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A Study of Uncertainty and Probability in Transportation Problem

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Abstract:

The transportation problem is a special class of problem. Cost is one of the affected factors in transportation. Determine the value of transportation cost, it is solved by using different methods like the North-West Corner Method, Least Cost Method,Row (Column) Minima Method, Vogel Approximation Method, etc. but these methods cannot say about the probability. Our aim is to analyze probability in various methods of transportation and each movement in transportation is taken as an event and understand the probability dimension in these methods. Uncertainty comes from a certainty situation. Because on the other side of certainty, there should be reflected uncertainty. In this paper, we also discuss the parameter of uncertainty in various methods of transportation problems using case study. The derived results in the paper is use to establish a concreate transportation system on basis of minimize the transportation cost and allocation.

Keywords: Uncertainty, Probability, Transportation problem, North-West Corner Method, Least Cost Method, Vogel Approximation Method

Introduction:

In Operations Research, the Transportation Problem is a fundamental topic that focuses on optimizing the allocation of resources to minimize transportation costs. Various methods, such as the North-West Corner Method, Least Cost Method, and Vogel's Approximation Method, are employed to solve this problem. Transportation is a vital part of human life and is essential for economic growth and societal development. As the global population continues to rise, the demand for transportation services increases, yet the infrastructure may not expand at the same rate. This raises a critical question for the future: how will we manage the growing demand for transportation over the next 20 years, given limited resources? This challenge highlights the importance of strategic planning and optimization techniques in ensuring efficient transportation systems for the future.

Transportation costs in India have been influenced by several factors, with road infrastructure projects playing a critical role. As per the Ministry of Road Transport and Highways, the expansion of road networks under schemes like BharatmalaPariyojana has been significant. By 2023, over approximately 1.45 lakh kilometers of state roads were upgraded to national highways. The increase in road construction, such as expressways and economic corridors, has aimed to improve freight movement and reduce transportation time, but it has also contributed to rising logistics and infrastructure costs. Additionally, fuel price fluctuations and toll fees have impacted transportation expenses. The National Highways Authority of India (NHAI) has also introduced measures like toll securitization, raising Rs 37,000 crore through this method, which supports further infrastructure projects but can increase the cost burden for road users.Freight transportation by road remains the dominant mode, accounting for approximately 71% of total freight in India, but rising costs, fluctuating fuel prices, and road maintenance continue to push transportation expenses higher.

The transportation problem is characterized by significant uncertainty, particularly in the allocation of resources, which plays a key role in minimizing transportation costs. Allocating the right amount to the appropriate cell in a transportation matrix is critical for cost reduction. Several methods, such as the North-West Corner Method, Least Cost Method, Row (Column) Minima Method, and Vogel's Approximation Method, are commonly used to find a basic feasible solution. Can probability theory be applied to these methods? Absolutely, since these methods involve events that can be analyzed probabilistically, making the process more predictable and efficient.

Literature Review:

Dev, Sultana, and Mitra(2011) paper present a fuzzy optimization approach to the transportation problem, utilizing a fuzzy extension of the simplex method. It addresses uncertainties in transportation costs through trapezoidal fuzzy numbers, enhancing decision-making for maximizing distributor benefits under fuzzy constraints.

Hussain, and Qayyum (2019) introduces an innovative approach to solving Linear Programming Problems (LPP), specifically the Transportation Problem (TP), by applying Probability Density Functions (PDF). This method enhances accuracy and reduces complexity, using Laplace Transform and statistical tools to optimize transportation costs and address challenges in random variables and constraints.

Gupta,Garg, and Chaudhary (2020) present a multi-objective capacitated stochastic transportation problem model incorporating gamma distribution for uncertain supply and demand parameters. Using maximum likelihood estimation and chance-constrained programming, it optimizes transportation decisions. The model is illustrated through a case study, demonstrating its computational effectiveness in uncertain environments.

Zheng and Yu (2023) propose a novel probability-based method to solve the multi-objective shortest path problem. By treating each objective as an individual event, they calculate favorable probabilities for optimization. The approach is tested on a transportation problem, yielding efficient solutions without linear weighting or constraint issues.

