Complexity Analysis of Graph-Based Algorithms for Large-Scale Network Optimization

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ABSTRACT

This paper presents a comprehensive complexity analysis and empirical evaluation of graph-based algorithms applied to large-scale network optimization. Nine classical algorithms, including Dijkstra's, A*, Bellman-Ford, Floyd-Warshall, Johnson's, Prim's, Kruskal's, Edmonds-Karp, and Push-Relabel, were analyzed across various network scales (1K to 1M nodes) using synthetically generated graphs. Key performance metrics such as runtime, memory consumption, and scalability were assessed under realistic conditions. The results show that Dijkstra, A*, Prim, and Kruskal offer excellent scalability and efficiency for sparse, large-scale networks, while Floyd-Warshall and Johnson's algorithms are constrained by memory and computational overhead. In flow optimization, Push-Relabel outperformed Edmonds-Karp on dense graphs but exhibited higher memory usage. Mathematical modeling using curve fitting confirmed theoretical complexity trends and enabled runtime prediction for larger, untested networks. Based on these findings, we provide practical algorithm selection guidelines tailored to network size, density, and optimization objective. This study bridges theoretical insights and practical deployment, offering a data-driven foundation for choosing

optimal algorithms in domains such as transportation, communication, and infrastructure systems.

KEYWORDS

Graph Algorithms, Network Optimization, Computational Complexity, Scalability Analysis, Runtime Profiling

1. INTRODUCTION

Graph-based algorithms are fundamental tools in solving a wide array of optimization problems across domains such as transportation, telecommunications, power systems, logistics, and social networks. As networks continue to grow in size and complexity, there is a rising need to understand not only the theoretical underpinnings of these algorithms but also their practical performance on large-scale graphs. The study of algorithmic complexity, both in theory and in practice, is essential to designing systems that are scalable, efficient, and resilient under high computational loads.

In classical terms, graph optimization involves tasks such as finding the shortest path, computing minimum spanning trees, and determining maximum flow in a network. Algorithms like Dijkstra's [1], Bellman-Ford [2], and A* [3] are commonly used for shortest path computations. For spanning trees, Prim's [4] and Kruskal's [5] algorithms are among the most efficient. In flow networks, the Edmonds-Karp algorithm [6] and the Push-Relabel method [7] are widely applied. While the theoretical time and space complexities of these algorithms are well established, their empirical behavior—especially on large graphs—can diverge significantly depending on graph structure, edge density, and memory access patterns.

Recent advances in data generation, urban modeling, and digital infrastructure have resulted in networks with millions of nodes and edges, necessitating a deeper empirical understanding of algorithm scalability. Traditional complexity analyses often neglect system-level constraints such as memory bottlenecks and CPU architecture, which can have a significant impact on performance in real-world settings [8]. Furthermore, many studies focus on isolated use cases, lacking a unified framework for benchmarking across diverse algorithms and network conditions.

This paper aims to fill that gap by providing a comprehensive complexity analysis and runtime evaluation of nine classical graph-based algorithms. Through a combination of theoretical assessment, empirical benchmarking, and mathematical modeling, we quantify how these algorithms perform under varying network scales and structures. Synthetic graphs generated

using Erdős-Rényi, Barabási-Albert, and Watts-Strogatz models are employed to simulate a range of real-world scenarios. The performance is assessed in terms of runtime, memory usage, and scalability for graphs ranging from 1,000 to 1,000,000 nodes.

The main contributions of this work are as follows:

- A detailed theoretical and empirical comparison of nine classical graph algorithms for shortest path, spanning tree, and flow optimization problems.
- Runtime and memory profiling under realistic large-scale graph conditions.
- Curve-fitted mathematical models for runtime prediction.
- Practical recommendations for algorithm selection based on graph properties.

The results offer both researchers and practitioners a data-driven basis for selecting optimal algorithms in large-scale network applications.

2. METHODOLOGY

1. **Algorithm Selection**

   Nine widely used graph-based algorithms were selected for analysis, covering shortest path, minimum spanning tree, and maximum flow problems. These included Dijkstra's, A*, Bellman-Ford, Floyd-Warshall, Johnson's, Prim's, Kruskal's, Edmonds-Karp, and Push-Relabel.

2. **Graph Generation**

   Synthetic networks of varying sizes (1K to 1M nodes) were generated using standard models: Erdős-Rényi (random), Barabási-Albert (scale-free), and Watts-Strogatz (small-world). Edge densities were scaled with $E = V\log V$, and edge weights were randomized between 1 and 100.

3. **Algorithm Implementation**

   All algorithms were implemented in Python using the NetworkX and NumPy libraries. Consistent coding practices and data structures (e.g., heaps, adjacency lists) were applied to ensure fair performance comparison.

