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POWER DOMINATOR EDGE COLORING OF LINE GRAPH AND DEGREE SPLITTING OF SOME CERTAIN CLASSES OF GRAPHS

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Abstract

A dominator edge coloring of G, is a proper edge coloring of G, in which every edge of some color class (possibly its own class) are dominated by every edge in G. The dominator edge chromatic number of G is the minimum number of distinct color classes required in a dominator edge coloring of G, denoted by$\chi'_d(G)$. We introduced the variate known as power dominator edge coloring based on the notion of power edge domination and determine the power dominator edge chromatic number for certain classes of graph. ***Power Dominator Edge Coloring*** of a graph G, refers to the proper edge coloring where each edge in the edge set power dominates every edge of at least one-color class. The Power Dominator Edge Chromatic number (PDEC-number)$\chi'_{pd}(G)$, is the minimum number of colors that a power dominator edge coloring of the graph requires [6]. Here we determine the PDEC-number for the line graph and for degree splitting of some certain classes of graphs.

Keywords:
Line Graph, Dominator edge coloring, Power dominating set, Power dominator edge coloring, degree splitting.

1. Introduction:

In this paper, we exclusively examine finite, undirected graphs $G(V,E)$ that do not contain parallel edges or self-loops. Here, V represents the set of vertices, and E denotes the set of edges in the graph. In a graph G, two vertices u and v are considered adjacent if there exists an edge directly connecting them. This edge is denoted as (u,v). Two edges that share a common vertex or a vertex along with its incident edge are also considered adjacent. An edge is said to dominate all edges adjacent to it. A path, a cycle, a complete graph, a star, and a wheel graph, each of order n, are represented by P_n, C_n, K_n, $K_{1,n}$, and $W_{1,n}$, respectively.

A subset $S \subseteq V(G)$ with every vertex not in S are adjacent to minimum one vertex in S are known as *dominating set*. The smallest cardinality among such dominating sets is called the domination number of G, denoted by $\gamma(G)$. A *proper coloring* of a graph G is A function $f: V(G) \longrightarrow \{1,\dots,k\}(k \in N)$ with $f(u) \neq f(v)$ for every adjacent vertices u and v is proper coloring of G and the minimum color required for such proper coloring is called the *chromatic number* of G, with the notation $\chi(G)$. A proper coloring of G for which every vertex in $G$is dominated by all vertices of at least one color class is called *dominator coloring* of G. The minimum number of such colors needed is called *dominator chromatic number* of G, denoted by$\chi_d(G)$. [12]

A subset M of $E(G)$ with every edge of $E - M$ are adjacent to a member of M, then the set is called edge dominating set. The number of elements of a minimum edge dominating set will be denoted by $\gamma'(G)$. A *proper edge coloring* of a graph G is a function $c: E(G) \longrightarrow \{1, \ldots, k\} (k \in N)$ with $c(e) \neq c(f)$ for any connected edges e and f. The fewest colors required to properly color the graph is known as *edge chromatic number* of $G$and denoted by $\chi'(G)$.

A *dominator edge (DE) coloring* of a graph G is a proper edge coloring where every edge in G is dominated by at least one edge from some color class, potentially including its own. The *dominator edge chromatic (DEC) number* of G is the smallest number of colors needed among all possible dominator edge colorings of G, which is represented by$\chi'_d(G)$.[10]

A set S qualifies as a power dominating set in a graph G if all vertices in V can be monitored according to these rules:

1. At the start, all vertices in $N[S]$ are considered observed.
2. If an observed vertex u has only one unobserved neighbor v, and all its other neighbors are already observed, then v becomes observed by u.

The minimum number of vertices required to monitor the entire graph under power domination rules is called the power domination number of G and is represented by $\gamma_p(G)$.

A *power dominator coloring* of a graph GGG is a proper vertex coloring in which every vertex has the ability to power dominate all vertices in at least one-color class. The *power dominator chromatic number*, denoted by $\chi_{pd}(G)$, is the smallest number of colors required to achieve such a coloring while maintaining the power domination property. [11]

2. PDEC- Number of a Graph.

Informed by the power dominator coloring concept for graphs, we have merged the two concepts namely, dominator edge coloring and power domination of graphs to introduce a new variation of coloring known as a power dominator edge coloring. This technique mandates that each edge in a color class power dominates every other edge. Prior to this, we revisit the concept of monitoring set for edges in graphs.

