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Solution of Third Order Kortewege-De Vries (KdV) Nonlinear Partial Differential Equations By Elzaki Adomian Decomposition Method

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Abstract: In this paper we have to find the solution of Third Order Kortewege-De Vries (KdV) nonlinear partial differential equation by Elzaki Adomian Decomposition Method. Elzaki transform is applied and then nonlinear term handled with the help of Adomian polynomial. We get exact solution of Kortewege-De Vries (KdV) nonlinear partial differential equations. It is believed that this work will make it easy to study the Kortewege-De Vries (KdV) nonlinear partial differential equations arising different areas of research and innovation. Therefore the current method can be extended for the solution of higher order Kortewege-De Vries (KdV) nonlinear partial differential equations and relevant problems. We give illustration through three problems.

Keywords: Elzaki transform, Elzaki Adomian Decomposition method, Kortewege-De Vries (KdV) nonlinear partial differential equations, Adomian polynomial etc.

AMS Classifications: 35A22, 35F20, 35F25

1. Introduction:

The study of exact solutions of nonlinear partial differential equation is very important in the study of nonlinear physical systems. The KdV equation arises from motion of big wave in shallow water. It has wide applications in quantum mechanics and nonlinear optics. It is well known that wave equation phenomena of plasma media and fluid dynamics are modeled by kink shaped tanh solution. Several approaches of finding Kortewege-De Vries (KdV) nonlinear partial differential equations In 1974 Sawada, Katura and Takeyasu Kotera [1] proposed method for finding N-Soliton Solution for KdV equations.Wang, Mingling[2] found exact solutions for KdV-Burgers equation. The extended tanh method [3]. In the literature, different nonlinear partial differential equations solved by various methods like Laplace Substitution Method[4] for nonlinear partial differential equations involving mixed partial derivatives, Bhrawy A. H. and et.al [5] solved the time-fractional coupled KdV equations Method. Adomian polynomial and Elzaki transform method[6-7] finds solution through Elzaki transform. Laplace Decomposition Method[8] forms solution to KdV nonlinear partial differential equations. Ramadan, Motaz Ahmed, and Hayah Samy Aly [9] proposed for solving extended KdV equation. Higher order time-fractional nonlinear equations obtained via Elzaki transform in [10]. This paper investigates the exact solution to the

KdVnonlinear partial differential equation by Elzaki Adomian Decomposition method.

The paper is organized as follows. The basic definition and some properties are discussed in section 2. Formation of Elzaki Aomian Decomposition Method is done in section 3. Finally applications are discussed in section 4 and conclusion in section 5.

2. Basic definition and properties of Elzaki Transform:

2.1 Definition:

If f(t) is continuous function and for all $t \geq 0$ in a region $(-1)^j \times [0, \infty)$, then Elzaki transform for kernel having exponential function is defined as

$$E[f(t)] = T(v) = v\int_0^\infty f(t)e^{\frac{-t}{v}}\, dt \text{ and } v \in (-k_1, k_2);\ k_1, k_2 > 0. \qquad (1)$$

2.2 Properties:

The properties are illustrated using definition of Elzaki transform [11]. We mention some in the following.

- If $f(t) = \sum_{n=0}^{\infty} a_n t^n$ then $E[f(t)] = T(v) = \sum_{n=0}^{\infty} n!\, v^{n+2}$
- If $f(t) = tf(t)$ then $E\{tf(t)\} = v^2 \frac{dT}{dv} - vT(v)$
- If $f(t) = t^2 f(t)$ then $E\{t^2 f(t)\} = v^4 \frac{d^2T}{dv^2}$
- If $f(t) = f'(t)$ then $E[f'(t)] = T'(v) = \frac{T(v)}{v} - v\, f(0)$
- If $f(t) = f^{(n)}(t)$ then $E\left[f^{(n)}(t)\right] = T^{(n)}(v) = \frac{T(v)}{v^n} - \sum_{k=0}^{n-1} v^{2-n-k}\ f^{(k)}(0),\ \ n \geq 1$.