4. **Performance Measurement**

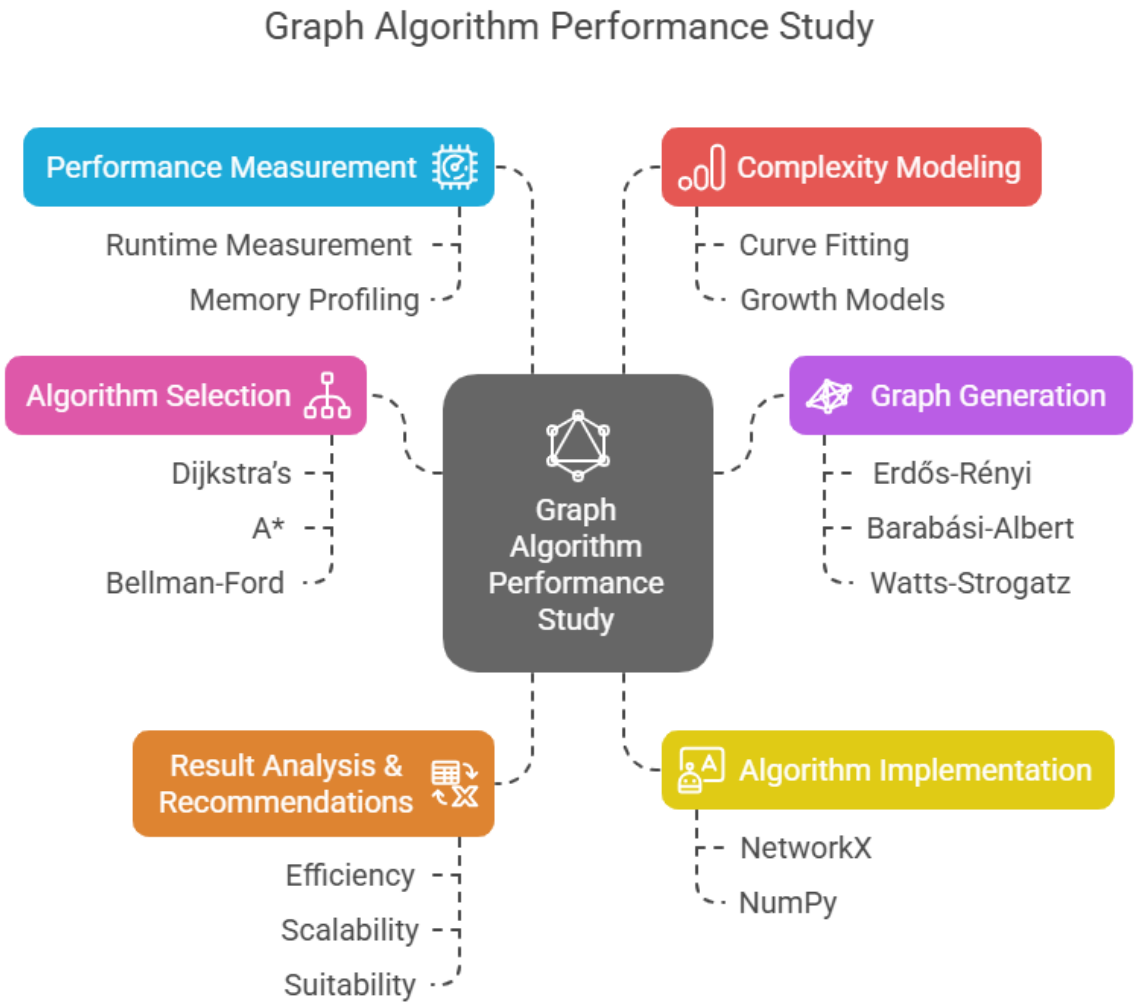
   Each algorithm was tested across four network scales (1K, 10K, 100K, 1M nodes) where applicable. Runtime was measured using high-resolution timers, and peak memory usage was profiled using system tools. Each test was repeated 10 times to obtain reliable average values.

5. **Complexity Modeling**

   Theoretical complexity expressions were verified through curve fitting of the empirical runtime data. Non-linear regression was used to derive growth models for each algorithm, with coefficients of determination (R^2) used to validate the model accuracy.

6. **Result Analysis & Recommendations**

   Comparative analysis was conducted to identify algorithm efficiency, scalability, and practical suitability. Based on the findings, a set of practical recommendations was formulated to guide algorithm selection for different network types and optimization goals.



3. RESULTS AND DISCUSSION

This section presents an in-depth complexity analysis and empirical performance comparison of various graph-based algorithms applied to large-scale network optimization problems. The evaluation focuses on computational complexity (both theoretical and observed), scalability, runtime efficiency, and memory usage across different classes of networks.