Definition 2.1 In a graph G, we correlate a set of monitoring set $M(e)$with an edge e, as follows:
Step: 1 $M(e) = N[e]$; the closed neighbourhood of e
Step: 2 If g is the neighbor of f and $M(e)$ already contains all of g's neighbors except for f, then add edge f to $M(e)$.
Step: 3 Once another edge can no longer be added to $M(e)$, repeat step 2:
The edges of $M(e)$ are therefore said to be Power Dominated by the e.

Definition 2.2

A *Power Dominator Edge Coloring* of a graph G is a proper edge coloring where each edge influences or power dominates all edges within at least one-color class. The *Power Dominator Edge Chromatic Number* (PDEC-number), denoted as $\chi'_{pd}(G)$, represents the minimum number of colors required for such a coloring.

Example: 2.3 A power dominator edge coloring of a graph G in Fig. 2.1 is discussed now.

For the graph G, $\chi'_{pd}(G) = 4$. In fact, color 1 is assigned to the edge e_1 while color 2 can be allocated to the edges e_2, e_5. The edges e_3, e_7can have color 3 and the remaining edge e_6 has the color 4. For the edge e_1; we see the monitoring set, $M(e_1) = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$ = $N[e_1]$. The color class-1 is power dominated by the edges e_1, e_2, e_3. For the color class-2 is power dominated by the edges e_1, e_2, e_3,e_4. And the remaining edges e_5, e_6, e_7 power dominates the color class – 4. Hence every edge of at least one-color class is power dominated by each edge. Hence $\chi'_{pd}(G) = 4$. It is clear that three colors are not enough to get a power dominator edge coloring for this graph.

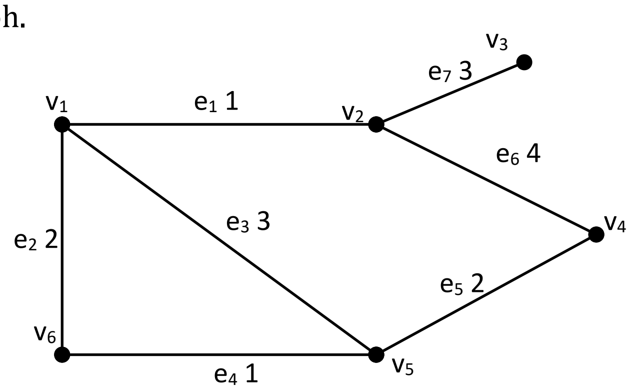


Fig 2.1

$$\chi'_{pd}(G) = 4$$

Lemmas: [1]

2.1 For a Path $P_n, n \geq 3, \chi'_{pd}(P_n) = 2$.

2.2 For a cycle $C_n, n \geq 3, \chi'_{pd}(C_n) = \begin{cases} 2, if\ n\ is\ even \\ 3, if\ n\ is\ odd \end{cases}$

2.3 For the Star graph $K_{1,n}, n \geq 2, \chi'_{pd}(K_{1,n}) = n$.

2.4 For the Wheel graph,$W_{1,n}, n \geq 3, \chi'_{pd}(K_{1,n}) = n$.

2.5 For the Sunlet graph S_n, $\chi'_{pd}(S_n) = \begin{cases} \left\lceil \frac{n}{2} \right\rceil + 3, if\ n\ is\ odd \\ \frac{n}{2} + 2,\ if\ n\ is\ even \end{cases}$

2.6 For the Book Graph B_m, $\chi'_{pd}(B_m) = m + 1, m \geq 3$

2.7 For a Tadpole graph $T_{m,n}, \chi'_{pd}(T_{m,n}) = \begin{cases} 3,\ if\ m\ is\ even\ and\ n \geq 1 \\ \begin{cases} 4,\ if\ m\ is\ odd\ and\ n \geq 2 \\ 3\ if\ m\ is\ odd\ and\ n = 1 \end{cases} \end{cases}$

2.8 For the Centipete graph, $Ct_n, \chi'_{pd}(Ct_n) = \begin{cases} \frac{n}{2} + 2, if\ n\ is\ even \\ \left\lceil \frac{n}{2} \right\rceil + 3, if\ n\ is\ odd \end{cases}, n \geq 4$

3. PDEC- number of Line graphs

The line graph L(G) of an undirected graph G is defined such that each vertex in L(G)corresponds to an edge in G, and two vertices in L(G) are adjacent if and only if their corresponding edges in G share a common endpoint. In this context, We establish the power dominator edge chromatic number for different classes of line graphs, analyzing their structural properties and deriving the minimum number of colors required for a valid power dominator edge coloring.

