

SOLUTION OF FULLY Z-TRANSPORTATION PROBLEMS

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Abstract

In this paper, we present an innovative approach to solving the transportation problem with fully Z-number parameters (FZTP). First, we convert the FZTP to an FFTP with a certain reliability. Then the FFTP problem can be tackled by using additive inverse methods. This approach ensures that information regarding reliability is not lost.

Keywords: Z-number, triangular Z-number, trapezoidal Z-number, fuzzy transportation problem, Z- transportations problem.

1. Introduction

The concept of fuzzy sets was introduced by Zadeh [20] in 1965, and it dealt with imprecision and vagueness in real-world situations. A lot of fuzzy transportation problems were investigated by Chiang Kao [5], Chanas et al. [3], Chanas and Kutcha [4], Kaur and Kumar [6,7], Lin [9], Chandran and Kandaswamy [2], Liu and Kao [10], and Li et al. [8]. The Fully Fuzzy Transportation Problem have been investigated by Uma Maheshwari and Ganesan [19], Ambadas Deshmukh et al. [1], Pandian and Natarajan [14], Shanmugasundari & Ganesan [17], Muruganandam and Srinivasan [12], Mohanaselvi and Ganesan [11], Nagoor Gani and others [13].

In our previous paper, we proposed a new approach to solving ZTP [15] and a new approach for solving transportation problem with Z-number parameters [16]. In this paper, we give an innovative approach to solving the TP with fully Z-number parameters (FZTP). FZTP can be considered as a generalisation of fully fuzzy transportation problem. We convert the fully Z-transportation problem (FZTP) to a fully fuzzy transportation problem (FFTP) with a certain reliability. Then by solving the FFTP problem the solution to FZTP is obtained.. The proposed method utilizes R-type operations introduced by Stephen [18].

2. Preliminaries

Definition 2.1 Fuzzy set

A fuzzy set in X is defined as a set of ordered pairs $A = \{(x, \mu_A(x)) | x \in X\}$, where $\mu_A(x)$ is referred to as the membership function for the fuzzy set. If X is a group of objects represented by the generic symbol X, then X is the definition of a fuzzy set A in X. Each element of X is assigned a membership value between 0 and 1 using the membership function.

Definition 2.2 Triangular fuzzy number

A fuzzy number $A = (a_1, a_2, a_3)$ is defined as a triangular fuzzy number only if the membership function of this is represented as:

$$\mu_A(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2} & a_2 \leq x \leq a_3 \\ 0 & x \geq a_3 \end{cases}$$

Definition 2.3 Arithmetic operations on triangular fuzzy number

Let $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$ be any two triangular fuzzy numbers. Then

- (i) $(a_1, a_2, a_3) + (b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$
- (ii) $(a_1, a_2, a_3) - (b_1, b_2, b_3) = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$
- (iii) $(a_1, a_2, a_3) \cdot (b_1, b_2, b_3) = (a_1 \cdot b_1, a_2 \cdot b_2, a_3 \cdot b_3)$

Definition 2.4 Additive inverse operation on triangular fuzzy numbers

Let $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$ be any two triangular fuzzy numbers. We denote this operation by I_+ .

$$(a_1, a_2, a_3) I_+ (b_1, b_2, b_3) = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$$

Definition 2.5 Trapezoidal fuzzy number

A fuzzy number $A = (a_1, a_2, a_3, a_4)$ is a trapezoidal fuzzy number if its membership function is given by

$$\mu_A(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & a_1 \leq x \leq a_2 \\ 1 & a_2 \leq x \leq a_3 \\ \frac{x-a_4}{a_3-a_4} & a_3 \leq x \leq a_4 \\ 0 & x \geq a_4 \end{cases}$$

Definition 2.6 Arithmetic operations on trapezoidal fuzzy number

Let $A = (a_1, a_2, a_3, a_4)$ and $B = (b_1, b_2, b_3, b_4)$ be any two triangular fuzzy numbers. Then