Methodology:

From ↓ To →	**Destination D_1**	**Destination D_2**	**.....**	**Destination D_n**	**Supply**
O_1	a_{11}	a_{12}	…	a_{1n}	s_1
O_2	a_{21}	a_{22}	…	a_{2n}	s_2
:	:	:	:	:	:
O_m	a_{m1}	a_{m2}	…	a_{mn}	s_m
Demand	d_1	d_2		d_n	Balanced i.e. $\sum_{i=1}^{m} s_i = \sum_{j=1}^{m} d_j$

[Table-1: Transportation Problem]

Probability approach in North-West Corner Method

Sr. No.	Events	Description of the Events in N-W Method	Possible Outcome	Probability
1	E_{1j}	The event of allocation among $(1,j)^{th}$ cells	$\{d_j\}$ or $\{s_1\}$ or $\{s_1 - d_j\}$	$P(E_{1j}) = \begin{cases} 1 & ; if\ E_{1j} = \{d_j\}\ or\ E_{1j} = \{s_1\} \\ \frac{1}{n(\{s_1 - d_j\})} & ; if\ E_{1j} = \{s_1 - d_j\} \\ 0 & ; Otherwise \end{cases}$
2	E_{i1}	The event of allocation among $(i,1)^{th}$ cells	$\{d_1\}$ or $\{s_i\}$ or $\{d_1 - s_i\}$	$P(E_{i1}) = \begin{cases} 1 & ; if\ E_{i'j} = \{d_j\}\ or\ E_{i'j} = \{s_1\} \\ \frac{1}{n(\{d_1 - s_i\})} & ; if\ E_{i'j} = \{d_1 - s_i\} \\ 0 & ; Otherwise \end{cases}$

Result-1: **In north-west corner method, if E_{1j} is the event of allocation among $(1,j)^{th}$ cells and E_{i1} is the event of allocation among $(i,1)^{th}$ cells then,**

$$\sum_{\substack{i=1\ and \\ j=1}}^{\substack{i=m\ and \\ j=n}} P\left(E_{1j} \bigcap E_{i1}\right) = 1$$

Proof-1:

Let us consider,

$$\left(\bigcup_{j=1}^{n} E_{1j}\right) \bigcap \left(\bigcup_{i=1}^{m} E_{i1}\right) = E_{11}$$

$$\bigcup_{\substack{i=1\ and \\ j=1}}^{\substack{i=m\ and \\ j=n}} \left(E_{1j} \bigcap E_{i1}\right) = E_{11}$$

By definition of probability,

$$\sum_{\substack{i=1\ and\\ j=1}}^{\substack{i=m\ and\\ j=n}} P\left(E_{1j} \bigcap E_{i1}\right) = P(E_{11})$$

Now, in first north-west corner cell of transportation problem table, has definitely require to allocation either demand or supply.

So, $P(E_{11}) = 1,$

$$\boldsymbol{\sum_{\substack{i=1\ and\\ j=1}}^{\substack{i=m\ and\\ j=n}} P\left(E_{1j} \bigcap E_{i1}\right) = 1}$$

Proof-2:

$$\text{L.H.S.} = \sum_{\substack{i=1\ and\\ j=1}}^{\substack{i=m\ and\\ j=n}} P\left(E_{1j} \bigcap E_{i1}\right)$$

$$= P\left(E_{11} \bigcap E_{11}\right) + P\left(E_{12} \bigcap E_{11}\right) + \cdots + P\left(E_{1n} \bigcap E_{11}\right) + \cdots + P\left(E_{mn} \bigcap E_{11}\right) + \cdots$$
$$+ P\left(E_{m2} \bigcap E_{m1}\right) + P\left(E_{m2} \bigcap E_{11}\right) + \cdots + P\left(E_{mn} \bigcap E_{m1}\right)$$

$$= P(E_{11}) + 0 \qquad \left(Because\ of\ P\left(E_{1j} \bigcap E_{i1}\right) = 0; i \neq j\right)$$

$$= 1 \qquad (Because\ of\ P(E_{11}) = 1)$$

$$= R.H.S.$$

$$\therefore \boldsymbol{\sum_{\substack{i=1\ and\\ j=1}}^{\substack{i=m\ and\\ j=n}} P\left(E_{1j} \bigcap E_{i1}\right) = 1}$$

Result-2: In transportation problem, the probability of allocation of last cell (in the sense of South-East point) is 1.

