The substitution $\square = 1$ in all the expressions involving fractional parameter $\square$will give the results for micropolar generalized thermoviscoelastic half space.The corresponding expressions for all physical fields can be procured from the equations(31)-(36)

(iii) **Response without Hall Current effect**: To neglect the hall current effects substitute $\square = 0$, we obtain the components of stresses,displacement and temperature distribution in a rotating fractional ordered magneto micropolar viscothermoelastichalf space without Hall current effect from the equations (31)-(36)

(iv) **Response without Rotation**: If $\Omega_{0} = 0$ in equations (31)-(36), we obtain the componentsof stresses displacement and temperature distribution in a fractional order micropolar magneto thermoelastic with one relaxation time with following changed values of $\square_{1}$,$\square_{2}$, $\square_{4}$ and $\square_{5}$

$$\square_{1} = -\left(\square^{2} + \frac{\square^{2}}{1 + \square_{0}\square} + \frac{\square\square}{(1 + \square^{2})(1 + \square_{0}\square)}\right), \square_{2} = \frac{\square\square}{(1 + \square_{0}\square)(1 + \square^{2})}, \square_{4} = \frac{-\left(\square^{2} + \frac{\square\square}{1 + \square^{2}}\right) - \square^{2}}{\square_{1}(1 + \square_{2}\square) + \square_{2}(1 + \square_{3}\square)}$$

$$\square_{5} = \frac{\left(\frac{\square}{1 + \square^{2}}\right)\square}{\square_{1}(1 + \square_{2}\square) + \square_{2}(1 + \square_{3}\square)}$$

6.Inversion of the transforms

The transformed components of displacements, stresses and temperature distribution are the functions of $\square_{3}$ and the parameter of Laplace and Fourier transform s and $\square$ respectively and hence of the form $(\square, \square_{3}, \square)$. We invert the Laplace and Fourier transforms by using the method described byKumar and Rani [39] to obtain the solution of the problem in the physical domain.

7. Numerical results and discussion

In order to illustrate the impactof various types thermal loads on fractional order magneto micropolar thermoviscoelastic half space under the combined effects hall current, rotation,a numerical analysis is carried out.We take the following values of relevant parameters for the case of a magnesium crystal like material as Eringen [40], Deswal and Kalkal [17]

Parameter	□ (□□□$^{-1}$□$^{-2}$)	□(□□□$^{-1}$□$^{-2}$)	□(□□□$^{-1}$□$^{-2}$)	□(□^{2})	□(□□□□$^{-2}$)
Value	9.4×10^{11}	4.0×10^{11}	1.0×10^{11}	0.2×10^{-19}	0.779×10^{-9}
Parameter	□(□□□$^{-3}$)	□$_□$(□$^{-1}$)	□*(□□□$^{-1}$□$^{-1}$)	□*(□□$^{-1}$□$^{-1}$)	□$_0$(□)
Value	1.74×10^{3}	2.36×10^{-5}	9.623×10^{2}	2.510	0.05
Parameter	□$_0$(□□□^{2}/□□□.□)	□$_0$(Am^{-1})	□$_0$(Hm^{-1})	□$_0$(□)	
Value	9.36×10^{5}	$(10^{7}/4$□$)$	4□$(10)^{-7}$	298	

For a particular model of micropolar viscothermoelastic solid the values of □$_□$ (□ $= 1-6$) areconsider as

$$□_1 = 0.01, □_2 = 0.02, □_3 = 0.01, □_4 = 0.03, □_5 = 0.02, □_6 = 0.01$$

Considering the above data, computations are carried out to evaluate non-dimensional field variables and results are depicted graphically at different position of □$_1$ at □ $= 0.1$, and □$_3 = 1$ .The computations are carried under hall currents effects (m=0.2) and rotation ($\Omega_0 = 0.6$)for □ $= 0.5$.
The solid line (——) in all the figures represents the effects of concentrated thermal loads, dashed lines (- - -) represents the effect of uniformly distributed thermal load and dashed dotted lines (- . - . -) shows the physical quantity under the effect of linearly distributed thermal load. Figure 1 display the effects of three different types of thermal loads on normal stress. As shown in figure the normal stress increases in case of uniformly distributed thermal loads as compared to others. With increase in distance from the point of application of the source all the normal stress curves approaching the same value. Figure 2 shows the oscillatory behavior of tangential stress due to thermal loads of various types. Maximum amplitude is observed in case of concentrated thermal loads as compared to uniformly and linearly distributed thermal loads.
To be completed for 3 -6

