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Fuzzy N-Graph structures in Topological Spaces

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Abstract: In this note we are introducing seems to focus on developing a new work to manage fuzzy information by connecting fuzzy sets, graph structures, and topology. The primary goal is to establish a Fuzzy N-Graph structure within topological spaces using graph theory, particularly focusing on the adjacency relation between vertices within fuzzy graph structures.

Keywords: Fuzzy N-Graph structures in topological spaces

1. Introduction and preliminaries

A graph mathematically represents a network, describing relationships between vertices (nodes) and edges. Graph theory models real-world phenomena but struggles with systems exhibiting uncertainty in attributes. Fuzzy graphs were introduced to address this limitation, motivated by real-world complexities. Kauffman (using Zadeh's fuzzy relation) pioneered the concept of fuzzy graphs. Fuzzy-graph theory is rapidly expanding, offering enhanced solutions across various fields, including networking, communication, data mining, clustering, image processing, planning, and scheduling, Graph structures are fundamental in computer science and computational intelligence. Fuzzy-graph structures effectively address uncertainty and ambiguity in real-world scenarios. The article introduces a new framework called Fuzzy N- Graph, combining fuzzy graph structures with topological space. In a Fuzzy N-Graph Relationships between vertices (edges) are fuzzy, not crisp. Edges are represented by fuzzy values in the range [0, 1], rather than binary (0 or 1). The study explores basic concepts of Fuzzy N- Graph structures in topological space and examines related properties.

Definition 1.1 (3)**.** *Let G be a nonempty simple graph and H be a subgraph of G. The operator below approximation and above approximation H of G. Let R(G) be the graph factorization or decomposition of G. Then the upper and lower approximation of subgraph H are indicate by* $N^*(H)$ *and* $N_*(H)$ *respectively, as follows:*
$N^*(H) = \{\cup H_i : H_i \in R(G) \text{ and } H_i \cap H \neq \phi\}$,
$N_*(H) = \{\cup H_i : H_i \in R(G) \text{ and } H_i \subseteq H\}$,
Also, the operator boundary region for subgraph H in G is defined as
$B_N(H) = N^*(H) - N_*(H)$

Definition 1.2. [3] *Consider a subgraph H of non-empty simple graph of G and R(G) be the graph factorization of Define* $\zeta_R(H) = \{G, \emptyset, N_*(H), N^*(H), B_N(H)\}$*.Then* $\zeta_R(H)$ *satisfies the following axioms*

(1) *The graph G (full graph) itself and the null graph* ϕ *are the members of the collection* $\zeta_R(H)$
(2) *The union of any members of* $\zeta_R(H)$*is a member of* $\zeta_R(H)$*.*
(3) *The finite intersection of any members of* $\zeta_R(H)$ *is in* $\zeta_R(H)$*.*

Then a collection of $\zeta_R(H)$ *is called N-graph topology on G. We call the pair* $(G, \zeta_R(H))$ *as the N-graph topological space. The members of* $\zeta_R(H)$ *are called the N-open subgraph of G and the complement* H^c *of the N-open subgraph H is the subset of G induced by the edge set* $E - E_H$ *is called an N-closed subgraph of G.A subgraph which is both an N-open subgraph and N-closed subgraph is called N-clopen subgraph.*

Definition 1.3 (3)**.** *Consider a subgraph H of non-empty simple graph G.Then the Indiscrete N-graph topology is defined as* $\zeta_R(H) = \{\phi, G\}$*, a collection of the trivial subgraphs of G.*

Definition 1.4. *Let H be a subgraph of non-empty simple graph G and* $\zeta_R(H)$ *is a N-graph topological space. The N-interior and N-closure of subgraph S is defined as follows: N-int of subgraph S is the union of all N-open subgraph which is an edge induced subgraph of subgraph S and it is noted by* $N_G Int(S)$*.The N-closure of subgraph S is the intersection of all N-closed subgraph which is supergraph of S and it is denoted by* $N_G Cl(S)$*.Also* $N_G Int(S)$ *is the largest N-open subgraph of S and* $N_G Cl(S)$ *is the smallest super graph N-closed subgraph of S.*