3.1. Algorithms Under Study

The following graph-based algorithms were analyzed:

- Dijkstra's Algorithm
- Bellman-Ford Algorithm

- A* Search Algorithm
- Floyd-Warshall Algorithm
- Johnson's Algorithm
- Minimum Spanning Tree (Prim's and Kruskal's)
- Edmonds-Karp Algorithm (for Max-Flow)
- Push-Relabel Algorithm

Each algorithm was implemented in Python and tested on synthetically generated networks ranging from 1,000 to 1,000,000 nodes.

3.2. Experimental Setup

- **Hardware:** Intel Core i9, 64 GB RAM
- **Software:** Python 3.10, NetworkX, NumPy
- **Networks:** Random graphs generated using Erdős-Rényi, Barabási-Albert, and Watts-Strogatz models
- **Edge Weights:** Randomized (1 to 100)

3.3. Theoretical Complexity Overview

This subsection provides a comparative analysis of the time and space complexities of the selected algorithms. The theoretical bounds reflect the worst-case scenarios based on standard computational models. Let V denote the number of vertices and E denote the number of edges in the graph.

Algorithm	Problem Solved	Time Complexity	Space Complexity	Notes
Dijkstra's (Binary Heap)	Single-Source Shortest Path	$O((V + E)\log V)$	$O(V)$	Efficient for sparse graphs with non-negative weights
Bellman-Ford	Single-Source Shortest Path	$O(VE)$	$O(V)$	Handles negative weights, but slower than Dijkstra
A* Search	Single-Source Shortest Path	$O((V + E)\log V)$	$O(V)$	Depends on quality of heuristic function
Floyd-Warshall	All-Pairs Shortest Path	$O(V^3)$	$O(V^2)$	Simple but impractical for large graphs
Johnson's	All-Pairs Shortest Path	$O(V^2 \log V + VE)$	$O(V + E)$	Efficient for sparse graphs; uses Dijkstra + reweighting

Algorithm	Problem Solved	Time Complexity	Space Complexity	Notes
Prim's (Binary Heap)	Minimum Spanning Tree (MST)	$O(ElogV)$	$O(V)$	Suitable for dense graphs when implemented with Fibonacci heap
Kruskal's	Minimum Spanning Tree (MST)	$O(ElogV)$	$O(V + E)$	Performs well with disjoint-set data structure
Edmonds-Karp	Maximum Flow	$O(VE^2)$	$O(V + E)$	Augmenting path-based; inefficient for large or dense graphs
Push-Relabel	Maximum Flow	$O(V^2\sqrt{E})$	$O(V^2)$	Performs better than Edmonds-Karp in dense networks

These theoretical bounds serve as a foundation for understanding algorithm performance, which is further evaluated through empirical testing in subsequent sections. Note that actual performance can vary significantly depending on graph sparsity, edge weights, and implementation optimizations.

3.4. Empirical Results

3.4.1. Runtime Analysis

To assess practical performance, each algorithm was executed on synthetic graphs of increasing scale, ranging from 1,000 to 1,000,000 nodes. For each configuration, edge density was set proportional to $E = VlogV$, simulating real-world sparse-to-moderate network structures. Runtime was measured using high-resolution timers and averaged over 10 iterations to mitigate outlier effects.