Theorem: 3.1

(i) For a Path$P_n, n \geq 4, \chi'_{pd}(L(P_n)) = 2$.

(ii) For a cycle $C_n, n \geq 3, \chi'_{pd}(C_n) = \begin{cases} 2, if\ n\ is\ even \\ 3, if\ n\ is\ odd \end{cases}$

Proof:

(i) For the path P_n on n vertices $n \geq 4$, the line graph is the path P_{n-1}. Let the vertices of the cycle C_n be $a_1, a_2, \ldots, a_{n-1}$ and the n edges be $f_1, f_2, \ldots, f_n$, where $f_i = a_i a_{i+1} 1 \leq i \leq n-2$. Clearly each edge $e_i, 1 \leq i \leq n-2$power dominates all the edges of the path P_{n-1}. For proper coloring, assign color 1 toe_i, for odd i and assign color 2 to e_i for even i. Hence each edge power dominates all the edges of both color class and so $\chi'_{pd}(L(P_n)) = \chi'_{pd}(P_n) = 2$

(ii) For the path C_n on n vertices $n \geq 3$, the line graph is the cycle C_n. Let the vertices of the cycleC_n be $a_1, a_2, \ldots, a_n$ and the n edges be $f_1, f_2, \ldots, f_n$, where $f_i = a_i a_{i+1}$, $1 \leq i \leq n-1$ and $f_n = a_n a_1$. Clearly each edge f_i, $1 \leq i \leq n$, power dominates all the edges of the cycle C_n.

<u>Case:1</u>n is even. Assign color 1 to each edge f_i, for odd $i 1 \leq i \leq n-1$and assign color 2 to f_i for even i, $1 \leq i \leq n-1$. this becomes proper edge coloring of C_n. Hence each edge power dominates all the edges of both color class and so $\chi'_{pd}(C_n) = 2$.

<u>Case:2</u>n is odd. Assign color 1 to each edge f_i, for odd i, $1 \leq i \leq n-2$ and assign color 2 to f_i for even i, $1 \leq i \leq n-1$. Assign color 3 to the remaining edge f_n, since f_n is adjacent to both f_1 and f_{n-1}. This becomes proper edge coloring of C_n. Hence each edge power dominates all the edges of both color class and so $\chi'_{pd}(C_n) = 3$.

Theorem: 3.2

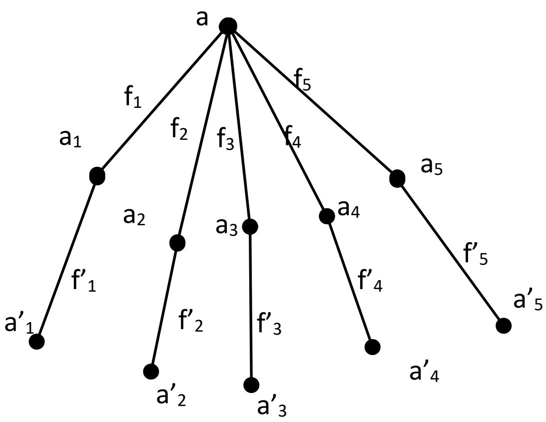
For the line graph of double star$K_{1,n,n}$, $n \geq 2$, $\chi'_{pd}(L(K_{1,n,n})) = \chi'_d(K_n) + 1$.

Proof:

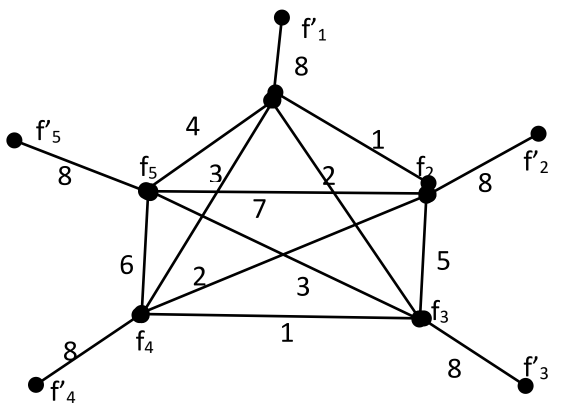
Let $K_{1,n,n}$be double star graph with vertex set $V = \{a\} \cup V_1 \cup V_2$ahere a is the root vertex, $V_1 = \{a_i : 1 \leq i \leq n\}$ and $V_2 = \{a'_i : 1 \leq i \leq n\}$and the edge set $E = E_1 \cup E_2$ where $E_1 = \{f_i : 1 \leq i \leq n\}$ and $E_1 = \{f'_i : 1 \leq i \leq n\}$ where the edge f_i connectes $a\ and a_i$ whereas the edge f'_i connects $a_i$and a'_i.