Definition 1.5 (3)**.** *Let H be a subgraph of N-graph topological space* $\zeta_R(H)$ *is called*

(1) *N-semi open subgraph if* $H \subseteq N_G Cl(N_G Int(H))$
(2) *N-α open subgraph if* $H \subseteq N_G Int(N_G Cl(N_G Int(H)))$
(3) *N-pre open subgraph if* $H \subseteq N_G Int(N_G Cl(H))$
(4) *N- β open subgraph if* $H \subseteq N_G Cl(N_G Int(N_G Cl(H)))$
(5) *N- γ open subgraph(N-b open subgraph) if* $H \subseteq N_G Cl(N_G Int(H)) \cup N_G Int(N_G Cl(H))$*.*
(6) *N- regular open subgraph if* $H = N_G Int(N_G Cl(H))$

The group of N- semi-open subgraphs (resp. N- α open subgraph, N- pre open subgraph, N- β open subgraph, N- γ open subgraph, N-regular open subgraph) from G will be denoted by $N_{S_G} - SO(H)$ *(resp.*$N_{S_G} - \alpha O(H)$*,*$N_{S_G} - PO(H)$*,* $N_{S_G} - \beta O(H)$*,* $N_{S_G} - \gamma O(H)$*,*$N_{S_G} - RO(H)$*).The complement of an N-semi open subgraph (resp. N- α open subgraph, N-pre open subgraph, N- β open subgraph, N- γ open subgraph, N- regular open subgraph) called a N-semi closed subgraph and the family of all N- semi closed subgraphs(resp. N- α closed subgraph, N- pre closed subgraph, N- β closed subgraph, N- γ closed subgraph, N- regular closed subgraph) from G will be denoted by* $N_{S_G} - SC(H)$*.(resp.*$N_{S_G} - \alpha C(H)$*,*$N_{S_G} - PC(H)$*,* $N_{S_G} - \beta C(H)$*,* $N_{S_G} - \gamma C(H)$*,*$N_{S_G} - RO(H)$*)*

Definition 1.6 (11). *A graph structure* $G = (V, R_1, ..., R_n)$ *is defined as V a nonempy set of verices which are mutually disjoint relation on* V*.Each relation* R_i, $1 \leq i \leq n$ *is satisfies*
symmery, $(a, b) \in R_i$ *then* $(b, a) \in R_i$
irreflexive, $(a, a) \notin R_i$ *for any* $a \in V$
This structure can be visualized as a graph with edges labeled $R1, ..., Rn$*, representing different types of relationships between vertices.*

Definition 1.7 (11). *Let S be a set. A map* $\sigma : S\ [0, 1]$ *is called a fuzzy subset of S. A fuzzy relation on S is a fuzzy subset of* $S \times S$*. Let S be a set , M and N be fuzzy sets on S. Then the join and the meet of M and N denoted by* $M \vee N$ *and* $M \wedge N$*, are defined as follows: for each* $x \in S$*,* $(M \vee N)(x) = max(M(x), N(x))$ *and* $(M \wedge N)(x) = min(M(x), N(x))$.

Definition 1.8 (11). *Let* $G = (V, \sigma, \mu)$ *is a triplet fuzzy graph where where V is a finite, nonempty set representing the vertices (or nodes) of the graph,* σ *is a fuzzy of* V*, which defines the membership function for the vertices, meaning each vertex has a degree of membership in the fuzzy set* $\sigma.v$ *is a fuzzy relation on* σ*, which represents the edges of the graph. The membership value* $\mu(a, b)$ *denotes the strength of the connection (or edge) between two vertices a and b. The condition for a fuzzy graph is that the fuzzy relation* μ *must satisfy* $\mu(a, b) \leq \sigma(a) \wedge \sigma(b) \forall a, b \in V$*. This ensures that the fuzzy edge set* μ *is a fuzzy relation on the fuzzy vertex set* σ *, meaning that the strength of the connection (edge) cannot exceed the minimum of the strengths of the connected vertices.*