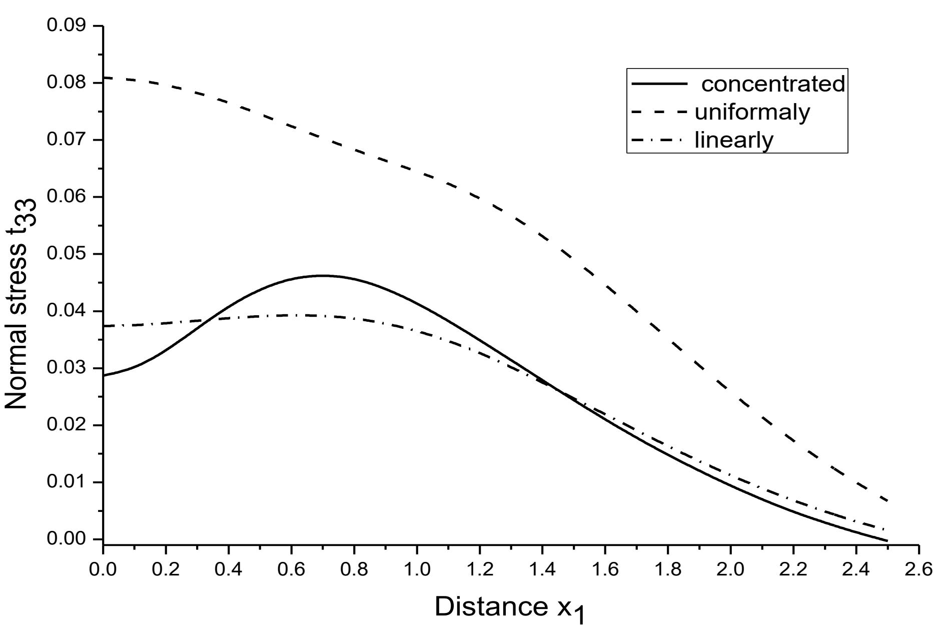

FIGURE 1 EFFECTS OF DIFFERENT TYPES OF THERMAL LOADS ON THE NORMAL STRESS IN A FRACTIONAL ORDER MAGNETO- MICROPOLAR THEMROELASTIC HALF SPACE

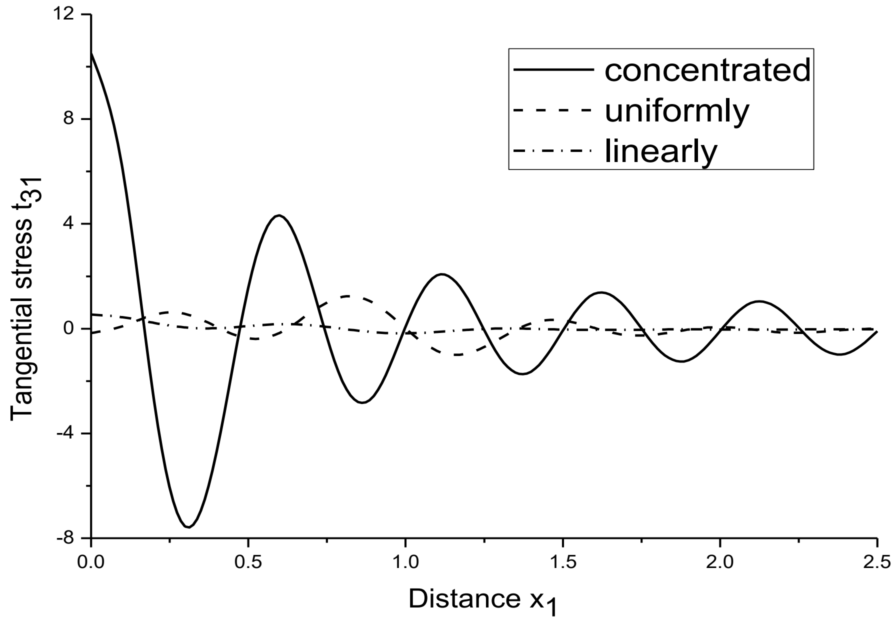

FIGURE 2 EFFECTS OF DIFFERENT TYPES OF THERMAL LOADS ON TANGENTIAL STRESS IN A FRACTIONAL ORDER MAGNETO- MICROPOLAR THEMROELASTIC HALF SPACE