2.3 Properties for Partial Differential Equation using Elzaki transform:

Let $u(x,t)$ be a function of two independent variables $x\ and\ t$ then

- If $f(t) = \frac{\partial u(x,t)}{\partial t}$ then $E\left\{\frac{\partial u(x,t)}{\partial t}\right\} = \frac{1}{v}T(x,v) - v\,u(x,0)$
- If $f(t) = \frac{\partial u(x,t)}{\partial x}$ then $E\left\{\frac{\partial u(x,t)}{\partial x}\right\} = \frac{d[T(x,v)]}{dx}$
- If $f(t) = \frac{\partial^2 u(x,t)}{\partial t^2}$ then $E\left\{\frac{\partial^2 u(x,t)}{\partial t^2}\right\} = \frac{1}{v^2}T(x,v) - u(x,0) - v\frac{\partial u(x,0)}{\partial t}$
- If $f(t) = \frac{\partial^2 u(x,t)}{\partial x^2}$ then $E\left\{\frac{\partial^2 u(x,t)}{\partial x^2}\right\} = \frac{d^2[T(x,v)]}{dx^2}$
- If $f(t) = \frac{\partial^n u(x,t)}{\partial t^n}$

  Then $E\left\{\frac{\partial^n u(x,t)}{\partial t^n}\right\} = \frac{1}{v^n}T(x,v) - \frac{u(x,0)}{v^{n-2}} - \frac{1}{v^{n-3}}\frac{\partial u(x,0)}{\partial t} - \cdots - v\frac{\partial^{n-1} u(x,0)}{\partial t^{n-1}}$.

3. Elzaki Adomian Decomposition Method

This section proposes the Elzaki Adomian Decomposition method to solve Korteweg-De Vries equations (KdV). Here we use Elzaki Adomian Decomposition for the general nonlinear partial differential equations. To show the concept we use KdV equations along with initial condition as below

$$Lu(x,t) + Nu(x,t) + Ru(x,t) = 0, \quad (2)$$

with the initial condition $u(x,0) = f(x)$ (3)

Where $L = \frac{\partial}{\partial t}$, $Nu(x,t)$ is nonlinear term and $Ru(x,t)$ is the term contains higher order partial derivatives, we get equation (2) as

$$\frac{\partial u(x,t)}{\partial t} + Nu(x,t) + Ru(x,t) = 0, \quad (4)$$

$$\frac{\partial u(x,t)}{\partial t} = -Nu(x,t) - Ru(x,t) \quad (5)$$

Taking Elzaki transform on both sides

$$E_t[u(x,t)] = -E[Nu(x,t) + Ru(x,t)] \quad (6)$$

$$\frac{u(x,v)}{v} - vu(x,0) = -E_x[Nu(x,t) + Ru(x,t)]$$

$$u(x,v) = v^2 u(x,0) - vE_x[Nu(x,t) + Ru(x,t)]$$

Using initial conditions

$$u(x,v) = v^2 f(x) - vE[Nu(x,t) + Ru(x,t)]\,. \quad (7)$$

Taking Inverse Elzaki transform on the both sides, we get

$$u(x,t) = E^{-1}[v^2 f(x)] - E^{-1}\big[vE[Nu(x,t) + Ru(x,t)]\big] \quad (8)$$

$$u(x,t) = f(x) - E^{-1}[vENu(x,t) + Ru(x,t)] \quad (9)$$

$$u(x,t) = u_0(x,t) - E^{-1}\big[vE[Nu(x,t) + Ru(x,t)]\big]. \quad (10)$$

Now let we consider nonlinear term $Nu(x,t) = \mathcal{A}_n$. This nonlinear term decomposed through the Adomian polynomial which is as

$$\mathcal{A}_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\, f\big[\textstyle\sum_{i=0}^{\infty} \lambda^i\, u_i\big]_{\lambda=0} \quad (11)$$

The equation (10) becomes

$$u(x,t) = u_0(x,t) - E^{-1}\big[vE[\mathcal{A}_n + Ru(x,t)]\big] \quad (12)$$

Here we get recurrence relation using $u(x,t)=\sum_{n=0}^{\infty}u_n(x,t)$, $Ru(x,t)=\frac{\partial^n u_n}{\partial x^n}$ in equation (12)as

$$\sum_{n=0}^{\infty}u_{n+1}(x,t)=-E^{-1}\left[vE\left[\mathcal{A}_n+\frac{\partial^n u_n}{\partial x^n}\right]\right] \tag{13}$$

We find $u_1,u_2,u_3,\ldots.$

Here the terms $u_1,u_2,u_3,\ldots.$ then it is using in series as.

$$u(x,t)=\sum_{n=0}^{\infty}u_{n+1}(x,t)=u_0+u_1+u_2+u_3+\cdots \tag{14}$$

We get the exact or approximate solution through (14).