2. Fuzzy lower and fuzzy upper subgraph,fuzzy boundary subgraphs

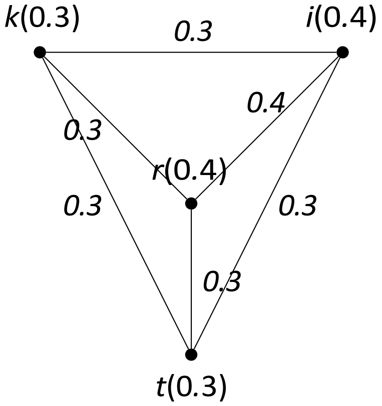
Definition 2.1. *Let* $G = \{\sigma, \mu_1, \mu_2, ...\mu_n\}$ *is a fuzzy graph structure of the graph structure* G^**, where* $\sigma : V \rightarrow [0, 1]$ *be the fuzzy set on* V *and* $\mu_1, \mu_2, ...\mu_n$ *be the fuzzy sets on the relations* $R_1, R_2,$ R_n *respectively.Let G be a fuzzy graph of vertices we define an adjacency relation* R_A *is* $R_A = \{(v, \sigma(v) \in v(G), \mu_i(vu) \geq 0\}$.

Definition 2.2. *Let* $G = \{\sigma, \mu_1, \mu_2, ...\mu_n\}$ *be a fuzzy graph structure of the graph structure* $G^* = (V, R_1, R_2, ...R_n)$*, where* $\sigma : V \rightarrow [0, 1]$ *be the fuzzy set on* V *and* $\mu_1, \mu_2, ...\mu_n$ *(which assign degrees of membership to the edges) be the fuzzy sets on the relations* $R_1, R_2,$ R_n *respectively.Let* K_f *be a subgraph of G and* R_A *be the adjacency relation then the fuzzy lower subgraph* K_f *are indicate* $\mathrm{F}_+[K_f] = \{\cup M_i : M_i \in R_A, M_i \subseteq [K_f]\}$ *.This means that we take the union of all fuzzy relations* M_i *that are contained in the adjacency relation* R_A *and whose fuzzy sets are subsets of the vertices* K_f *,* $\mathrm{F}^+[K_f] = \{\cup M_i : M_i \subseteq R_A$ *and* $M_i \cap [K_f] \neq \phi$*.This represents the union of all fuzzy relations* M_i *whose intersection with* K_f *is non-empty and which are contained in the adjacency relation* R_A*.Also the fuzzy boundary subgraph* K_f *in a* *fuzzy graph structure* $G = \{\sigma, \mu_1, \mu_2, ...\mu_n\}$ *is defined as* $F_B[K_f] = \mathrm{F}_+[K_f] - \mathrm{F}^+[K_f]$

Example 2.3. *Consider graph structure involving a set* $V = u, v, w, x$, *and two relations* $R1$ *and* $R1$ *i.e.,* $R1 = uw, uv, wx$ *and* $R2 = ux, vw, vx$ *such that these relations are different. Moreover, they do not have elements of same vertices like uu, vv, ww, xx, hence, then R1 and R2 are irreflexive. Given these properties, the graph structure with vertex set*

V and relations $R1$ *and* $R2$ *forms a simple nondirected graph in which:*

The relations $R1$ *and* $R2$ *are symmetric (bidirectional). The relations* $R1$ *and* $R2$ *are disjoint (no overlap between the two types of relations). Here* $R1$ *and* $R2$ *are irreflexive (no self-loops are allowed). The graph is fuzzy in nature, with membership values defining the degree of connectivity between vertices, subject to the properties of the fuzzy relations . Hence, vertex set V with relations R1 and R2 is a graph structure* $G^* = (V, R1, R2)$