The graph $K_{1,n,n}$ contains $2n$ edges. Then its line graph contains $2n$ vertices of which the vertices $f_i$forms a clique of order n, (that is K_n). The vertex f'_i form a pendant vertex which is adjacent to f_i.

For the proper coloring, we assign $\chi'_d(K_n)$ colors to the edges in the cliques. The remaining edges which are connect with f'_i getting same color. Thus $\chi'_{pd}(K_{1,n,n}) = \chi'_d(K_n) + 1$.



$K_{1,5,5}$



$\chi'_{pd}(L(K_{1,n,n}))$ =8

Theorem: 3.3

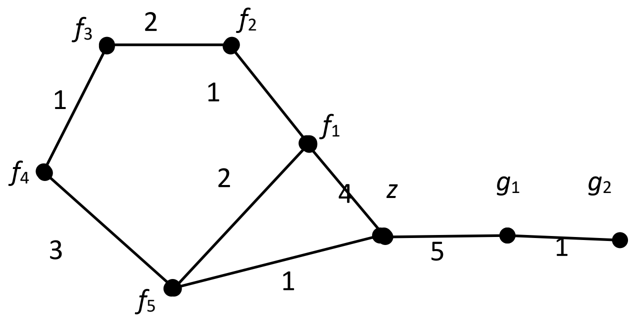
For a Tadpole graph $T_{m,n}$, $\chi'_{pd}\big(L(T_{m,n})\big) = 5, m \geq 4$ *and* $n \geq 2$

Proof:

By the definition of Tadpole graph, $T_{m,n}$ is obtained by joining a vertex of $C_m, m \geq 4$ by an edge and an end vertex of the path $P_n, n \geq 2$. Let $V\big(T_{m,n}\big) = \{a_1, a_2, \dots, a_m\} \cup \{b_1, b_2, \dots, b_n\}$, where $a_i, 1 \leq i \leq m$, are on the cycle and $b_i, 1 \leq i \leq n$, are on the path. Let a_m be the vertex joined to the end vertex b_1 of the path in the tadpole graph.

In the line graph of $T_{m,n}$, let the vertex $f_i, 1 \leq i \leq m-1$, correspond to the edges in the cycle and let $g_i, 1 \leq i \leq n-1$, be the edges in the path, P_n *in* $T_{m,n}$. Let z be the end vertex of the path which is adjacent to $f_1$and f_m.Thus, we get C_3 which has z as a vertex.

Assign colors 1, 2 and 3 to the edges in the clique. For the edges in the cycle, we required 3 colors, namely, 1,2 successively, and color one of the edges, which are incident with the vertices of C_3 with color 4, for which each edge dominates at least one-color class. Now color the edges in path, we color the first edge by using the color 5 and remaining edges by color 1 and color 4. Hence each edge dominates the non-repeated color class. Thus, $\chi'_{pd}\big(L(T_{m,n})\big) = 5$



$\chi'_{pd}(L(T_{5,3})) = 5$

Theorem: 3.4

For the Sunlet graph S~n~, $\chi'_{pd}(L(S_n)) = \begin{cases} \left\lfloor \frac{n}{2} \right\rfloor + 4, if\ n\ is\ odd \\ \frac{n}{2} + 3,\ if\ n\ is\ even \end{cases}$

Proof:

Let $a_1, a_2, \dots, a_n$ be the vertices of the n-cycle in the line graph of Sunlet graph Sn and let$a'_1, a'_2, \dots, a'_n$be the vertices in the line graph corresponding to the pendant vertices of S_n. The line graph of S_n, is the graph containing cycle C_n for which each edge of the cycle forms a clique C_3 with a vertex associated with the pendant edge in the S_n.