- (i) $(a_1, a_2, a_3, a_4) + (b_1, b_2, b_3, b_4) = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$
- (ii) $(a_1, a_2, a_3, a_4) - (b_1, b_2, b_3, b_4) = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$
- (iii) $(a_1, a_2, a_3, a_4) \cdot (b_1, b_2, b_3, b_4) = (a_1 \cdot b_1, a_2 \cdot b_2, a_3 \cdot b_3, a_4 \cdot b_4)$

Definition 2.7 Additive inverse operation on trapezoidal fuzzy numbers

Let $A = (a_1, a_2, a_3, a_4)$ and $B = (b_1, b_2, b_3, b_4)$ be any two trapezoidal fuzzy numbers. We denote this operation by I_+ .

$$(a_1, a_2, a_3, a_4) I_+ (b_1, b_2, b_3, b_4) = (a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4)$$

Definition 2.8 Zadeh’s definition of z-number

A Z-number is an ordered pair of fuzzy numbers $Z=(A,B)$, associated with the uncertain real-valued variable X , with the first component A , a restriction on the possible values for X , as well as the second component B , a measure of the first component’s reliability.

Definition 2.9 Triangular Z-number

In the Z-number $Z= (A, B)$, if both components A and B are triangular fuzzy numbers, then the corresponding Z-number is called a Triangular Z-number.

Definition 2.10 Trapezoidal Z-number

In the Z-number $Z= (A, B)$, if the two component A and B are trapezoidal fuzzy numbers, then the corresponding Z-number is called a Trapezoidal Z-number.

Definition 2.11 MIN R Type operation

Let $* \in \{+, -, \times, /\}$, the MIN R operation on the set of all continuous Z-number is defined to be

$$(A, B) (*, MIN)(C, D) = (A * C, MIN (B, D)), \text{ where the extension principle is used to calculate } A * C \text{ and } MIN (B, D) = B \text{ if } R_k(B) < R_k(D) \text{ and } D \text{ if } R_k(D) < R_k(B)$$

Definition 2.12 Sum of two Triangular Z-numbers by MIN R

Let $Z_1= (A_1, B_1)$ & $Z_2= (A_2, B_2)$ be any two Triangular Z-numbers, then $Z_1(+, MIN)Z_2 = (A_1, B_1)(+, MIN)(A_2, B_2) = (A_1 + A_2, MIN(B_1, B_2))$

Definition 2.13 Product R Type operation

Let $* \in \{+, -, \times, /\}$, the Product R operation on the set of all continuous Z-number is defined to be $(A, B) (*, \cdot) (C, D) = (A * C, B \cdot D)$, where $A * C$ and $B \cdot D$ are calculated using extension principle

Definition 2.14 Sum of two Triangular Z-numbers by Product R

Let $Z_1 = (A_1, B_1)$ & $Z_2 = (A_2, B_2)$ be any two Triangular Z-numbers, then $Z_1(+, \cdot)Z_2 = (A_1, B_1)(+, \cdot)(A_2, B_2) = (A_1 + A_2, B_1 \cdot B_2)$

3. Fully Z-Transportation Problem (FZTP):

Here we will consider Fully Z-Transportation Problem (FZTP)- a transportation problem where the costs, supply, and demand are Z-numbers.

The FZTP can be mathematically formulated as follows

$$\text{Minimize } Y = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}, \quad i=1,2,\dots,m \quad j=1,2,\dots,n$$

Subject to

$$\sum_{j=1}^n x_{ij} = a_i, \quad i=1,2,\dots,m$$

$$\sum_{i=1}^m x_{ij} = b_j, \quad j=1, 2, \dots, n$$

and x_{ij} are fuzzy numbers with support in $[0, \infty]$

where $c_{ij} = (c_{ij1}, c_{ij2})$, $a_i = (a_{i1}, a_{i2})$ and $b_j = (b_{j1}, b_{j2})$ are all Z-numbers.

The given Z-Transportation Problem is said to be balanced if

$$\sum_{i=1}^m a_{i1} = \sum_{j=1}^n b_{j1}$$

i.e., if the total supply equal to the total demand.

3.1 Converting FZTP to FFTP

In this section, a novel approach to solving the Fully Z-Transportation problem is suggested.

Let us discuss two distinct cases related to the FZTP.

Case (i):

Here the summation is interpreted as a (i_+, \min) operation.