Consider a fuzzy graph structure $G = (\sigma, \mu_1, \mu_2)$, *we define a fuzzy set* $\sigma(k) = 0.3$, $\sigma(r) = 0.3$, $\sigma(i) = 0.4$, $\sigma(t) = 0.4$ *and define a fuzzy sets* μ_1, μ_2 *on R1,R2 is* $\mu_1(k, i) = 0.3$, $\mu_1(k, t) = 0.3$, $\mu_1(t, r) = 0.3$, $\mu_1(i, r) = 0.4$, $\mu_1(k, r) = 0.3$, $\mu_1(v, w_3) = 0.3$.*Let* $G = (\sigma, \mu_1, \mu_2)$ *be the fuzzy graph structure with vertices* $V(G) = \{k(0.3), t(0.3), i(0.4), x(0.4)\}$ *and the adjacency relation is* $R[k(0.3)] = \{t(0.3), i(0.4), r(0.4)\}$, $R[t(0.3)] = \{k(0.3), i(0.4), r(0.4)\}$, $R[i(0.4)] = \{k(0.3), t(0.3), r(0.4)\}$, $R[r(0.4)] = \{k(0.3), t(0.3), i(0.4)\}$ *is* R_f *is subgraph with vertices* $V(R_f) = \{k(0.3), i(0.3)\}$ *then* $\mathrm{F}_+[R_f] = \{\phi\}$, $\mathrm{F}^+[R_f] = \{G\}$,$F_B[R_f] = \{G\}$

Property 2.4. *Let* $G = \{\sigma, \mu_1, \mu_2, ...\mu_n\}$ *is a fuzzy graph structure. Let* R_f *be a subgraph of G with vertices and* R_A *be the adjacency relation then*

(1) $\mathrm{F}_+[R_f] \subseteq R_f \subseteq \mathrm{F}^+[R_f]$

(2) $\mathrm{F}_+[G] \subseteq G \subseteq \mathrm{F}^+[G]$

Proposition 2.5. *Let* $G = \{\sigma, \mu_1, \mu_2, ...\mu_n\}$ *is a fuzzy graph structure. Let* R_f *and* N_f *be a subgraph of G with vertices and* R_A *be the adjacency relation then*

(1) *If* $R_f \subseteq N_f$ *then* $\mathsf{F}_+[R_f] \subseteq \mathsf{F}_+[N_f]$
(2) *If* $R_f \subseteq N_f$ *then* $\mathsf{F}^+[R_f] \subseteq \mathsf{F}^+[N_f]$
(3) $\mathsf{F}_+[R_f] \cup \mathsf{F}_+[N_f] \subseteq \mathsf{F}_+[R_f \cup N_f]$
(4) $\mathsf{F}_+[R_f \cap N_f] = \mathsf{F}_+[R_f] \cap \mathsf{F}_+[N_f]$
(5) $\mathsf{F}^+[R_f] \cup \mathsf{F}^+[N_f] = \mathsf{F}^+[R_f \cup N_f]$
(6) $\mathsf{F}^+[R_f \cap N_f] \subseteq \mathsf{F}^+[R_f] \cap \mathsf{F}^+[N_f]$

Proposition 2.6. *Let* $G = \{\sigma, \mu_1, \mu_2, ...\mu_n\}$ *be a fuzzy graph structure and* $(V(G), \mathfrak{I}_F)$ *be the Fuzzy N-graph topological space.Then*

$$\mathsf{F}_+(\mathsf{F}_+(R_f)) \subseteq \mathsf{F}_+(R_f) \subseteq \mathsf{F}^+(\mathsf{F}_+(R_f)) \subseteq R_f \subseteq$$
$$\mathsf{F}_+(\mathsf{F}^+(R_f)) \subseteq \mathsf{F}^+(R_f) \subseteq \mathsf{F}^+(\mathsf{F}^+(R_f)) .$$

Proof: We will prove that $\mathsf{F}^+(\mathsf{F}_+(R_f)) \subseteq R_f \subseteq \mathsf{F}_+(\mathsf{F}^+(R_f))$. *Let* $w_1 \in \mathsf{F}^+(\mathsf{F}_+(R_f))$. *Implies, there exist* $w_2 \in \mathsf{F}_+(R_f)$ *such that* $w_1w_2 \in E(G)$. *Therefore,* $v1 \in R_A \subseteq R_f$. *Now, since* $w_1 \in R_f$ *then* $R_A(w_1) \subseteq \mathsf{F}^+(R_f)$. *Implies,* $w_1 \in \mathsf{F}_+(\mathsf{F}^+(R_f))$