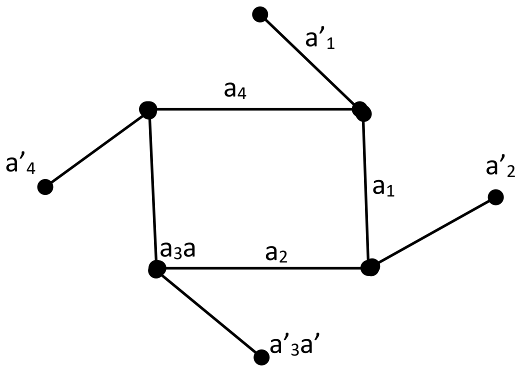
Case:1 n is odd

Since n is odd, the number of edge pairs are $\left\lfloor \frac{n}{2} \right\rfloor + 1$. Assign distinct $\left\lfloor \frac{n}{2} \right\rfloor + 1$ colors to an edge in a couple of edges. Now we assign a new color$\left\lfloor \frac{n}{2} \right\rfloor + 2$, to the remaining edges in the cycle and assign colors $\left\lfloor \frac{n}{2} \right\rfloor + 3$ and $\left\lfloor \frac{n}{2} \right\rfloor + 4$to the edges in C_3, which are incident with the edges in the cycle. Since each edge in C_3 are adjacent to the edge having colored by a non-repeated color and this way of coloring is proper edge coloring, each edge of G dominates all edges in at least one - color class.

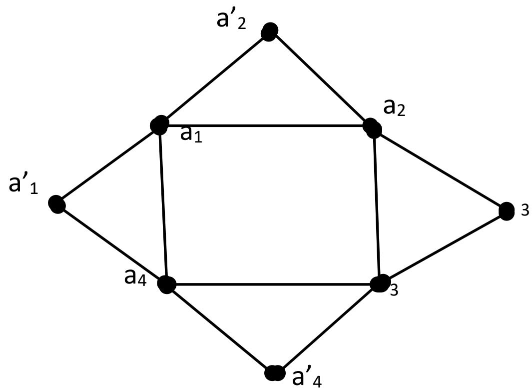
Therefore $\chi'_{pd}(L(S_n)) = \left\lfloor\frac{n}{2}\right\rfloor + 4$

Case:2 n is even

As we done in Case:1, we divide the edges in the cycle as $\frac{n}{2}$ pairs. Assign distinct $\frac{n}{2}$ colors to the edges in the cycle to an edge in each pair. Now we assign a new color $\frac{n}{2}+1$, to the remaining all edges in the cycle and assign color $\frac{n}{2}+2$ to the edges in C_3. Since each edge in C_3 are adjacent to the edge having colored by a non-repeated color and this way of coloring is proper edge coloring, each edge of G dominates all edges in at least one - color class. Therefore $\chi'_{pd}(L(S_n)) = \frac{n}{2}+2$.



$\chi'_{pd}(L(S_4)) = 4$



$\chi'_{pd}(L(S_4)) = 5$

Theorem:3.5

For the Centipete graph, Ctn, $\chi'_{pd}(L(Ct_n)) = \begin{cases} \frac{n}{2}+3\ , & if\ n\ is\ even \\ \left\lceil\frac{n}{2}\right\rceil + 3, & if\ n\ is\ odd \end{cases}, n \geq 4$

Proof:

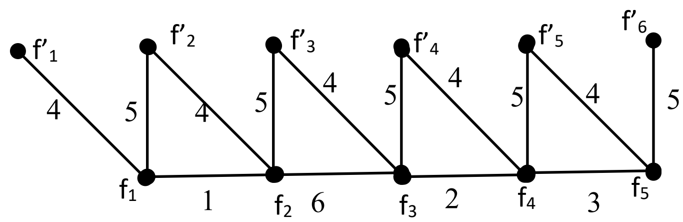
Let $a_1, a_2, \ldots, a_n$ be the vertices of the path and let $a'_1, a'_2, \ldots, a'_n$be the pendant vertices for which $a'_i$is adjacent to $a_i 1 \leq i \leq n$ in Ct_n. Let $f_1, f_2, \ldots, f_{n-1}$ be the edges in path and $f'_1, f'_2, \ldots, f'_n$. The line graph of the centipete graph, is the graph containing cycle $P_{n-1}$with the end vertices have a leaf corresponding to the pendant edge in Ct_n and the edges in P_{n-1} being an edge in separate C_3.

Case:1

If n is even, then the line graph has path of odd length assign distinct colors to odd e_i's. Since n is even, the number of odd e_i's are $\frac{n}{2}$and hence we need $\frac{n}{2}$ distinct color. And color the remaining edges in the path by color $\frac{n}{2}+1$. Now color the edges incident with the vertex, corresponding to the pendant edge in the centipete graph by colors $\frac{n}{2}+2$ and $\frac{n}{2}+3$, since theses edges are adjacent. Therefore, each edge of Ctn power dominates all the edges in at least one color class and so $\chi'_{pd}(L(Ct_n)) = \frac{n}{2}+3$.