Consider the FZTP

	D₁	D₂	...	D_n	Supply
S₁	(c_{11}, c'_{11})	(c_{12}, c'_{12})	...	(c_{1n}, c'_{1n})	(a_1, a'_1)
S₂	(c_{21}, c'_{21})	(c_{22}, c'_{22})	...	(c_{2n}, c'_{2n})	(a_2, a'_2)
⋮	⋮	⋮	⋮	⋮	⋮
S_m	(c_{m1}, c'_{m1})	(c_{m2}, c'_{m2})	...	(c_{mn}, c'_{mn})	(a_m, a'_m)
Demand	(b_1, b'_1)	(b_2, b'_2)	...	(b_n, b'_n)	

The above FZTP is converted into the following FFTP

	D₁	D₂	...	D_n	Supply
S₁	c_{11}	c_{12}	...	c_{1n}	a_1
S₂	c_{21}	c_{22}	...	c_{2n}	a_2
⋮	⋮	⋮	⋮	⋮	⋮
S_m	c_{m1}	c_{m2}	...	c_{mn}	a_m
Demand	b_1	b_2	...	b_n	

The reliability of this system is

$$\text{Min } (c'_{11}, c'_{12}, \dots, c'_{1n}, c'_{21}, c'_{22}, \dots, c'_{2n}, \dots, c'_{m1}, c'_{m2}, \dots, c'_{mn}, a'_1, a'_2, \dots, a'_m, b'_1, b'_2, \dots, b'_n)$$

Solving the FFTP:

Consider the case where $c_{11}, c_{12}, \dots, c_{1n}, c_{21}, c_{22}, \dots, c_{2n}, \dots, c_{m1}, c_{m2}, \dots, c_{mn}, a_1, a_2, \dots, a_m, b_1, b_2, \dots, b_n$ are all triangular fuzzy numbers.

Then the FFTP can be solved by the additive inverse operation.

Example 3.1:

Consider the fully ZTP, in which the cost matrix, supply, and demand are all triangular Z- numbers in the following table.

	D₁	D₂	D₃	Supply
A	(approx.16, sure)	(approx.20, Very sure)	(approx.12, sure)	(approx.200, completely sure)
B	(approx.14, very sure)	(approx.8, sure)	(approx.18, sure)	(approx.160, very sure)
C	(approx.26, completely sure)	(approx.24, very sure)	(approx.16, very sure)	(approx.90, sure)
Demand	(approx.180, very sure)	(approx.120, sure)	(approx.150, very sure)	

The linguistic terms can be converted to fuzzy terms in the following table:

	D₁	D₂	D₃	Supply
A	((15,16,17), (.75, .8, .85))	((19,20,21), (.85, .9, .95))	((11,12,13), (.75, .8, .85))	((190,200,210), (1,1,1))
B	((12,14,16), (.85, .9, .95))	((7,8,9), (.75, .8, .85))	((17,18,19), (.75, .8, .85))	((150,160,170), (.85, .9, .95))
C	((24,26,28), (1,1,1))	((23,24,25), (.85, .9, .95))	((15,16,17), (.85, .9, .95))	((85,90,95), (.75, .8, .85))
Demand	((175,180,185), (.75, .8, .85))	((110,120,130), (.75, .8, .85))	((140,150,160), (.85, .9, .95))	

Solution:

First, we convert the given FZTP into FFTP

	D₁	D₂	D₃	Supply
A	(15,16,17)	(19,20,21)	(11,12,13)	(190,200,210)
B	(12,14,16)	(7,8,9)	(17,18,19)	(150,160,170)
C	(24,26,28)	(23,24,25)	(15,16,17)	(85,90,95)
Demand	(175,180,185)	(110,120,130)	(140,150,160)	(425,450,475)

The reliability of this FFTP system is Min (very sure, sure, completely sure)

$$= \text{Min } ((.85, .9, .95), (.75, .8, .85), (1,1,1))$$

$$= (.75, .8, .85) = \text{sure}$$

Since $\sum a_{i1} = \sum b_{j1} = (425,450,475)$, the problem is balanced FFTP. Hence, there exists a feasible solution.