4. Applications:

Example 1.

Solve the third order KdV equation $u_t+6uu_x+u_{xxx}=0$ having initial conditions $u(x,0)=x$

Solution: Consider

$$u_t+6uu_x+u_{xxx}=0 \tag{15}$$

$$u_t=-6uu_x-u_{xxx} \tag{16}$$

Taking Elzaki transform on both sides

$$E_t(u(x,v))=E[-6uu_x-u_{xxx}] \tag{17}$$

$$\frac{u(x,v)}{v}-vu(x,0)=E[-6uu_x-u_{xxx}] \tag{18}$$

$$u(x,v)=v^2u(x,0)-\ vE[6uu_x+u_{xxx}] \tag{19}$$

initial conditions, we get

$$u(x,v)=v^2x-vE[6uu_x+u_{xxx}] \tag{20}$$

Taking inverse Elzaki transform on both sides, we get

$$u(x,t)=E^{-1}[v^2x]-vE^{-1}[6uu_x+u_{xxx}] \tag{21}$$

$$u(x,t)=x-vE^{-1}[6uu_x+u_{xxx}] \tag{22}$$

From above equation (22)

$$u_0(x,t)=x\ . \tag{23}$$

Here nonlinear terms appear in the equation. We can find by using Adomial polynomial.

Now consider

$$u\frac{\partial u}{\partial x} = \sum_{n=0}^{\infty} \mathcal{A}_n \quad . \tag{24}$$

Here we have recurrence relation from equation (22). We find solution of each term through it

$$u_{n+1}(x,t) = -vE^{-1}[6\mathcal{A}_n + u_{n_{xxx}}] \ . \tag{25}$$

Where $\mathcal{A}_n$ is Adomian polynomial and use to find $u_0, u_1, u_2, \dots, u_n, n \geq 0$

Calculate terms as

$$\sum_{n=0}^{\infty} \mathcal{A}_n = u\frac{\partial u}{\partial x} \tag{26}$$

$$\mathcal{A}_0 = u_0 u_{0x} \tag{27}$$

$$\mathcal{A}_1 = u_0 u_{1x} + u_1 u_{0x} \tag{28}$$

$$\mathcal{A}_2 = u_0 u_{2x} + 2u_1 u_{1x} + u_2 u_{0x} \tag{29}$$

:

:

etc

Use in the above equation, we get

$$u_{n+1}(x,t) = -vE^{-1}[6\mathcal{A}_n + u_{n_{xxx}}].$$

Taking $n = 0$ in (25) we get

$$u_1(x,t) = -vE^{-1}[6\mathcal{A}_0 + u_{0_{xxx}}] \, . \tag{30}$$

Here $\mathcal{A}_0$ calculate through equation (27) and substitute in equation (30) we get

$$u_1(x,t) = -6xt \, . \tag{31}$$

For $n = 1$ in (25) we get

$$u_2(x,t) = -vE^{-1}[6\mathcal{A}_1 + u_{1_{xxx}}] \, .$$

Here $\mathcal{A}_1$ calculate through equation (28) and substitute in equation (30) we get

$$u_2(x,t) = 36xt^2 \, . \tag{32}$$

For $n = 2$ in (25) we get

$$u_3(x,t) = -vE^{-1}[6\mathcal{A}_2 + u_{2_{xxx}}].$$

Here $\mathcal{A}_2$ calculate through equation (29) and substitute in equation (30) we get

$$u_3(x,t) = -216xt^3 \tag{33}$$

:

:

Etc.

We get the solution using values in recurrence relation.

$$u(x,t) = \sum_{n=0}^{\infty} u_{n+1}(x,t)$$

$$u(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t) + \cdots$$

$$u(x,t) = x - 6xt + 36xt^2 - 216xt^3 + \cdots$$

$$u(x,t) = x[1 - 6t + (6t)^2 - (6t)^3 + \cdots].$$

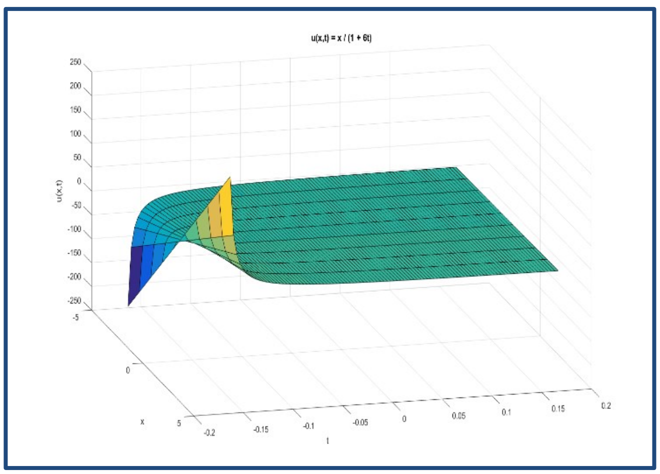
This is a geometric Taylor series expansion. Hence we get