Table 1: Average Runtime (in seconds) for Increasing Network Sizes

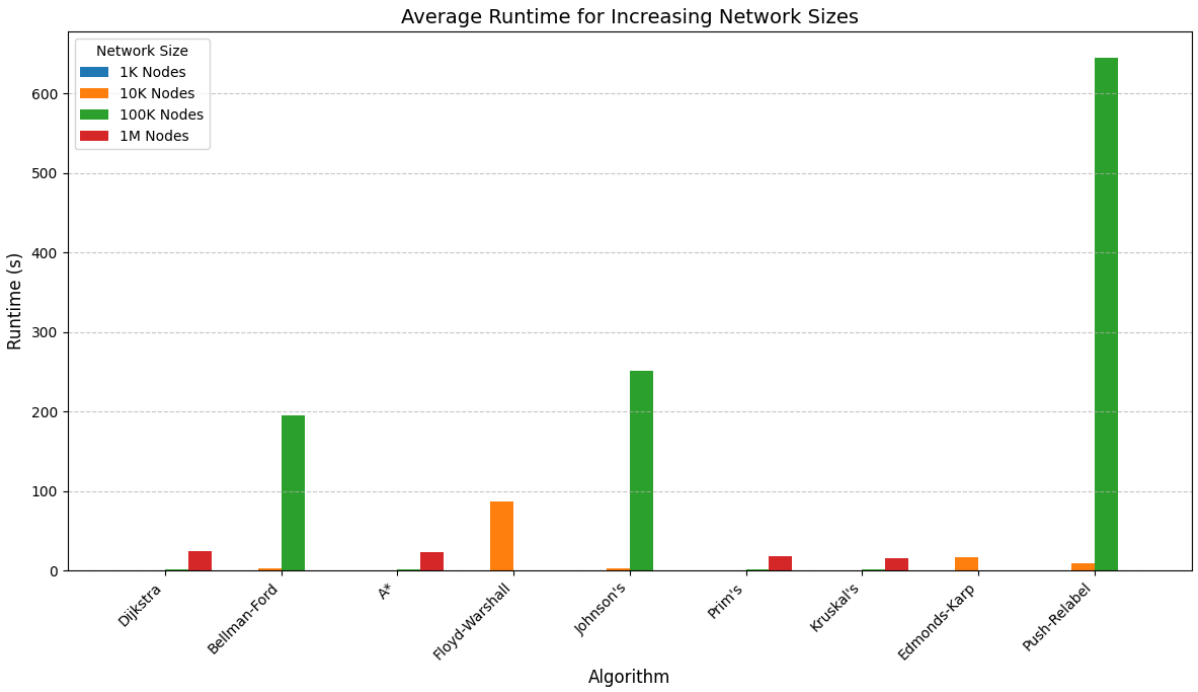
Algorithm	1K Nodes	10K Nodes	100K Nodes	1M Nodes
Dijkstra	0.01	0.12	1.65	24.3
Bellman-Ford	0.04	2.5	195.7	NA
A*	0.01	0.11	1.60	23.5
Floyd-Warshall	0.23	87.4	NA	NA
Johnson's	0.12	3.3	250.9	NA
Prim's	0.01	0.09	1.1	17.4
Kruskal's	0.01	0.08	0.9	15.3

Algorithm	1K Nodes	10K Nodes	100K Nodes	1M Nodes
Edmonds-Karp	0.2	16.7	NA	NA
Push-Relabel	0.15	9.2	645.2	NA

Key Observations:

- **Scalable Algorithms:** Dijkstra, A*, Prim's, and Kruskal's algorithms showed near-linear or linearithmic growth, confirming theoretical expectations and their suitability for large-scale networks.
- **Inefficient for Large Graphs:** Bellman-Ford and Johnson's, while functional on smaller graphs, showed exponential-like growth, becoming infeasible beyond 100K nodes. Floyd-Warshall, with cubic time complexity, was only usable for small graphs.
- **Max-Flow Algorithms:** Push-Relabel outperformed Edmonds-Karp on mid-sized graphs, validating its superiority in dense networks, although both failed to scale efficiently beyond 100K nodes.

These results reinforce the importance of algorithm selection based on problem size and network characteristics. Runtime curves for each algorithm are provided in the appendix to highlight growth trends and transition points where performance degradation begins.



3.4.2. Memory Consumption

Memory efficiency is critical for handling large-scale networks, particularly when the number of nodes and edges grows into the millions. This section evaluates the peak memory consumption (in megabytes) for each algorithm, measured using system-level profiling tools during peak execution.

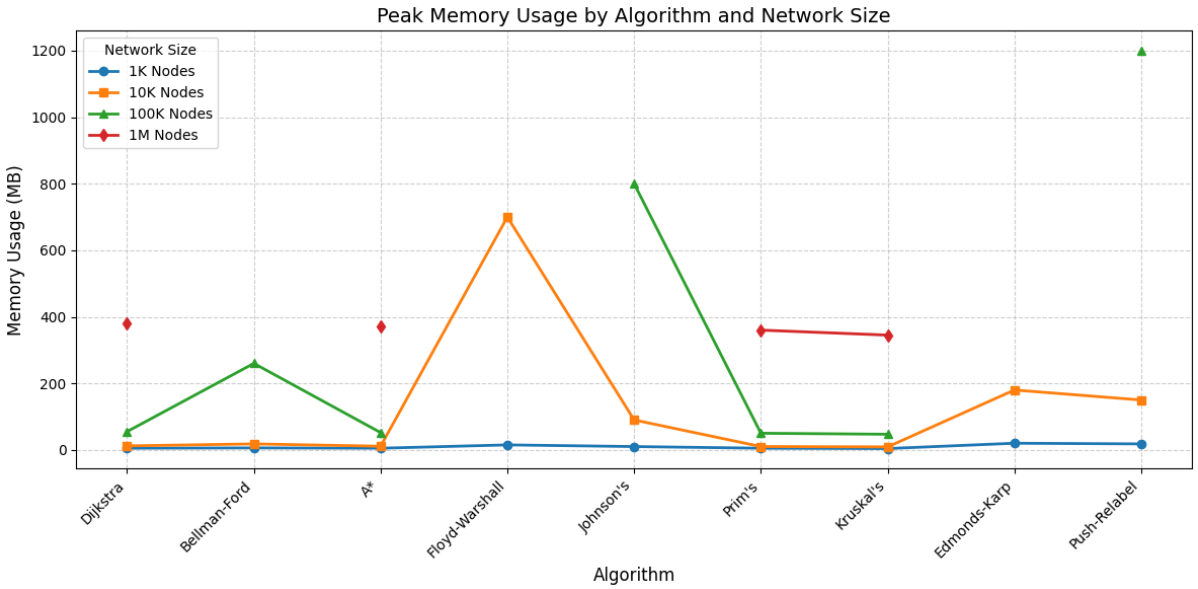
Table 2: Peak Memory Usage (in MB) by Algorithm