The supply at A is (190,200,210) and the demand at D_1 is (175,180,185).

Since the demand is less than the supply. So we allot (175,180,185) to (A, D_1) cell.

Now after this allotment what is remaining origin at A?

$$\begin{aligned} \text{Remaining supply at A} &= (190,200,210) - (175,180,185) \\ &= (190 - 175, 200 - 180, 210 - 185) \\ &= (15,20,25) \end{aligned}$$

Since this is less than the demand (110,120,130) at the destination D_2 . This amount is allotted to (A, D_2) cell.

Now the total allotment from A = (175,180,185) + (15,20,25) = (190,200,210) = total supply at A

Proceeding like this, we get the following initial table by using the north-west corner method.

	D_1	D_2	D_3	Supply
A	(15,16,17) (175,180,185)	(19,20,21) (15,20,25)	(11,12,13)	(190,200,210) (15,20,25)
B	(12,14,16)	(7,8,9) (95,100,105)	(17,18,19) (55,60,65)	(150,160,170) (55,60,65)
C	(24,26,28)	(23,24,25)	(15,16,17) (85,90,95)	(85,90,95)
Demand	(175,180,185)	(110,120,130) (95,100,105)	(140,150,160) (85,90,95)	

Solving the FFTP, we get the following fuzzy optimal solution is

$$\begin{aligned} x_{11} &= (135,140,145); \quad x_{13} = (55,60,65); \quad x_{21} = (40,40,40); \quad x_{22} = (110,120,130); \\ x_{33} &= (85,90,95) \end{aligned}$$

The fully fuzzy transportation cost

$$\begin{aligned} &= [(15,16,17) \times (135,140,145)] + [(11,12,13) \times (55,60,65)] + [(12,14,16) \times (40,40,40)] \\ &\quad + [(7,8,9) \times (110,120,130)] + [(15,16,17) \times (85,90,95)] \\ &= [(2625,2880,3145) + (605,720,845) + (480,560,640) + (770,960,1170) + \\ &\quad (1275,1440,1615)] \\ &= (5755,6560,7415) \end{aligned}$$

The solution for the FZTP is $x_{11} = (135,140,145); x_{13} = (55,60,65); x_{21} = (40,40,40);$

$$x_{22} = (110,120,130); \quad x_{33} = (85,90,95)$$

The Z-transportation cost = ((5755, 6560, 7415), (.75, .8, .85))

Case (ii)

Here the summation is interpreted as a (+, ·) operation.

The reliability of this system is

$$\text{Product } (c'_{11}, c'_{12}, \dots, c'_{1n}, c'_{21}, c'_{22}, \dots, c'_{2n}, \dots, c'_{m1}, c'_{m2}, \dots, c'_{mn}, a'_1, a'_2, \dots, a'_m, b'_1, b'_2, \dots, b'_n)$$

Solving the FFTP:

FFTP can be solved as in Case (i).

Example 3.2:

Consider the ZTP in which the cost matrix, supply and demand are all trapezoidal fuzzy numbers in the following table.

	D₁	D₂	D₃	D₄	Supply
A	(approx.19, sure)	(approx.30, sure)	(approx.50, very sure)	(approx.. 10, almost sure)	(approx.7, sure)
B	(approx.70, sure)	(approx.30, very sure)	(approx.40, sure)	(approx.. 60, completely sure)	(approx.9, very sure)
C	(approx.40, very sure)	(approx.8, completely sure)	(approx.70, sure)	(approx.20, sure)	(approx.18, sure)
Demand	(approx.5, very sure)	(approx.8, sure)	(approx.7, very sure)	(approx. 14, almost sure)	

The linguistic terms can be converted to fuzzy terms in the following table:

	D₁	D₂	D₃	D₄	Supply
A	((17,18,20,21), (.75, .8, .85, .9))	((28,29,31,32), (.75, .8, .85, .9))	((48,49,51,52), (.85, .9, .95, 1))	((7,9,11,13), (.65, .7, .75, .8))	((1,4,10,13), (.75, .8, .85, .9))
B	((68,69,71,72), (.75, .8, .85, .9))	((26,29,31,34), (.85, .9, .95, 1))	((38,39,41,42), (.75, .8, .85, .9))	((56,59,61,64), (1,1,1,1))	((2,6,12,16), (.85, .9, .95))
C	((36,39,41,44), (.85, .9, .95, 1))	((5,7,9,11), (1,1,1,1))	((66,69,71,74), (.75, .8, .85, .9))	((17,19,21,23), (.75, .8, .85, .9))	((13,16,20,23), (.75, .8, .85, .9))
Demand	((1,3,7,9), (.85, .9, .95, 1))	((2,5,11,14), (.75, .8, .85, .9))	((1,5,9,13), (.85, .9, .95, 1))	((12,13,15,16), (.75, .8, .85, .9))	

Solution:

First, we convert the given FZTP into FFTP

	D₁	D₂	D₃	D₄	Supply
A	(17,18,20,21)	(28,29,31,32)	(48,49,51,52)	(7,9,11,13)	(1,4,10,13)
B	(68,69,71,72)	(26,29,31,34)	(38,39,41,42)	(56,59,61,64)	(2,6,12,16)
C	(36,39,41,44)	(5,7,9,11)	(66,69,71,74)	(17,19,21,23)	(13,16,20,23)
Demand	(1,3,7,9)	(2,5,11,14)	(1,5,9,13)	(12,13,15,16)	(16,26,42,52)

The reliability of this FTP system is Product (very sure, sure, completely sure, almost sure)

$$= ((.85, .9, .95, 1) \cdot (.75, .8, .85, .9) \cdot (1,1,1,1), (.65, .7, .75, .8))$$

$$= (0.414375, 0.504, 0.605625, 0.72)$$

Since $\sum a_{i1} = \sum b_{j1} = (16,26,42,52)$, the problem is balanced FFTP. Hence, there exists a feasible solution.

The supply at A is (1,4,10,13) and the demand at D_1 is (1,3,7,9).

Since the demand is less than the supply. So we allot (1,3,7,9) to (A, D_1) cell.

Now after this allotment what is remaining origin at A?

$$\begin{aligned} \text{Remaining supply at A} &= (1,4,10,13) - (1,3,7,9) \\ &= (1 - 1, 4 - 3, 10 - 7, 13 - 9) \\ &= (0,1,3,4) \end{aligned}$$

Since this is less than the demand (2,5,11,14) at the destination D_2 . This amount is allotted to (A, D_2) cell.

Now the total allotment from A = (1,3,7,9) + (0,1,3,4) = (1,4,10,13) = total supply at A

Proceeding like this, we get the following initial table by using the north-west corner method.

	D_1	D_2	D_3	D_4	Supply
A	(17,18,20,21) (1, 3, 7, 9)	(28,29,31,32) (0, 1, 3, 4)	(48,49,51,52)	(7,9,11,13)	(1,4,10,13) (0,1,3,4)
B	(68,69,71,72)	(26,29,31,34) (2, 4, 8, 10)	(38,39,41,42) (0,2,4,6)	(56,59,61,64)	(2,6,12,16) (0,2,4,6)
C	(36,39,41,44)	(5,7,9,11)	(66,69,71,74) (1,3,5,7)	(17,19,21,23) (12, 13, 15, 16)	(13,16,20,23) (12,13,15,16)
Demand	(1,3,7,9)	(2,5,11,14)	(1,5,9,13)	(12,13,15,16)	

Solving the FFTP, we get the following fuzzy optimal solution is

$$x_{11} = (1,3,7,9); x_{14} = (0,1,3,4); x_{22} = (1,1,3,3); x_{23} = (1,5,9,13); x_{32} = (1,4,8,11);$$

$$x_{34} = (12,12,12,12)$$

The fuzzy transportation cost

$$\begin{aligned} &= (17,18,20,21) \times (1,3,7,9) + (7,9,11,13) \times (0,1,3,4) + (26,29,31,34) \times (1,1,3,3) + \\ &\quad (38,39,41,42) \times (1,5,9,13) + (5,7,9,11) \times (1,4,8,11) + (17,19,21,23) \times (12,12,12,12) \\ &= (17,54,140,189) + (0,9,33,52) + (26,29,93,102) + (38,195,369,546) + (5,28,72,121) + \\ &\quad (204,228,252,276) \\ &= (290,543,959,1286) \end{aligned}$$