3. Fuzzy N-graph structures in topological space

In this section we have studied the concept of Fuzzy N-graph structures in topological space

Definition 3.1. *Let* $G = \{\sigma, \mu_1, \mu_2, ...\mu_n\}$ *be a fuzzy graph structure and take* M_f *be a subgraph of G and* R_A *be the adjacency relation then* $\mathfrak{I}_F = \{G, \varphi, \mathsf{F}_+[R_f], \mathsf{F}^+[R_f], F_B[R_f]\}$ *forms a topology on* $V(G)$ *is called* F_+ *graph structures on G with vertices with respect to* R_f *.we call* $(G, \mathfrak{I}_F)$ *as the Fuzzy N-Graph topological space.The elements in* $\mathfrak{I}_F$ *is called Fuzzy N-open subgraphs and the complement of* $\mathfrak{I}_F^1$ *is called Fuzzy N-closed subgraphs.*

Example 3.2. *Consider example 2.3 with a fuzzy graph structure* $G = (\sigma, \mu_1, \mu_2)$, *we define a fuzzy set* $\sigma(k) = 0.3, \sigma(t) = 0.3, \sigma(i) = 0.4, \sigma(r) = 0.4$ *and define a fuzzy sets* μ_1, μ_2 *on R1,R2 is* $\mu_1(k, t) = 0.3$, $\mu_1(k, r) = 0.3, \mu_1(t, r) = 0.3, \mu_1(i, r) = 0.4, \mu_1(k, i) = 0.3, \mu_1(t, i) = 0.3$. *Let* $G = (\sigma, \mu_1, \mu_2)$ *be the fuzzy graph structure with vertices* $V(G) = \{k(0.3), t(0.3), i(0.4), r(0.4)\}$ *and the adjacency relation is* $R[k(0.3)] = \{t(0.3), r(0.4), i(0.4)\}$, $R[t(0.3)] = \{k(0.3), i(0.4), r(0.4)\}$, $R[t(0.4)] = \{k(0.3), t(0.3), r(0.4)\}$, $R[r(0.4)] = \{k(0.3), t(0.3), i(0.4)\}$ *is* R_f *is subgraph with vertices* $V(R_f) = \{k(0.3), t(0.3), r(0.4)\}$ *then* $\mathsf{F}_+[V(R_f)] = \{i(0.4)\}$, $\mathsf{F}^+[V(R_f)] = \{G\}$, $F_B[V(R_f)] = \{k(0.3), t(0.3), r(0.4)\}$ *and the fuzzy N-Graph topological space is* $\mathfrak{I}_F = \{G, \varphi, \{i(0.4)\}, \{k(0.3), t(0.3), r(0.4)\}\}$ *and its complement is* $\mathfrak{I}_F^C = \{G, \varphi, \{i(0.4)\}, \{k(0.3), t(0.3), r(0.4)\}\}$

Definition 3.3. *Let* $G = \{\sigma, \mu_1, \mu_2, ...\mu_n\}$ *be a fuzzy graph structure and* $(G, \mathfrak{I}_F)$ *be the Fuzzy N-graph topological space and consider T be* *the subgraph of* $V(G)$ *then its Fuzzy N-graph interior and Fuzzy N-graph closure is defined by* $\mathrm{F}_{Int}(T) = \bigvee\{U \in \mathfrak{I}_F : U \leq T\}$*,* $\mathrm{F}_{CL}(T) = \wedge\{U \in \mathfrak{I}_F^1 : U \geq T\}$*,where* $\mathfrak{I}_F^1$ *is the complement of* $\mathfrak{I}_F$*.That is the Fuzzy N-graph closure of T, denoted by* $\mathrm{F}_{CL}(T)$ *is the smallest fuzzy N- closed subgraph that contains T and The fuzzy N- graph interior of T denoted by* $\mathrm{F}_{Int}(T)$ *is the largest fuzzy N-open subgraph that is contained within T.*