$$u(x,t) = \frac{x}{1+6t}, \quad (34)$$

and it is convergent if $|t| < -\frac{1}{6}$ for all (x,t) belongs to the domain of $u(x,t)$

$$u(x,t) = \frac{x}{1+6t}, \ |t| < -\frac{1}{6}. \quad (35)$$

This is an exact solution of the homogeneous KdV equation.



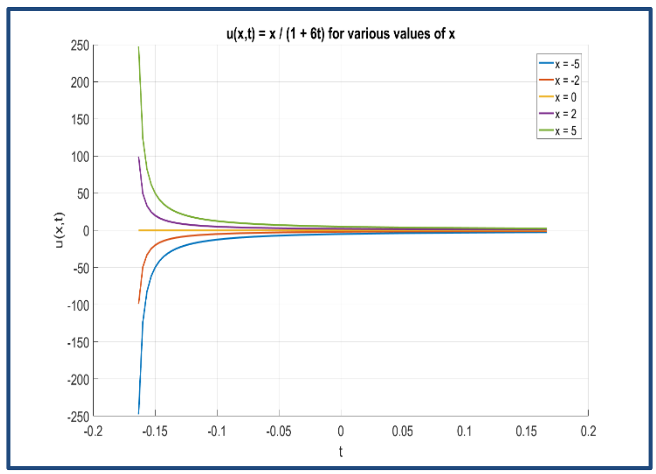


Figure (a) Figure (b)

Figure (a) is the plot for the solution of Example 1 (Equation 35), and Figure (b) is the plot of Equation (35) for x=-5, -2, 0, 2 ,5 and |t|<1/6.

Example 2. Solve the third-order KdV equation $u_t + 6uu_x + u_{xxx} = 0$, having initial conditions $u(x,0) = 6x$.

Solution: Consider

$$u_t + 6uu_x + u_{xxx} = 0 \quad (36)$$

$$u_t = -6uu_x - u_{xxx}. \quad (37)$$

Taking the Elzaki transform on both sides

$$E_t(u(x,v)) = E[-6uu_x - u_{xxx}] \tag{38}$$

$$\frac{u(x,v)}{v} - vu(x,0) = E[-6uu_x - u_{xxx}]\,. \tag{39}$$

$$u(x,v) = v^2u(x,0) - \; vE[6uu_x + u_{xxx}]. \tag{40}$$

initial conditions, we get

$$u(x,v) = v^2 6x - vE[6uu_x + u_{xxx}]\,. \tag{41}$$

Taking inverse Elzaki transform on both sides, we get

$$u(x,t) = E^{-1}[v^2 6x] - vE^{-1}[6uu_x + u_{xxx}] \tag{42}$$

$$u(x,t) = 6x - vE^{-1}[6uu_x + u_{xxx}] \tag{43}$$

From above equation (43)

$u_0(x,t) = 6x.$

Here nonlinear terms appear in the equation(43). We can find by using the Adomian polynomial.

Now consider

$$u\frac{\partial u}{\partial x} = \sum_{n=0}^{\infty}\mathcal{B}_n\,. \tag{44}$$

Here we have recurrence relation. We find solution of each term through it

$$u_{n+1}(x,t) = \; -vE^{-1}[6\mathcal{B}_n + u_{n_{xxx}}]. \tag{45}$$

Where $\mathcal{B}_n$ is Adomian polynomial and use to find $u_0, u_1, u_2, \dots, u_n, n \geq 0$

Calculate terms as

$$\sum_{n=0}^{\infty}\mathcal{B}_n = u\frac{\partial u}{\partial x} \tag{46}$$

$$\mathcal{B}_0 = u_0u_{0x} \tag{47}$$

$$\mathcal{B}_1 = u_0u_{1x} + u_1u_{0x} \tag{48}$$

$$\mathcal{B}_2 = u_0u_{2x} + 2u_1u_{1x} + u_2u_{0x} \tag{49}$$

:

:

etc

Use in the above equation(45), we get

$u_{n+1}(x,t) = -vE^{-1}[6\mathcal{B}_n + u_{n_{xxx}}].$

Taking $n = 0$ in (45) we get

$$u_1(x,t) = -vE^{-1}[6\mathcal{B}_0 + u_{0_{xxx}}] \tag{50}$$

Here $\mathcal{B}_0$ calculate through equation (47) and substitute in equation (50) we get

$$u_1(x,t) = 216xt\ . \tag{51}$$

For $n = 1$ in (45) we get

$$u_2(x,t) = -vE^{-1}[6\mathcal{B}_1 + u_{1_{xxx}}]\ . \tag{52}$$

Here $\mathcal{B}_1$ calculate through equation (48) and substitute in equation (52) we get

$$u_2(x,t) = 7776xt^2\ . \tag{53}$$

For $n = 2$ in (45) we get

$$u_3(x,t) = -vE^{-1}[6\mathcal{B}_2 + u_{2_{xxx}}]\ . \tag{54}$$

Here $\mathcal{B}_2$ calculate through equation (49) and substitute in equation (54), we get

$$u_3(x,t) = 279936xt^3 \tag{55}$$

:

:

Etc

We get the solution using values in recurrence relation.

$$u(x,t) = \sum_{n=0}^{\infty} u_{n+1}(x,t)$$

$$u(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t) + \cdots$$

$$u(x,t) = 6x + 216xt + 7776xt^2 + 279936xt^3 + \cdots$$

$$u(x,t) = 6x[1 + 36t + (36t)^2 + (36t)^3 + \cdots].$$

This is a geometric Taylor series expansion. Hence we get

$$u(x,t) = \frac{6x}{1-36t} \tag{56}$$

and it is convergent if $|t| < \frac{1}{36}$ for all (x,t) belongs to the domain of $u(x,t)$

$$u(x,t) = \frac{x}{1+6t},\ |t| < \frac{1}{36}\ . \tag{58}$$

This is exact solution of the homogeneous KdV equation.

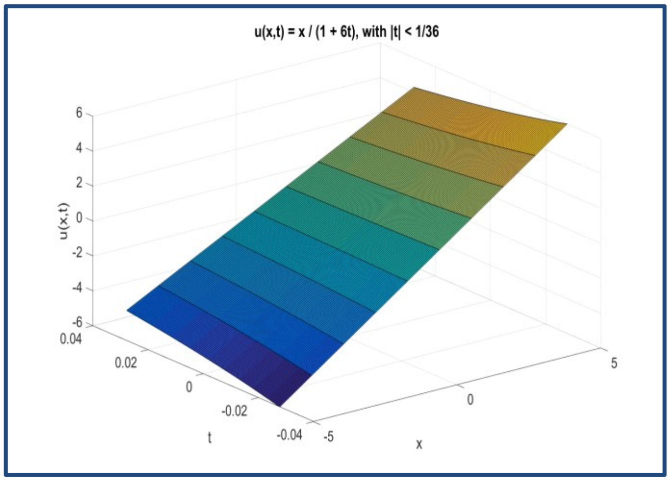


Figure (c)

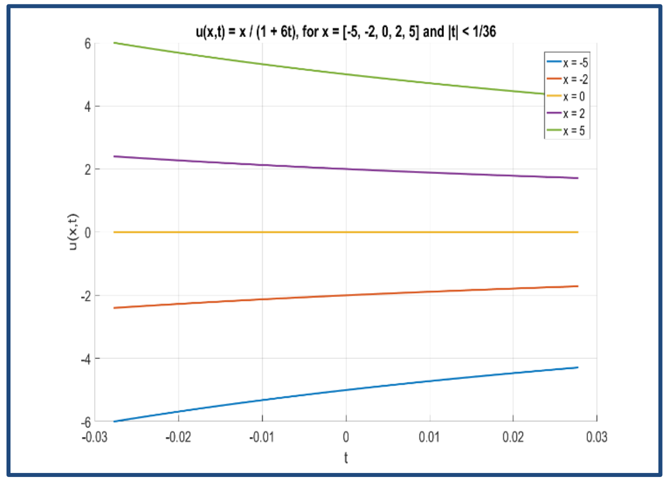


Figure (d)

Figure (c) is the plot for the solution of Example 2 (Equation 58), and Figure (d) is the plot of Equation (35) for x=-5, -2, 0, 2 ,5 and |t|<1/36.