In mathematical, we can write as $\boldsymbol{\sum_{\substack{i=1\ and\\ j=1}}^{\substack{i=m\ and\\ j=n}} P(E_{mj} \cap E_{in}) = 1}$

Proof-1: Let us consider

$$\left(\bigcup_{j=1}^{n} E_{mj}\right) \bigcap \left(\bigcup_{i=1}^{m} E_{in}\right) = E_{mn}$$

If transportation problem is in square matrix, then $E_{mn} = E_{mm}\ or\ E_{nn}$

$$\bigcup_{\substack{i=1\ and\\ j=1}}^{\substack{i=m\ and\\ j=n}} \left(E_{mj} \bigcap E_{in}\right) = E_{mn}$$

By definition of probability,

$$\sum_{\substack{i=1 \, and \\ j=1}}^{\substack{i=m \, and \\ j=n}} P\left(E_{mj} \bigcap E_{in}\right) = P(E_{mn})$$

Now,

$$P(E_{mn}) = \begin{cases} 1 & ; if \ E_{mn} = \{d_m\} \ or \ E_{mn} = \{s_n\} \\ \dfrac{1}{n(\{d_m - s_n\})} & ; if \ E_{i'j} = \{d_n - s_m\} \\ 0 & ; Otherwise \end{cases}$$

$if \ E_{mn} = \{d_j\} \ or \ E_{mn} = \{s_1\}$then $P(E_{mn}) = 1$ as per probability table in north-west corner method. Last cell has at least own demand. So, it is remaining allocation while we are reaching at allocation in that cell. So, it is definitely that we should allocate some value either totally demand or the value after subtraction of total allocate
In general, $n(\{d_m - s_n\}) = 1$

Probability Approach InLeast Cost Method:

Sr. No.	Events	Description of the Events in Least Cost Method	Possible Outcome	Probability
1	$E_{i'j}$	The event of allocation among $(i', j)^{th}$ cells; i' indicates the row of first least cost's row in table and i' is fixed natural number in the problem.	$\{d_j\} \ or \ \{s_{i'}\} \ or \ \{s_{i'} - d_j\}$	$P(E_{i'j}) = \begin{cases} 1 & ; if \ E_{i'j} = \{d_j\} \ or \ E_{i'j} = \{s_{i'}\} \\ \dfrac{1}{n(\{s_{i'} - d_j\})} & ; if \ E_{i'j} = \{s_{i'} - d_j\} \\ 0 & ; Otherwise \end{cases}$
2	E_{i^*j}	The event of allocation among $(i^*, j)^{th}$ cells; i^* indicates the row of later allocation after first least cost's row andi^* is fixed natural number.	$\{d_j\} \ or \ \{s_{i^*}\} \ or \ \{d_j - s_{i^*}\}$ or$\{s_{i^*} - d_j\}$	$P(E_{i^*j}) = \begin{cases} 1 & if \ E_{i^*j} = \{d_j\} \ or \ \{s_{i^*}\} \\ \dfrac{1}{n(\{s_{i^*} - d_j\})}, & if \ E_{i^*j} = \{s_{i^*} - d_j\} \\ \dfrac{1}{n(\{d_j - s_{i^*}\})}, & if \ E_{i^*j} = \{d_j - s_{i^*}\} \\ 0 & ; Otherwise \end{cases}$