Case:2

If n is odd, then we assign distinct colors to odd e_i's in the path. Since n is odd, the number of odd e_i's are $\left\lceil\frac{n}{2}\right\rceil$ and hence we need $\left\lceil\frac{n}{2}\right\rceil$ distinct colors to color the odd edges in the path and color the remainingcentral edges in the path by the color $\left\lceil\frac{n}{2}\right\rceil+1$. Now color the edges incident with the vertex which is the pendant edges in the centipete graph by the colors $\left\lceil\frac{n}{2}\right\rceil+2$ and $\left\lceil\frac{n}{2}\right\rceil+3$, since these are adjacent to each other.Hence each edge of L(Ctn) power dominates all the edges in at least one color class and so $\chi'_{pd}(L(Ct_n)) = \left\lceil\frac{n}{2}\right\rceil+3$



$\chi'_{pd}(L(Ct_6)) = 6$

4. Degree splitting of some classes of Graphs

Definition: 4.1

Give a graph $G = (V, E)$ with $V = S_1 \cup S_2 \cup \dots \cup S_t \cup T$, where t is an integer ≥ 1, each $S_i, 1 \leq t \leq t$, is a set of at least two vertices of G of the same degree and $T = V - \cup S_i$, then the degree splitting graph $DS(G)$ of the graph G is defined as a graph which is obtained from G by adding vertices $w_1, w_2, \dots, w_t$ and joining w_i, for each $i, 1 \leq i \leq t$, to each vertex of S_i. Note that if $V(G) = \cup_{i=1}^{t} S_i$ and $T = \phi$. [5]

Theorem: 4.2

For a path $P_n, n \geq 5$, $\chi'_{pd}(DS(P_n)) = \begin{cases} n-1, n \geq 5 \\ 4,\ n=4 \\ 3,\ n=2 \\ 2,\ n=3 \end{cases}$

Proof:

<u>Case : 1</u> $n = 2,3$

Initially we note that the degree splitting graphs $DS(P_2)$ and $DS(P_3)$ are respectively isomorphic to C_3 and C_4, so that $\chi'_{pd}(DS(P_2)) = \chi'_{pd}(DS(C_3)) = 3\ and\ \chi'_{pd}(DS(P_3)) = \chi'_{pd}(DS(C_4)) = 2$.

<u>Case: 2</u> $n = 4$

Let $G = P_4, V = \{v_1, v_2, v_3, v_4\}$ be a path with $V = S_! \cup S_2$, where $S = \{v_1, v_4\} and\ S_2 = \{v_2, v_3\}$ the degree splitting $DS(P_4)$ is obtained by adding two vertices $w_1\ and\ w_2$ corresponding to $S_1\ and\ S_2$ respectively. The graph $DS(P_4)$ contains C_3. To color this cycle, we need 3 colors and one new color 4 to color an edge incident with w_2 and color the remaining edges by the colors of the cycle. This coloring is a proper coloring and so all edge of $DS(P_4)$ power dominates the color class 4. Thus $\chi'_{pd}\big(DS(P_4)\big) = 4$.

<u>*Case: 3*</u> $n \geq 5$

Let $G = P_n, V = \{v_1, v_2, .., v_n\}$ be a path on n vertices with $V = S_! \cup S_2$, where $S = \{v_i: \deg(v_i) = 1\} and\ S_2 = \{v_i: \deg(v_i) = 2\}$ the degree splitting $DS(P_n)$ is obtained by adding two vertices $w_1\ and\ w_2$ corresponding to $S_1\ and\ S_2$ respectively. Thus $V[DS(P_n)] = V(P_n) \cup \{w_1, w_2\}$ and $E[DS(P_n)] = E(P_n) \cup \{w_1v_1, w_1v_2\} \cup \{w_2v_i: 2 \leq i \leq n-2\}$. $|V[DS(P_n)]| = n+2$ and $|E[DS(P_n)]| = 2n-1$Since $\deg(w_2) = n-2$, we need $n-2$ colors to color the edges incident with w_2. Also v, w_1, we need a color so that the apart from incident with w_2 are dominated by the new color, say $'a'$. For coloring the edges in the path P_n, we can use any colors from $n-2$ colors. The edges incident with w_2 and the edges in the path P_n is power dominated by the color $'a'$. Also the remaining edges are power dominated by the color $'a'$, which was a non-repeated color. Thus the minimum number of colors we need is $n-2+1 = n-1$ colors. Hence $\chi'_{pd}\big(DS(P_n)\big) = n-1, n \geq 5$.