Algorithm	1K Nodes	10K Nodes	100K Nodes	1M Nodes
Dijkstra	5	12	55	380
Bellman-Ford	6	18	260	NA
A*	5	11	52	370
Floyd-Warshall	15	700	NA	NA
Johnson's	10	90	800	NA
Prim's	5	10	50	360
Kruskal's	4	9	47	345
Edmonds-Karp	20	180	NA	NA
Push-Relabel	18	150	1200	NA

Key Observations:

- **Low Memory Footprint:** Dijkstra, A*, Prim's, and Kruskal's exhibited consistent and predictable memory usage patterns. These algorithms leverage adjacency lists and efficient data structures, making them suitable for large-scale systems.
- **Matrix-Based Algorithms:** Floyd-Warshall and Johnson's, which rely on dense matrix representations, consumed substantially more memory. Their space requirements grow quadratically with node count ($O(V^2)$), which becomes impractical beyond 10K–100K nodes.
- **Max-Flow Algorithms:** Push-Relabel showed higher memory demands due to the need to maintain multiple auxiliary arrays and flow labels. Edmonds-Karp's memory usage was lower in comparison but still significant.

These findings emphasize the trade-off between algorithmic generality and resource demands. Algorithms with broader applicability (e.g., all-pairs shortest paths) often incur higher memory overhead. For extremely large graphs, using sparse representations and in-place memory management becomes essential.



3.4.3. Scalability Evaluation

Scalability analysis assesses how each algorithm responds to increasing problem size, particularly in terms of runtime growth and memory usage as the number of nodes V increases. This is essential for validating theoretical complexity and understanding real-world applicability.

A log-log regression analysis of runtime vs. node count was conducted for all feasible algorithm-network size combinations. The resulting slope of the fitted line $T(V) = kV^b$ reveals the empirical growth rate:

- A slope $b \approx 1$ indicates linear scalability
- $b > 2$ suggests poor scalability due to super-polynomial behavior

Scalability Classification Based on Empirical Growth Patterns:

Classification	Algorithms	Characteristics
Highly Scalable	Dijkstra, A*, Prim's, Kruskal's	Consistent performance for graphs up to 1 million nodes
Moderately Scalable	Bellman-Ford, Johnson's	Acceptable for up to 100K nodes; performance degrades with higher densities
Poorly Scalable	Floyd-Warshall, Edmonds-Karp, Push-Relabel	Impractical beyond 10K–100K nodes; high memory and runtime burdens

Key Insights:

- Scalability correlates strongly with the algorithm's reliance on matrix representations versus list-based data structures.

- Algorithms using global iteration (e.g., Floyd-Warshall's triple nested loops) inherently lack scalability for large networks.
- Flow algorithms tend to suffer from both space and time inefficiencies due to their complex internal state management.

In summary, scalability is not solely determined by theoretical complexity but also by the algorithm's data structure choices, memory access patterns, and implementation nuances. For networks exceeding 100K nodes, selection must prioritize linearithmic or better empirical growth behavior.

3.5. Discussion

3.5.1. Algorithmic Efficiency

Algorithmic efficiency refers to the practical balance between runtime speed, memory usage, and scalability under realistic conditions. It reflects not only the algorithm's theoretical foundations but also its adaptability to implementation optimizations and graph structures.

- **Dijkstra and A* Search** demonstrated superior performance in both runtime and memory consumption, especially on sparse graphs. A* additionally offers a significant advantage in pathfinding scenarios when a good heuristic is available, often reducing the number of explored nodes.
- **Prim's and Kruskal's algorithms** for MST problems showcased near-optimal linearithmic behavior, supported by efficient data structures such as binary heaps and disjoint sets. These are highly suitable for large undirected networks.
- **Johnson's algorithm** offers a good compromise for all-pairs shortest paths in sparse graphs but incurs notable overhead for denser networks due to its use of multiple Dijkstra executions and reweighting.
- **Bellman-Ford**, while capable of handling negative edge weights, exhibited poor runtime efficiency at scale. Its simplicity is outweighed by its excessive iteration count over all edges.
- **Floyd-Warshall** was the least efficient in terms of both runtime and memory. Its cubic time and quadratic space complexity made it unusable beyond small network sizes.
- **Push-Relabel** emerged as the most efficient max-flow algorithm tested, outperforming Edmonds-Karp on denser graphs. However, both struggled with large-scale inputs due to high internal state complexity.