Sr. No.	Events	Description of the Events in Least Cost Method	Possible Outcome	Probability
3	E_{ij^*}	The event of allocation among $(i,j^*)^{th}$ cells; j^* indicates the column of later allocation after first least cost's column and j^* is fixed natural number.	$\{d_{j^*}\}$ *or* $\{s_i\}$ *or* $\{d_{j^*}-s_i\}$ or$\{s_i-d_{j^*}\}$	$P(E_{ij^*}) = \begin{cases} 1 & ; if\ E_{ij^*}=\{d_{j^*}\}\ or\ \{s_i\} \\ \dfrac{1}{n(\{d_{j^*}-s_i\})} & ;\ if\ E_{ij^*}=\{d_{j^*}-s_i\} \\ \dfrac{1}{n(\{s_i-d_{j^*}\})} & ;\ if\ E_{ij^*}=\{s_i-d_{j^*}\} \\ 0 & ; Otherwise \end{cases}$
4	$E_{ij'}$	The event of allocation among $(i,j')^{th}$ cells; j' indicates the column of first least cost's column in table and j' is fixed natural number.	$\{d_{j'}\}$ *or* $\{s_i\}$ *or* $\{d_{j'}-s_i\}$	$P(E_{i'j}) = \begin{cases} 1 & ; if\ E_{i'j}=\{d_{j'}\}\ or\ E_{i'j}=\{s_i\} \\ \dfrac{1}{n(\{d_{j'}-s_i\})} & ; if\ E_{i'j}=\{d_{j'}-s_i\} \\ 0 & ; Otherwise \end{cases}$

Let $C(i',j'), C(i,j'), C(i',j)$ and $C(i,j)$ be the cost associated with cell $(i',j'), (i,j'), (i',j)$ and (i,j) respectively in the table where $i'$and $j'$are fixed natural numbers representing the row and column indices of the cell and X represent the total cost of allocation based on these events. Then
We can express X mathematically as:

$$X = C(i',j') + C(i,j') + C(i',j) + C(i,j)$$

Result-3:$\boldsymbol{P((\bigcup_{j=1}^{n} E_{i'j}) \cap (\bigcup_{i=1}^{m} E_{ij'})) = 1}$

Let's consider the union of all these events, denoted by U:

$$U = \left(\bigcup_{j=1}^{n} E_{i'j}\right) \cup \left(\bigcup_{j=1}^{n} E_{i^*j}\right) \cup \left(\bigcup_{i=1}^{m} E_{ij^*}\right) \cup \left(\bigcup_{i=1}^{m} E_{ij'}\right)$$

Thus, each event is subset of U.
So,

$$\left(\bigcup_{j=1}^{n} E_{i'j}\right) \cap \left(\bigcup_{i=1}^{m} E_{ij'}\right) = E_{i^*j^*};$$

$$E_{i^*j^*}\ is\ the\ event\ of\ allocation\ of\ first\ least\ cost\ cell.$$

By definition of probability (or probability measure),

$$P\left(\left(\bigcup_{j=1}^{n} E_{i'j}\right) \cap \left(\bigcup_{i=1}^{m} E_{ij'}\right)\right) = P(E_{i^*j^*})$$

Since, $P(E_{i^*j^*})$ is probability of allocation of the first least cost cell. $E_{i^*j^*}$represents intersection event of union of $E_{i'j}$ and union of $E_{ij'}$. According to least-cost method, we always start with first least-cost cell in transportation cost table. Therefore, the probability of allocation of the first least-cost cell in least-cost-method is certain. A Certain event has probability of 1. Thus, $P(E_{i^*j^*}) = 1$.
Now, Equation (1) is,

$$P\left(\left(\bigcup_{j=1}^{n} E_{i'j}\right) \cap \left(\bigcup_{i=1}^{m} E_{ij'}\right)\right) = 1$$

Hence the proof.

Illustration:
Imagine a company that needs to distribute goods from three warehouses (W1, W2, W3) to four stores (S1, S2, S3, S4). Each warehouse has a specific supply, and each store has a certain demand. That are as below: Warehouse Supplies: W1: 20, W2: 30, W3: 25 and Store Demands: S1: 15, S2: 25, S3: 20, S4: 15. Using North-West Corner Method, Find the probability of the event E_{1j} (Allocation from Warehouse 1 to Store j) and event E_{i1} (Allocation from Warehouse i to Store 1).