Example 3.4. *Consider example 2.3 with a fuzzy graph structure* $G = (\sigma, \mu_1, \mu_2)$*, we define a fuzzy set* $\sigma(k) = 0.3$*,* $\sigma(t) = 0.3$*,* $\sigma(i) = 0.4$*,* $\sigma(r) = 0.4$ *and define a fuzzy sets* μ_1, μ_2 *on R1,R2 is* $\mu_1(k, t) = 0.3$*,* $\mu_1(k, r) = 0.3$*,* $\mu_1(t, r) = 0.3$*,* $\mu_1(i, r) = 0.4$*,* $\mu_1(k, i) = 0.3$*,* $\mu_1(t, i) = 0.3$*. Let* $G = (\sigma, \mu_1, \mu_2)$ *be the fuzzy graph structure with vertices* $V(G) = \{k(0.3), t(0.3), i(0.4), r(0.4)\}$ *and the adjacency relation is* $R[k(0.3)] = \{t(0.3), r(0.4), i(0.4)\}$*,* $R[t(0.3)] = \{k(0.3), i(0.4), r(0.4)\}$*,* $R[i(0.4)] = \{k(0.3), t(0.3), r(0.4)\}$*,* $R[r(0.4)] = \{k(0.3), t(0.3), i(0.4)\}$ *is* R_f *is sub-graph with vertices* $V(R_f) = \{k(0.3), i(0.3), r(0.4)\}$ *then* $\mathrm{F}_{+}[R_f] = \{i(0.4)\}$*,* $\mathrm{F}^{+}[V(R_f)] = \{G\}$ *,* $F_B[R_f] = \{k(0.3), t(0.3), r(0.4)\}$ *and the fuzzy N-Graph topological space is* $\mathfrak{I}_F = \{G, \varphi, \{i(0.4)\}, \{k(0.3), t(0.3), r(0.4)\}\}$ *and its complement is* $\mathfrak{I}^1 = \{G, \varphi, \{i(0.4)\}, \{k(0.3), t(0.3), r(0.4)\}\}$*.Let* $T = \{t(0.3), r(0.4)\}$ *be the subgraph of G then the fuzzy N-graph interior of T is* $\{\phi\}$*,and the fuzzy N-graph closure of T is* $\{k(0.3), t(0.3), r(0.4)\}$

Property 3.5. *Let* $G = \{\sigma, \mu_1, \mu_2, ...\mu_n\}$ *be a fuzzy graph structure and* $(G, \mathfrak{I}_F)$ *be the Fuzzy N-graph topological space and let T and U be the subgraph of* $V(G)$ *then*

(1) $\mathrm{F}_{Int}(T) \leq T$
(2) $\mathrm{F}_{Int}(\mathrm{F}_{Int}(T)) = \mathrm{F}_{Int}(T)$
(3) $T \leq U \Longrightarrow \mathrm{F}_{Int}(T) \leq \mathrm{F}_{Int}(U)$
(4) $\mathrm{F}_{Int}(T \wedge U) = \mathrm{F}_{Int}(T) \wedge \mathrm{F}_{Int}(U)$
(5) $T \leq \mathrm{F}_{cl}(T) \Longrightarrow \mathrm{F}_{cl}(\mathrm{F}_{cl}(T)) = \mathrm{F}_{cl}(T)$
(6) $T \leq U \Longrightarrow \mathrm{F}_{cl}(T) \leq F_{cl}(U)$
(7) $\mathrm{F}_{cl}(T \vee U) = \mathrm{F}_{cl}(T) \vee \mathrm{F}_{cl}(U)$

Property 3.6. *Let* $G = \{\sigma, \mu_1, \mu_2, ...\mu_n\}$ *be a fuzzy graph structure and* $(G, \mathfrak{I}_F)$ *be the Fuzzy N-graph topological space and let T and U be the subgraph of G then*