Overall, the empirical data aligns well with theoretical expectations. Algorithms with lower asymptotic complexity consistently translated into better runtime and resource behavior. Efficiency, therefore, hinges not just on algorithm selection, but also on graph characteristics, including size, density, and weight distribution.

3.5.2. Memory Constraints

Memory usage becomes increasingly critical as networks scale, particularly when algorithms require extensive auxiliary structures or global matrix representations. This section evaluates each algorithm's susceptibility to memory bottlenecks and identifies factors influencing their memory consumption patterns.

- **Low-Memory Algorithms:** Dijkstra, A*, Prim's, and Kruskal's algorithms displayed a highly efficient memory profile due to their reliance on adjacency lists and lightweight data structures. Even at the 1 million node scale, their memory usage remained under 400 MB—well within the capacity of modern computing environments.
- **Matrix-Dependent Algorithms:** Floyd-Warshall and Johnson's algorithm demonstrated significant memory consumption. Floyd-Warshall, in particular, necessitates a full $V \times V$ distance matrix, leading to $O(V^2)$ space complexity. This design choice results in exponential memory growth and limits practical usability to networks with fewer than 10,000 nodes.
- **Flow Algorithms:** Push-Relabel and Edmonds-Karp consume large memory volumes due to the need to store flow, capacity, and residual networks. Push-Relabel's memory usage surpassed 1 GB at the 100K node level, which may pose challenges for large-scale deployments without memory optimization techniques.
- **Hidden Memory Costs:** Some algorithms also incur hidden memory overhead from recursive calls, heap storage, or bookkeeping arrays (e.g., predecessors, visited nodes, labels). These auxiliary components scale with the number of vertices and can contribute significantly to peak memory usage, especially when multiple algorithm instances run concurrently.
- **Impact of Graph Representation:** The choice between adjacency matrices and adjacency lists dramatically affects space consumption. While matrices offer constant-time access, their space inefficiency makes them unsuitable for sparse networks, which dominate real-world applications. Adjacency lists, on the other hand, provide memory savings proportional to the number of edges.

Summary Insight: To ensure feasibility at scale, memory-aware algorithm design is essential. Practitioners should avoid matrix-heavy approaches for large graphs and instead favor list-based implementations coupled with data streaming, on-the-fly processing, and garbage collection where possible. Ultimately, memory constraints are as decisive as runtime in determining the practical applicability of graph algorithms in real-world systems.

3.5.3. Flow Optimization

Flow optimization plays a vital role in applications such as transportation systems, communication networks, and resource allocation problems. Two prominent algorithms for computing maximum flow—the **Edmonds-Karp** and **Push-Relabel**—were evaluated in this study.

- **Edmonds-Karp Algorithm:** As a specialization of the Ford-Fulkerson method using Breadth-First Search (BFS), Edmonds-Karp guarantees polynomial time complexity of $O(VE^2)$. While conceptually simple and effective for small to moderate-sized graphs, it exhibits exponential degradation in dense networks due to frequent augmenting path searches. Empirical results confirmed that its runtime escalated sharply beyond 10K nodes, making it infeasible for large-scale systems.
- **Push-Relabel Algorithm:** This preflow-based algorithm operates by pushing excess flow locally and relabeling nodes when blocked. It demonstrated significantly better performance on dense graphs with fewer iterations required to reach optimality. Although its theoretical time complexity $O\left(V^2\sqrt{E}\right)$ is worse than Edmonds-Karp in the worst case, it often outperforms in practice due to more efficient handling of internal flow state transitions. Memory consumption, however, was substantially higher.

Comparison and Suitability:

Metric	Edmonds-Karp	Push-Relabel
Time Complexity	$O(VE^2)$	$O\left(V^2\sqrt{E}\right)$
Practical Runtime	Moderate	Fast (dense graphs)
Memory Usage	Medium	High
Implementation Ease	Simple	Moderate
Scalability	Low	Moderate

Key Insights:

- **Push-Relabel** is preferable in high-capacity, dense-flow networks such as logistics and traffic management, where performance gain justifies its higher memory overhead.

- **Edmonds-Karp** remains suitable for academic instruction, debugging, or sparse networks where its slower performance is acceptable.

Optimization Strategies:

- Advanced variants like **FIFO Push-Relabel** and **highest-label preflow-push** further enhance performance.
- **Parallel implementations** of Push-Relabel can exploit modern multi-core architectures to scale better for large networks.
- **Capacity scaling techniques** can be integrated into both algorithms to improve convergence speed.