Warehouse/ Store	**Store 1**	**Store 2**	**Store 3**	**Store 4**
Warehouse 1	50	64	87	94
Warehouse 2	55	45	87	75
Warehouse 3	85	98	75	35

Solution:

Event	**Description**	**Possible Outcomes**	**Probability**
E_{1j}	Allocation from Warehouse 1 to Store jjj	1. Allocate all goods from W1 to S1 2. Allocate part from W1 to S1 and remainder to S2 3. Allocate remaining supply from W1 after S1's demand is met	$P(E_{1j}) = \begin{cases} 1 & ; if\ \{d_j\}\ is\ met. \\ \frac{1}{n} & ; if\ partially\ allocation \\ 0 & ; Otherwise \end{cases}$

E_{i1}	Allocation from Warehouse iii to Store 1	1. Allocate all goods from Wi to S1 2. Allocate part from Wi to S1 and remainder to S2 3. Allocate remaining supply from Wi after S1's demand is met	$P(E_{i1}) = \begin{cases} 1 & ; if\ \{d_1\}\ is\ met. \\ \frac{1}{n} & ; if\ partially\ allocation \\ 0 & ; Otherwise \end{cases}$

	Store 1	**Store 2**	**Store 3**	**Store 4**
Warehouse 1	50 $\boxed{15}$	64	87	94
Warehouse 2	55 $\boxed{5}$	45	87	75
Warehouse 3	85	98	75	35

At (1,1) cell we need to allocate minimum between 20 and 15, is 15. So, Total demand '20' is divided into two part '15' and remaining of 15 (i.e. '5'). So, Probability of event E_{11} with respect to the allocation is, $P(E_{11}) = 1$.
Now, Supply of 'Warehouse 1' is no one. By North-West Corner Method concept, there is no probability of allocation in the cell (1,2,), (1,3) and (1,4).
So, $P(E_{12}) = 0, P(E_{13}) = 0, P(E_{14}) = 0$.
Next north-west cornet in table is at (2,1) cell. At (2,1) cell, we need to allocate minimum between '5' and 25, is 5. It is partially allocated with respect to supply. So, Probability of event E_{21} with respect to the allocation is, $P(E_{21}) = \frac{1}{2}$.
Now, Demand of 'Store 1' is no one. By North-West Corner Method concept, there is no probability of allocation in the cell (3,1). So, $P(E_{31}) = 0$.
Hence, the probability of event E_{1j} (Allocation from Warehouse 1 to Store j) and ev**ent** E_{i1} (Allocation from Warehouse i to Store 1) are as below:

$$P(E_{11}) = 1\ P(E_{12}) = 0, P(E_{13}) = 0, P(E_{14}) = 0 \text{ and } P(E_{21}) = \frac{1}{2}, P(E_{31}) = 0$$

Application of the Study:
The application of this study lies in improving the reliability and efficiency of transportation logistics by integrating probability theory into traditional methods like the North-West Corner and Least-Cost Methods. This approach ensures that allocations are not only cost-efficient but also predictable, providing more reliable solutions and improving decision-making in solving transportation problems.

Scope of the Study:
The future scope of this study includes extending the probabilistic framework to other transportation methods and optimization techniques, allowing for more comprehensive analysis and broader applicability. Further research can explore dynamic and real-time transportation problems, integrating uncertainty factors like fluctuating costs and demand. Additionally, the approach could be adapted for more complex logistics networks, enhancing decision-making in larger, real-world transportation systems.

Conclusion:

In this research paper, we explored the application of probabilistic approaches in solving the Transportation Problem, specifically for the North-West Corner (NWC) Method and Least Cost Method based on allocation. Through this analysis, we derived several key results related to probability. Especially, we proved that the probability of an allocation occurring at the North-West corner in the NWC method is always 1. Similarly, we established that the probability of allocation at the least cost in the first iteration of the Least-Cost Method may also 1. These findings enhance our understanding of the probabilistic behavior of allocation methods.This study integrates probability theory into traditional transportation problem-solving methods, providing a probabilistic framework for understanding and verifying the allocation process. This approach makes sure that allocations are both cost-efficient and backed by probability, making the solutions stronger and more reliable in transportation logistics.

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