(1) $\mathrm{F}_{Int}(G) \subseteq \mathrm{F}_{Cl}(G)$
(2) *If* $T \subseteq G$ *then* $\mathrm{F}_{int}(T) \subseteq \mathrm{F}_{int}(G) \subseteq V(G)$
(3) $\mathrm{F}_{Int}(T \cap U) = \mathrm{F}_{Int}(T) \cap \mathrm{F}_{Int}(U)$
(4) $\mathrm{F}_{Int}(T \cup U) \supset \mathrm{F}_{Int}(T) \cup \mathrm{F}_{Int}(U)$
(5) *If* $T \subset G$ *then* $\mathrm{F}_{Cl}(T) \subset V(G) \subset \mathrm{F}_{Cl}(G)$
(6) $\mathrm{F}_{Cl}(T \cap U) \cap \mathrm{F}_{Cl}(T) \cap \mathrm{F}_{Cl}(U)$

Theorem 3.7. *Let* $G = \{\sigma, \mu_1, \mu_2, ...\mu_n\}$ *be a fuzzy graph structure for a subgraph* T *of* G *then* $p \in \mathrm{F}_{Cl}(T) \Longleftrightarrow G_N \cap T \neq \phi$ *for every Fuzzy N- open subgraph* G_N *containing t,where* $T \subseteq G$

Proof:

Consider $t \in \mathrm{F}_{Cl}(T)$*.Let* G_N *be a fuzzy N- open subgraph containing t.Since* G_N *is fuzzy N- open subgraph ,* $G - G_N$ *is a fuzzy N- closed subgraph.If* $T \cap G_N = \phi$*, then* $T \subseteq G - G_N$ *,* $G - G_N$ *is a fuzzy N- closed subgraph containing T.Thus* $\mathrm{N}_h Cl[T] \subseteq G - G_N$ *, which is a contradiction to the fact that* $t \in \mathrm{N}_h Cl[T]$ *but* $a \notin G - G_N$ *.Therefore* $T \cap G_N \neq \phi$ *for every fuzzy N-open subgraph* G_N *containing t.conversly,Let* $P \cap G_N \neq \phi$ *for every fuzzy N-open subgraph* G_N *contaning t.If* $t \notin \mathrm{F}_{Cl}(T)$ *then* $a \in G - \mathrm{F}_{Cl}(T))$*,*$G - (\mathrm{F}_{Cl}(T))$ *is fuzzy N- open subgraph and thus* $G - ((\mathrm{F}_{Cl}(T)) \cap T) \neq \phi$ *(By assumption)* $P \subseteq \mathrm{F}_{Cl}(T) \Longrightarrow \mathrm{F}_{Cl}(T)^c \subseteq T^c \Longrightarrow G - (\mathrm{F}_{Cl}(T)) \subseteq G - T \Longrightarrow G - ((\mathrm{F}_{Cl}(T)) \cap T) \subseteq (G - T) \cap T$*.which is contradiction.Therefore* $t \in \mathrm{F}_{Cl}(T)$*.*

Definition 3.8. *Let* $G = \{\sigma, \mu_1, \mu_2, ...\mu_n\}$ *be a fuzzy graph structure and* R_A *be the adjacency relation . Consider* D_i*,* $i = 1, 2, 3, 4, 5$ *be any five different subgraphs of* G *, Some forms of fuzzy N-graph topology defined as follows:*

FG-Form I	$F_*(D_i) \neq F^*(D_i)$, $F_*(D_i) = F^*(D_i)$, where $F_*(D_i) \neq \phi, F^*(D_i) \neq G$
FG-Form II	$F_*(D_i) = \phi, F^*(D_i) \neq G$
PG-Form III	$F_*(D_i) \neq \phi$ and $F^*(D_i) = G$
PG-Form IV	$F_*(D_i) = \phi, F^*(D_i) = G$