Theorem: 4.3

For a path $C_n, n \geq 3$, $\chi'_{pd}\big(DS(C_n)\big) = n$

Proof:

In the cycle graph C_n, each vertex is of degree 2. By adding a new vertex w and made the vertex adjacent to all vertices of C_n to form a degree spillting of C_n. The resultant graph is just a wheel graph $W_{1,n}$, where w is the apex and remaining n vertices are the vertices of C_n. Thus $\chi'_{pd}\big(DS(C_n)\big) = \chi'_{pd}\Big(DS\big(W_{1,n}\big)\Big) = n$

Theorem: 4.4

For the star graph, $\chi'_{pd}\Big(DS\big(K_{1,n}\big)\Big) = n, n \geq 2$

Proof:

Consider the graph $K_{1,n}$ with $V = S \cup T$, where $S = \{v_1, v_2, \dots, v_n\}$ are pendant vertices and $T = \{v\}$ be the apex vertex of $K_{1,n}$ with degree n. By adding a new vertex w corresponding to S, we get the degree splitting of $K_{1,n}$. Since $\deg(v) = n$, we need n distinct colors to color the edges incident with v by assigning color i to the edge $vv_i, i = 1,2, \dots, n$. Now assign the same n colors to the edges incident with w in such a way that, assign color i to wv_{i+1} and color n to wv_1. Note that this is a proper edge coloring, ensuring that no two adjacent edges share the same color, and additionally, each edge power dominates all edges within at least one-color class. Thus $\chi'_{pd}\Big(DS\big(K_{1,n}\big)\Big) = n$

Theorem: 4.5

For the wheel graph, $\chi'_{pd}\Big(DS\big(W_{1,n}\big)\Big) = n + \chi'_{pd}(C_n), n \geq 3$

Proof:

In $W_{1,n}, n \geq 3$, let the vertex set be $V = \{u, v_1, v_2, \dots, v_n\}$, where u is the apex vertex. Also, the remaining vertices$v_i, 1 \leq i \leq n$ are of degree 3. Thus $V = S \cup T$, where $S = \{v_i |\ 1 \leq i \leq n\}, T = \{u\}$.

The degree splitting of $W_{1,n}$ is obtained by adding a new vertex w corresponding to S so the vertex set of $DS(W_{1,n})$ is $V_1 = S \cup T \cup \{w\}$ and the edge set is $E_1 = E \cup \{wv_i: 1 \leq i \leq n\}$. For proper coloring, we need n color to color the edges incident with u. Also to color the edges incident with w, we use the same n colors since $\deg(w) = n$. Now the rim of the wheel is isomorphic to the cycle graph C_n. Thus, to color the rim, we need$\chi^2_{pd}(C_n)$ colors distinct from previous n colors so that this would be proper coloring and each edge of power dominates all the edges of at least one-color class. Thus $\chi'_{pd}\left(DS(W_{1,n})\right) = n + \chi'_{pd}(C_n)$

Theorem: 4.6
For the complete bipartite graph, $\chi'_{pd}\left(DS(K_{n,n})\right) = 3n, n \geq 3$
Proof:
For a complete bipartite graph $K_{n,n}$ with a bipartition $V_1 \cup V_2$ of its vertex set. The degree splitting of $K_{n,n}$, $DS(K_{n,n})$ is obtained by introducing a new vertex w and joining this vertex to all the vertices of $K_{n,n}$, since all vertices are of same degree n. For coloring $K_{n,n}$, we need n distinct colors to color the edges joining $v_i\ of\ V_1$ to $v'_1\ of\ V_2$ so that each edge of $K_{n,n}$ power dominate all vertices of atleast one color class (each edge dominates the edge having that n colors). Now the vertex w has degree $2n$, we need $2n$ colors to color the edges of w which are different from the previous n colors. Hence $\chi'_{pd}\left(DS(K_{n,n})\right) = 3n$.