Example 3.

Consider third order KdV equation $u_t - 6uu_x + u_{xxx} = 0$ with initial conditions $u(x,0) = -\frac{2k^2e^{kx}}{(1+e^{kx})^2}$.

Solution: Consider

$$u_t - 6uu_x + u_{xxx} = 0\,. \tag{59}$$

$$u_t = 6uu_x - u_{xxx}\,, \tag{60}$$

Taking the Elzaki transform on both sides

$$E_t(u(x,t)) = E[6uu_x - u_{xxx}] \tag{61}$$

$$\frac{u(x,v)}{v} - vu(x,0) = E[6uu_x - u_{xxx}] \tag{62}$$

$$u(x,v) = v^2u(x,0) + vE[6uu_x - u_{xxx}]\,. \tag{63}$$

Initial conditions, we get

$$u(x,v) = v^2\left(-\frac{2k^2e^{kx}}{(1+e^{kx})^2}\right) + vE[6uu_x - u_{xxx}]\,. \tag{64}$$

Taking inverse Elzaki transform on both sides, we get

$$u(x,t) = E^{-1}\left[v^2\left(-\frac{2k^2e^{kx}}{(1+e^{kx})^2}\right)\right] + vE^{-1}[6uu_x - u_{xxx}] \tag{65}$$

$$u(x,t) = \left(-\frac{2k^2e^{kx}}{(1+e^{kx})^2}\right) + vE^{-1}[6uu_x - u_{xxx}]\,. \tag{66}$$

From above equation (66)

$$u_0(x,t) = -\frac{2k^2 e^{kx}}{(1+e^{kx})^2} \quad . \tag{67}$$

Here nonlinear terms appear in the equation. We can find by using Adomial polynomial.

Now consider

$$u\frac{\partial u}{\partial x} = \sum_{n=0}^{\infty} \mathcal{C}_{n.} \tag{68}$$

Here we have recurrence relation from equation (66). We find solution of each term through it

$$u_{n+1}(x,t) = \ vE^{-1}\left[6\mathcal{C}_n - u_{n_{xxx}}\right]. \tag{69}$$

Where $\mathcal{C}_n$ is Adomian polynomial and use to find $u_0, u_1, u_2, \dots, u_n, n \geq 0$

Calculate terms as

$$\sum_{n=0}^{\infty} \mathcal{C}_n = u\frac{\partial u}{\partial x} \tag{70}$$

$$\mathcal{C}_0 = u_0 u_{0x} \tag{71}$$

$$\mathcal{C}_1 = u_0 u_{1x} + u_1 u_{0x} \tag{72}$$

$$\mathcal{C}_2 = u_0 u_{2x} + 2u_1 u_{1x} + u_2 u_{0x} \tag{73}$$

:

:

Etc.

We have recurrence relation

$$u_{n+1}(x,t) = vE^{-1}[6\mathcal{C}_n + u_{n_{xxx}}].$$

Taking $n = 0$ in (69) we get

$$u_1(x,t) = vE^{-1}[6\mathcal{C}_0 - u_{0_{xxx}}]\,. \tag{74}$$

Here $\mathcal{C}_0$ calculate through equation (71) and substitute in equation (74) we get

$$u_1(x,t) = -\frac{2k^5 e^{kx}(e^{kx}-1)}{(1+e^{kx})^3}t\,. \tag{75}$$

For $n = 1$ in (69) we get

$$u_2(x,t) = -vE^{-1}[6\mathcal{C}_1 + u_{1_{xxx}}]. \tag{76}$$

Here $\mathcal{C}_1$ calculate through equation (72) and substitute in equation (76) we get

$$u_2(x,t) = -\frac{2k^8 e^{kx}(e^{2kx}-4e^{4kx}+1)}{(1+e^{kx})^4}\frac{t^2}{2!}\ . \tag{77}$$

For $n = 2$ in (69) we get

$$u_3(x,t) = -vE^{-1}[6C_2 + u_{2_{xxx}}]. \tag{78}$$

Here C_2 calculate through equation (73) and substitute in equation (78) we get

$$u_3(x,t) = -\frac{2k^{11}e^{kx}(e^{3kx}-11e^{2kx}+11e^{kx}-1)}{(1+e^{kx})^5}\frac{t^3}{3!}. \tag{79}$$

:

:

Etc

We get the solution using values in recurrence relation.

$$u(x,t) = \sum_{n=0}^{\infty} u_{n+1}(x,t)$$

$$u(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t) + \cdots$$

$$u(x,t) = -\frac{2k^2e^{kx}}{(1+e^{kx})^2} - \frac{2k^5e^{kx}(e^{kx}-1)}{(1+e^{kx})^3}t - \frac{2k^8e^{kx}(e^{2kx}-4e^{4kx}+1)}{(1+e^{kx})^4}\frac{t^2}{2!}$$
$$-\frac{2k^{11}e^{kx}(e^{3kx}-11e^{2kx}+11e^{kx}-1)}{(1+e^{kx})^5}\frac{t^3}{3!} + \cdots.$$

This is geometric Taylor's series expansion. Hence we get

$$u(x,t) = -\frac{2k^2e^{k(x-k^2t)}}{\left(e^{k(x-k^2t)}-1\right)^2}. \tag{80}$$

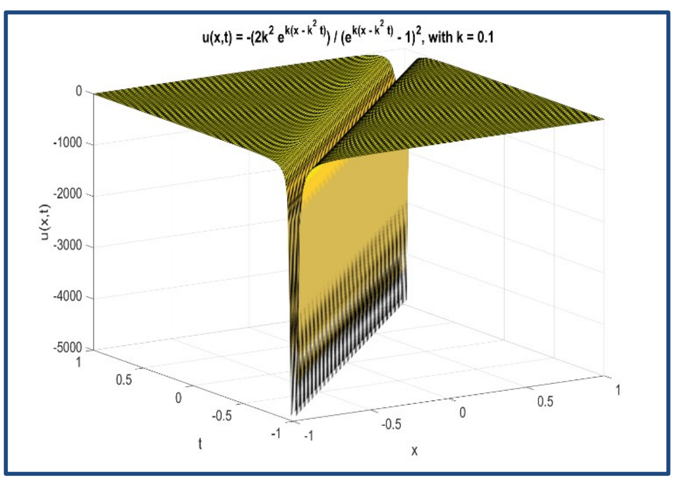


Figure (e)

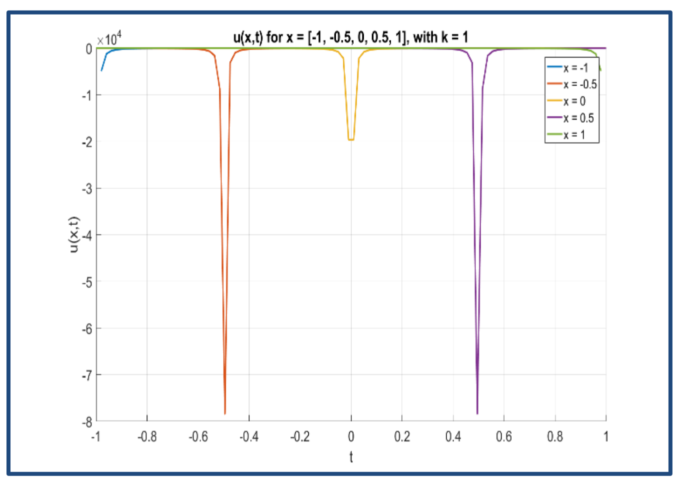


Figure (f)

Figure (e) is the plot for the solution of Example 3 (Equation 80), and Figure (f) is the plot of Equation (35) for x=-1, -0.5, 0, 0.5,1and |t|<1.

5. Conclusion:

This paper explores the Elzaki Adomian Decomposition Method (EADM) for solving nonlinear partial differential equations, specifically the Korteweg-De Vries (KdV) equation. By employing Adomian polynomials, the nonlinear terms are efficiently handled, enabling the method to derive approximate and exact solutions. The application of EADM provides a systematic approach to solving complex nonlinear problems in a wide range of scientific and engineering fields.

Through this investigation, we demonstrated the capability of the Elzaki Adomian Decomposition Method to manage the intricacies of the KdV equation, offering a reliable alternative for obtaining solutions where traditional methods may struggle. This method is effective for this particular equation and proves valuable for solving other nonlinear systems. The insights gained from this study will contribute to the broader understanding of applying decomposition techniques to nonlinear partial differential equations, enhancing their applicability in real-world problems. Thus, the results of this work will be beneficial for advancing various scientific and engineering applications, particularly those involving wave dynamics and other nonlinear phenomena.

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