Result of FG-Form I	$F(D_i) = \{G, \phi, F_*(D_i)\}$
Result of FG-Form II	$F(D_i) = \{G, \phi, F^*(D_i)$
Result of FG-Form III	$F(D_i) = \{G, \phi, F_*(D_i), B_G\{(D_i)\}$
Result of FG-Form IV	$F(D_i) = \{G, \phi\}$

3.0.1. **Remark.** *After classifying the fuzzy N-forms we will get the following results:*

	S_i	S_i^c
Result 1	ηF_1	ηF_1
Result 2	ηF_2	ηF_3
Result 3	ηF_3	ηF_2
Result 4	ηF_4	ηF_4

4. CONCLUSION AND APPLICATION

Graph theory plays a crucial role in solving problems across a wide range of fields, besides socialing, communication, determining data, as in gathering, image capturing, image segmentation, planning, and scheduling. In many real-world scenarios, the systems we model and analyze using graphs involve uncertainty and imprecision. This is where *fuzzy graph theory comes into play, providing a powerful framework for handling situations where elements of a graph—such as vertices and edges—are not precisely defined but instead exhibit varying degrees of membership.*

Fuzzy Graph Theory: Dealing with Uncertainty*:*

Fuzzy graph theory extends classical graph theory by incorporating fuzzy logic to represent uncertainty or vagueness within the system. Unlike traditional graphs, where the connections (edges) between vertices are binary (either an edge exists or it does not), fuzzy graphs allow for edges to have degrees of membership. This means that the existence of an edge between two vertices is represented by a fuzzy value, typically ranging from 0 (no connection) to 1 (full connection). This flexibility is particularly useful when dealing with systems that exhibit ambiguous or imprecise relationships between components.

Applications of Fuzzy Graph Theory*:*

Fuzzy graph theory has a broad spectrum of applications, particularly in domains where systems are inherently uncertain or imprecise. Some of the key applications include:

Networking and Communication:

In communication networks, there may be varying degrees of connectivity between nodes due to signal strength, bandwidth, or other factors. Fuzzy graph theory helps model these varying levels of connectivity, enabling better network design, optimization, and fault-tolerant routing.

Data Mining and Clustering:

In data mining, objects are often grouped into clusters based on their similarity. However, not all objects fit neatly into a single cluster. Fuzzy graph theory is useful for clustering, where each object may belong to multiple clusters to different extents. This enables more accurate representations of complex relationships in data.

Image Processing (Image Capturing and Segmentation):

In image processing, particularly in segmentation tasks, fuzzy graphs are used to capture the uncertainty inherent in segmenting an image into regions. The edges between image pixels (or regions) can have fuzzy values, representing the degree to which a boundary exists between them. This approach improves the quality of segmentation, especially in cases where boundaries are not clearly defined.

Planning and Scheduling:

In real-time planning and scheduling problems, activities or tasks may have different degrees of priority or may be uncertain due to resource limitations or external conditions. Fuzzy graphs can model these varying degrees of certainty, helping optimize schedules and plans in dynamic environments.

Fuzzy N-Graphs and Topological Spaces*:*

One of the more advanced areas of fuzzy graph theory is the concept of Fuzzy N-Graphs in Topological Spaces. This approach combines the *principles of fuzzy logic, graph theory, and topology to model complex systems in a highly flexible way. By incorporating the topological structure of a space, fuzzy N-graphs allow for the representation of continuous relationships and transitions between different parts of a system, with fuzzy logic providing the mechanism to handle uncertainty in these transitions.*

Fuzzy Subgraphs:

Subgraphs within the larger graph that also carry fuzzy relationships, allowing for local uncertainty in different parts of the system. Boundaries: In topological terms, boundaries in fuzzy graphs can be represented by fuzzy edges, indicating uncertain transitions or boundaries between regions. Adjacency Relations: These relations can be defined with varying degrees of strength, where vertices may be adjacent with different levels of certainty, depending on the context.

By studying fuzzy N-graphs, researchers can model and analyze real-world systems where the relationships between components are not deterministic but have varying degrees of confidence or fuzziness.

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