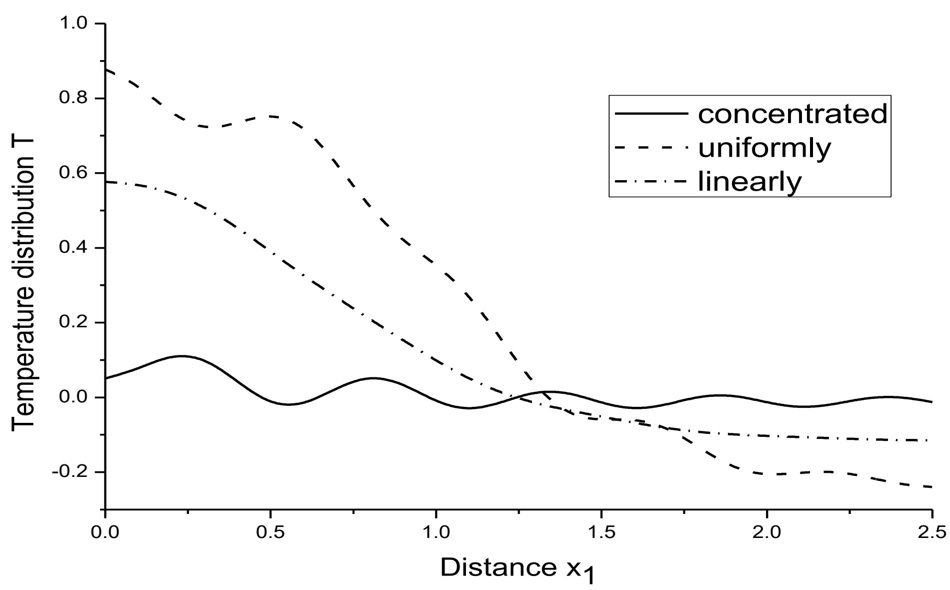

FIGURE 3 EFFECTS OF DIFFERENT TYPES OF THERMAL LOADS ON TEMPERATURE DISTRIBUTION IN A FRACTIONAL ORDER MAGNETO- MICROPOLAR THEMROELASTIC HALF SPACE

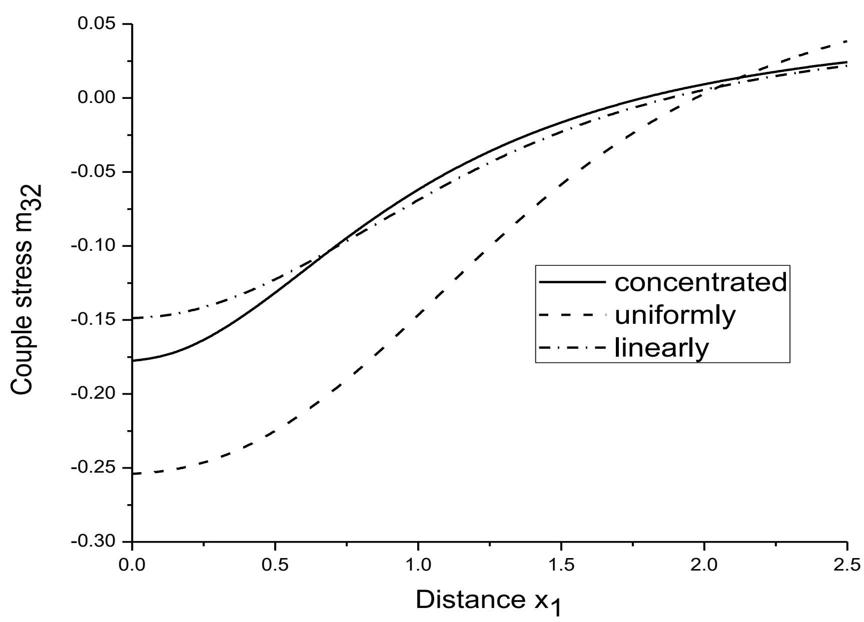

FIGURE 4 EFFECTS OF DIFFERENT TYPES OF THERMAL LOADS ON THE COUPLE STRESS IN A FRACTIONAL ORDER MAGNETO- MICROPOLAR THEMROELASTIC HALF SPACE