The solution for the FZTP is $x_{11} = (1,3,7,9); x_{14} = (0,1,3,4); x_{22} = (1,1,3,3); x_{23} = (1,5,9,13);$

$$x_{32} = (1,4,8,11); x_{34} = (12,12,12,12)$$

The Z-transportation cost = ((290, 543, 959, 1286),(0.414375, 0.504,0.605625,0.72))

4. Conclusion

An innovative approach to solving FZTP by using additive inverse methods has been provided in this paper. Here we demonstrate approach using the numerical examples.

Reference

- [1] Ambadas Deshmukh, Ashok Mhaske, Chopade P. U and Bondar K.L, "Fuzzy transportation problem by using trapezoidal fuzzy numbers", International Journal of Research & Analytical Reviews.
- [2] Chandran S and Kandaswamy G (2016), "A fuzzy approach optimization problem", Optimization and Engineering, 17(4), 965-980.
- [3] Chanas S, Kolodziejczyk W and Machay A, "A fuzzy approach to the transportation problem", Fuzzy Sets and System, 13(1984), 211-221.
- [4] Chanas S and Kutcha D, "A concept of the optimal solution of the transportation problem with fuzzy cost coefficient", Fuzzy Sets and System, 82(1996), 299-305.
- [5] Chiang Kao, Shiang-Tai Liu, "Solving fuzzy transportation problems based on extension principle", Journal of physical science, 10(2006), 63-69.
- [6] Kaur A and Kumar K. (2011), "A new method for solving fuzzy transportation problems using ranking function," Applied Mathematical Modelling, 35, 5652-5661.
- [7] Kaur A, Kacprzyk J, Kumar K and Analyst C.F. (2020), "Fuzzy transportation and transhipment problems", Springer International Publishing.
- [8] Li L, Huang Z, Da Q and Hu J (2008), "A new method based on goal programming for solving transportation problem with fuzzy cost", International Symposiums on Information processing, (pp-3-8), IEEE.
- [9] Lin F.T (2009), "Solving the transportation problem with fuzzy coefficients using genetic algorithms", IEEE International Conference on fuzzy systems, (pp. 1468-1473).
- [10] Liu S.T and Kao C (2004), "Solving fuzzy transportation problem based on extension principle", European Journal of Operation Research, 153(3), 661-674.
- [11] Mohanaselvi S and Ganesan K, "Fuzzy optimal solution to fuzzy transportation problem: A new approach", International journal on Computer Science and Engineering.
- [12] Muruganandam S and Srinivasan R, "A new algorithm for solving fuzzy transportation problem with trapezoidal fuzzy numbers," International Journal of Research Trends in Engineering and Research, 2 (3) (2016) pp.428-437.
- [13] Nagoor Gani A, Razak K.A, "Two-stage fuzzy transportation problem", Journal of physical sciences, 10 (2006), 63-69.
- [14] Pandian P & Natarajan G (2010), "A fuzzy new algorithm for finding a fuzzy optimal solution for fuzzy transportation problems", Applied Mathematical Sciences, 4, 79-90.
- [15] Poornima Devi A and Velammal G, "A New Approach for Solving Z Transportation Problems" AIP Conference Proceedings.
- [16] Poornima Devi A and Velammal G, "A New Approach for Solving Transportation Problem with Z-number Parameters", The Advances and Application of Mathematical Science.
- [17] Shanmugasundari M & Ganesan K (2013), "A novel approach for the fuzzy optimal Solution of fuzzy transportation problem," International Journal of Engineering Research and Applications, 3, 1416-1421.
- [18] Stephen S, "Novel Binary Operations on Z-numbers and their Applications in Fuzzy Critical Path Method", Advances in Mathematics: scientific Journal, 9 (2020), 3111-3120.
- [19] Uma Maheswari P and Ganesan K, "A modified method to solve fuzzy transportation problem involving trapezoidal fuzzy numbers", AIP Conf. Proc. 2277.
- [20] Zadeh L.A, "Fuzzy sets," Information and Control, 8 (1965), 338-353.