Theorem: 4.7
$\chi'_{pd}\left(DS(BT_n)\right) = 2n + 3$, where $BT(n)$ is a binary tree on diameter $2n$.
Proof:
Let $BT(n)$ be a complete binary tree with diameter $2n$, where $n \geq 1$. Each of the vertices $v_1, v_2, \dots, v_n$of $BT(n)$, other than the root vertex v_0 is of degree one or three. The vertex set of $BT(n)$ can be classified as $V = S_1 \cup S_2 \cup T$, where $T = \{v_0\}, S_1 = \{v_i : \deg(v_i) = 3\}$ and $S_2 = \{v_i : \deg(v_i) = 1\}$.
The degree splitting, $DS(BT(n))$ is constructed by introducing the new vertices $w_1 and\ w_2$ corresponding to $S_1 and\ S_2$respectively. The edge set of $DS(BT(n))$ is $E' = E \cup E_1 \cup E_2$, where $E = E\left(BT(n)\right), E_1 = \{v_i w_1 : 1 \leq i \leq n\} and\ E_2 = \{v_i w_2 : 1 \leq i \leq n\}$
Since the binary tree has $2n$ pendant vertices, $\deg(w_2) = 2n$ and and since $|S_1| = 2(n - 1), \deg(w_1) = 2(n - 1)$. To color the edges of w_2, we need $2n$ distinct colors. Out of these $2n$ colors, we use $2(n - 1)$ colors except the colors $1\ and\ n$ to color the edges incident with w_1. Use 2 colors $a\ and\ b$ to color the edges of the path in the binary tree and color the pendant edges of $BT(n)$ wit h a new color c. Thus we used $2n, a, b, c$ colors to color $DS(BT(n))$ properly. Furthermore, every edge ensures the power domination of all edges within at least one color class. Hence $\chi'_{pd}\left(DS(BT_n)\right) = 2n + 3$.

Theorem: 4.8
For bi-star $B_{n,n}$, $\chi'_{pd}\left(DS(B_{n,n})\right) = 2n + 1, n \geq 2$.
Proof:
Let $B_{n,n}$ be a bi-star graph with $V = S_1 \cup S_2$, where $S_1 = \{x, y : x\ and\ y\ adjacent\}$ and $S_2 = \{x_i, y_i : 1 \leq i \leq n\}$. Here for $1 \leq i \leq n, x_i and\ y_i$ are pendant vertices with all x_i joined to x and all y_i joined to y. We obtain the degree splitting graph of $B_{n,n}$, we introduce new vertices $w_1\ and\ w_2$ corresponding to $S_1\ and\ S_2$ respectively. The vertex w_1 is joined to $x\ and\ y$ while w_2 joined to all the remaining vertices of $B_{n,n}$. Since $\deg(w_2) = 2n$, we need $2n$ distinct colors to color the edges incident with w_2. Also use the same $2n$ colors to color the edges incident with $x\ and\ y$. Assign a new color $2n + 1$ to the edge xy, so the edges incident with w_1 is power dominated by this new color. Use any 2 colors from the $2n$ colors to color edges incident with

w_1. Therefore, this constitutes a proper edge coloring, where each edge also power dominates all edges within at least one color class. Thus $\chi'_{pd}\left(DS\left(B_{n,n}\right)\right) = 2n + 1, n \geq 2$.

Conclusion

In this paper, we introduced the concept of Power Dominator Edge Coloring (PDEC) and determined the PDEC-number for various classes of graphs, including line graphs and degree splitting graphs. By merging dominator edge coloring with power domination principles, we established results for specific graph structures such as paths, cycles, stars, wheels, tadpole graphs, centipede graphs, and sunlet graphs.

we explored the Power Dominator Edge Coloring (PDEC) of subdivision graphs and determined the PDEC-number for various classes of subdivided graphs. By analyzing how the subdivision of edges influences the power dominator edge chromatic number, we derived results for different graph families. The subdivision operation alters the adjacency relationships in a graph, impacting both the edge domination properties and the coloring requirements. Our findings highlight how introducing subdivision affects the structural properties of graphs and provides a deeper understanding of edge domination in graph coloring. The results obtained serve as a foundation for future investigations into the combinatorial properties of power dominator edge coloring in more complex graph classes.

Our findings provide insights into the minimum number of colors required to achieve power dominator edge coloring while ensuring that every edge in a graph power dominates at least one color class. The results derived for degree splitting graphs demonstrate how structural modifications influence the PDEC-number. These contributions lay the groundwork for further studies in combinatorial optimization, graph theory applications, and real-world problems related to network monitoring and communication systems.

Future research can extend this work to additional graph classes and explore algorithmic approaches for efficient power dominator edge coloring in large-scale graphs.

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