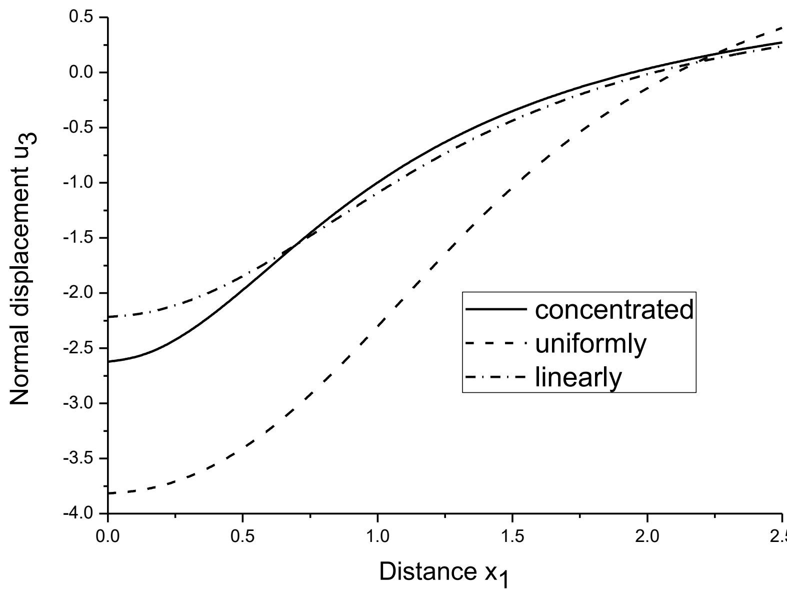

FIGURE 5 EFFECTS OF DIFFERENT TYPES OF THERMAL LOADS ON THE NORMAL DISPLACEMENT IN A FRACTIONAL ORDER MAGNETO- MICROPOLAR THEMROELASTIC HALF SPACE

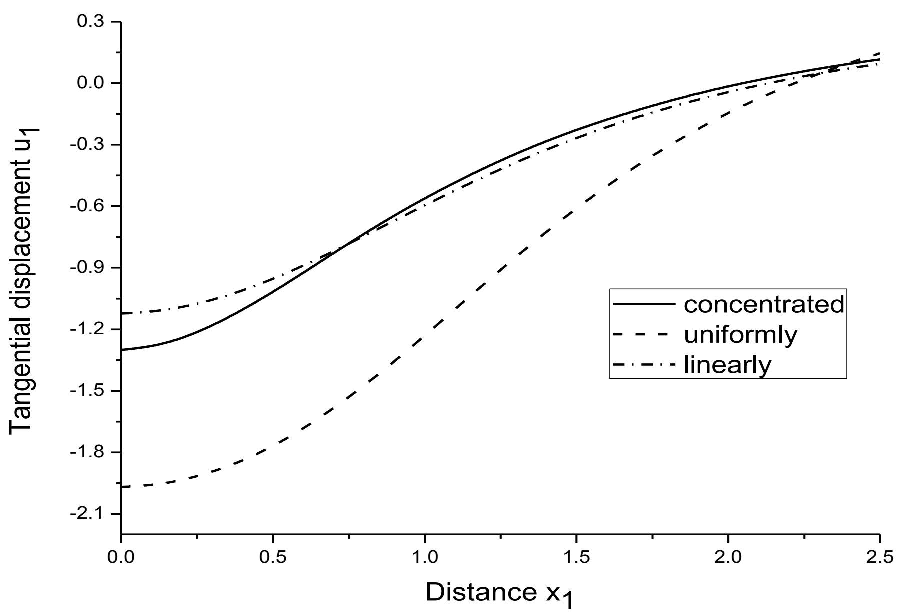

FIGURE 6 EFFECTS OF DIFFERENT TYPES OF THERMAL LOADS ON THE TANGENTIAL DISPLACEMENT IN FRACTIONAL ORDER MAGNETO- MICROPOLAR THEMROELASTIC HALF SPACE

8. Conclusion

In the present paper the interactions in amagneto-micropolar thermo-viscoelastic half space under ramp type heating source in the context of fractional order theory of thermoelasticity has been analyzed. The behavior of non-dimensional temperature field, normal stress, tangential stress and couple stress are examined due to the presence of various effects such as viscosity, hall current, rotation, and fractional order parameter. From the theoretical and numerical analysis, we can conclude the following points.

- The presence of viscosity term has strongly influenced the stresses and temperature field in the half space.The temperature field falls significantly near the point of application of ramp type heating source in case viscous micropolar half space as compared to non-viscous micropolar half spacewhileall the stresses initially are on higher side (in magnitude) in case of non-viscous micropolar half space.
- Hall current parameter has caused a little effect on temperature and normal stress field while tangential and couple stresses are affected significantly.
- Rotation has a salient effect on the all the fields
- Effects of fractional order parameter are quite pertinent on the fields near the point of application of the source. With increases in distance the results of fractional order theory of thermoelastcity are quite in agreements with generalized theory of thermoelasticity
- The Effects of all the considered parameters die out as we move away from the position of sources which is

It is concluded that the interaction in a rotating fractional ordered magneto-micropolar thermoelastic half space due to ramp type heating which induced hall currents is a significant problem of continuum mechanics. The results obtained in this theoretical model will be beneficial for the researcher working in the field of magneto micropolar fractional order thermoelasticity.

Declaration
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Mamatha Kumari
Literature view, solved the research problem, writing the discussion of the numerical results
Rama
Conceptualization, formulation, methodology, supervision, visualization.
Rajneesh
Problem formulation, methodology, editing, software, data evaluation, conclusion, editing.
All the authors read and agreed to submit for publication

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