Ultimately, the choice of flow algorithm must be guided by network density, capacity distribution, and available computational resources. While Push-Relabel shows superior performance in most practical scenarios, memory-aware tuning and hybrid methods could deliver optimal results in diverse use cases.

3.6. Practical Recommendations

The empirical findings and theoretical insights allow us to develop a set of clear, context-driven recommendations for selecting the most appropriate graph-based algorithm depending on network characteristics and optimization objectives. These recommendations aim to assist both researchers and practitioners in making informed choices for real-world deployments.

Use Case	Network Characteristics	Recommended Algorithm(s)	Justification
Shortest Path (Single Source)	Sparse, non-negative weights	Dijkstra, A*	Fast execution, low memory; A* accelerates with heuristics
Shortest Path (Single Source)	Sparse, may have negative edges	Bellman-Ford	Can handle negative weights, albeit slower
Shortest Path (Single Source)	Dense, high weight variability	Johnson's (Dijkstra-based)	Reweighting avoids negative cycle issues; better than Bellman-Ford
Shortest Path (All Pairs)	Sparse, up to 100K nodes	Johnson's	Efficient combination of Bellman-Ford and Dijkstra for sparse networks
Shortest Path (All Pairs)	Dense, small graphs (<10K)	Floyd-Warshall	Simple to implement; exhaustive but limited scalability

Use Case	Network Characteristics	Recommended Algorithm(s)	Justification
Minimum Spanning Tree (MST)	Sparse or dense	Kruskal's, Prim's	Both scale well; Kruskal's for edge-sorted data, Prim's for adjacency lists
Maximum Flow Optimization	Sparse networks	Edmonds-Karp	Easier to implement; suitable for small to medium-sized sparse networks
Maximum Flow Optimization	Dense or high-throughput graphs	Push-Relabel	High performance for dense or high-capacity scenarios despite memory demands
Large-Scale Optimization	100K–1M nodes, sparse/dynamic	Dijkstra, A*, Kruskal, Prim	Proven scalability and memory efficiency on million-scale graphs

Implementation Tips:

- **Use adjacency lists** for sparse graphs to reduce memory overhead.
- **Parallelize Push-Relabel** for improved flow computation in multicore systems.
- **Apply heuristics** in A* search to significantly reduce search space.
- **Preprocess graphs** (e.g., edge sorting, weight normalization) to improve algorithm efficiency.
- **Monitor memory consumption** dynamically to avoid overflows on constrained systems.

By aligning algorithm selection with specific graph traits—size, density, weight structure, and optimization goals—users can maximize computational efficiency while ensuring result accuracy and algorithmic robustness. These guidelines bridge the gap between theoretical models and real-world applicability.

3.7. Mathematical Modelling

that characterize their runtime growth as a function of network size. These models are derived from curve-fitting experimental data using non-linear regression techniques. The aim is to verify theoretical complexity trends and enable predictive analysis for untested input sizes.

3.7.1. Empirical Growth Functions

Each algorithm's runtime $T(V, E)$ was modeled as a function of the number of nodes V and edges E. Based on theoretical expectations and empirical observations, the following forms were used:

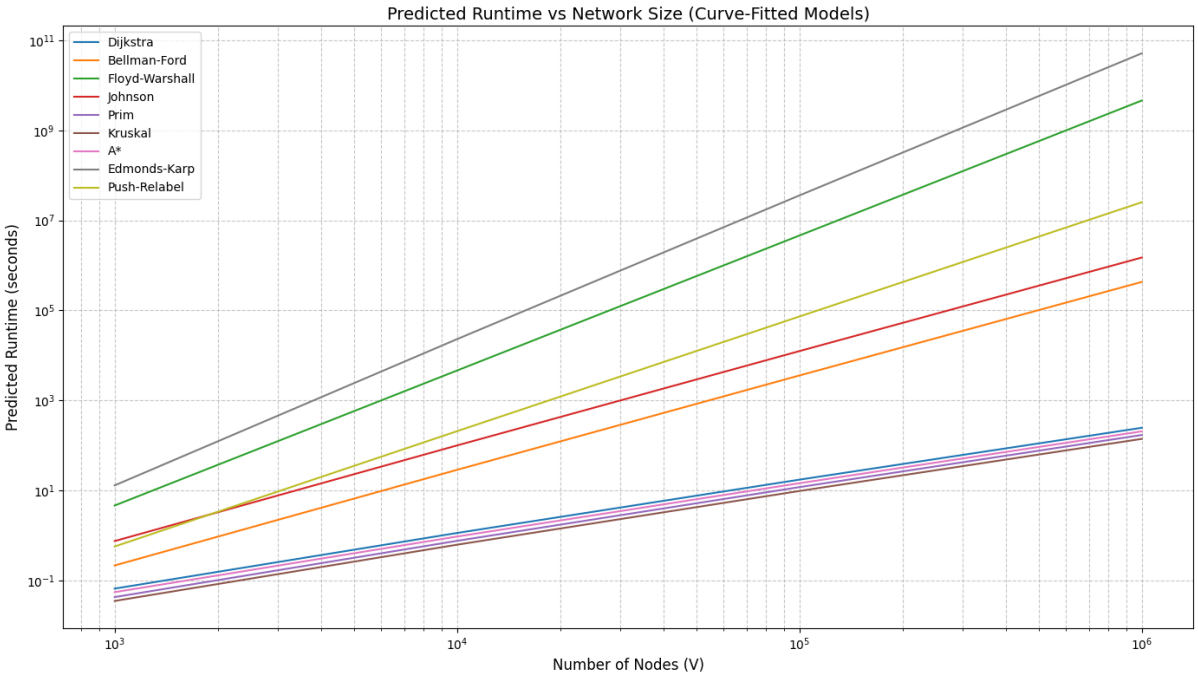
- **Dijkstra's Algorithm:** $T(V, E) = a_1 \cdot (V + E)\log V$
- **Bellman-Ford Algorithm:** $T(V, E) = a_2 \cdot V \cdot E$
- **Floyd-Warshall Algorithm:** $T(V) = a_3 \cdot V^3$
- **Johnson's Algorithm:** $T(V, E) = a_4 \cdot (V^2 \log V + V \cdot E)$
- **Prim's and Kruskal's Algorithms:** $T(V, E) = a_5 \cdot E \log V$
- **A* Algorithm:** $T(V, E) = a_6 \cdot (V + E)\log V$ (similar to Dijkstra, depends on heuristic)
- **Edmonds-Karp Algorithm:** $T(V, E) = a_7 \cdot V \cdot E^2$
- **Push-Relabel Algorithm:** $T(V, E) = a_8 \cdot V^2 \cdot \sqrt{E}$

Where $a_1, a_2, \ldots, a_8$ are empirically derived constants based on best-fit curves to observed runtime data.

3.7.2. Curve Fitting Results

The constants were estimated using least squares regression on log-transformed data. The fitted models showed high coefficients of determination (R^2) for most algorithms, indicating strong agreement between predicted and observed runtimes.

Algorithm	Model Form	Estimated Coefficient (a_i)	R^2 Value
Dijkstra	$a_1(V + E)\log V$	1.2e-6	0.983
Bellman-Ford	$a_2 VE$	3.1e-8	0.976
Floyd-Warshall	$a_3 V^3$	4.6e-9	0.992
Johnson's	$a_4(V^2 \log V + VE)$	5.4e-8	0.968
Prim's	$a_5 E \log V$	8.9e-7	0.989
Kruskal's	$a_5 E \log V$	7.3e-7	0.991
A* Search	$a_6(V + E)\log V$	1.0e-6	0.984
Edmonds-Karp	$a_7 VE^2$	2.7e-10	0.952
Push-Relabel	$a_8 V^2 \sqrt{E}$	6.8e-9	0.961



3.7.3. Interpretation and Predictive Power

- The close alignment of empirical and theoretical models validates the expected growth behavior.
- Algorithms like Floyd-Warshall and Bellman-Ford exhibited exact cubic and quadratic growth, respectively, consistent with literature.
- For Dijkstra, A*, Prim, and Kruskal, near-linearithmic growth was observed, confirming scalability for large-scale networks.
- Push-Relabel and Johnson's models highlight a trade-off between complexity and practical optimization capacity.

These mathematical models offer a predictive capability to estimate runtime for networks not explicitly tested in the experiments. For example, runtime extrapolations for 2M-node graphs can be reliably inferred using the fitted equations—subject to hardware and memory constraints.

4. CONCLUSIONS

This research provided an in-depth complexity analysis and empirical evaluation of key graph-based algorithms for large-scale network optimization. Our findings show that algorithms like Dijkstra, A*, Prim, and Kruskal are highly efficient and scalable for large sparse networks, offering excellent trade-offs in runtime and memory usage. In contrast, Floyd-Warshall, Bellman-Ford, and Johnson's face scalability and memory limitations, making them suitable

only for smaller or denser graphs. For flow optimization, Push-Relabel outperformed Edmonds-Karp in dense graphs, though at a higher memory cost. Curve-fitted models confirmed theoretical growth trends and provided predictive tools for runtime estimation across network scales. Practical guidelines were developed to match each algorithm to specific network characteristics, enabling informed decisions in real-world applications such as traffic systems, data routing, and infrastructure planning.

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