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Block k-Domination in Graphs

Suhas P. Gade[1], Dayanand K. Vyavahare[2,*]

[1]Department of Mathematics, Sangameshwar College (Autonomous), Solapur-413001, Maharashtra.

[2]School School of Computational Sciences, P.A.H. Solapur University, Solapur-413255, Maharashtra.

[*]Corresponding author: dayavyavahare77@gmail.com

Abstract: In this paper we introduce the concept block k-domination in graph. The block k-dominating number γ_{kb}(G) is a minimum cardinality of k-dominating set of block graph B(G) of graph G. In this work we established upper and lower bounds on γ_{kb}(G) in terms of elements of G and we obtain relationship between γ_{kb}(G) and other dominating parameters of G.

Keywords: Graph, Block graph, Domination number, k-domination number, Block k domination number.

MSC Classification: 05C69; 05C70; 05C76

1. Introduction

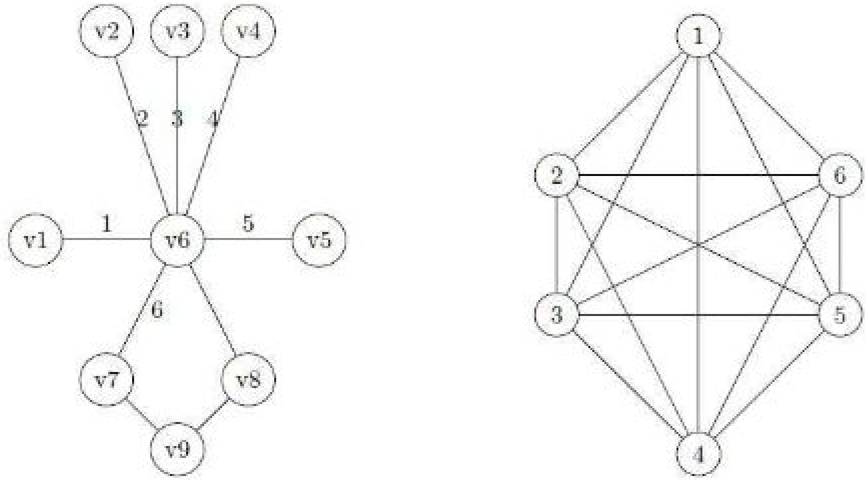
In this work, we consider G = (V, E) to be a simple connected graph with vertex set V (G) and edge set E(G). For any notation and term, we refer Harary [1]. The order and size of the graph G are given by |V (G)| = p and |E(G)| = q respectively. For a vertex u ∈ G, the number of edges incident on u is its degree, denoted as deg(u), with the minimum and maximum degrees denoted by δ and △, respectively. The degree of an edge e = uv in G is defined as deg(e) = deg(u) + deg(v) − 2, with the minimum and maximum edge degrees denoted by δ and △ respectively. The open neighborhood of a vertex u, denoted by N (u), consists of all vertices adjacent to u. The closed neighborhood, denoted by N [u], is defined as N [u] = N (u) ∪ {u}. For details on the concept of neighborhoods, refer to [2]. The vertex (edge) covering number α_0(G) (α_1(G)) is defined as the minimum cardinality of a vertex (edge) cover. A set M of vertices is independent if no two vertices in M are adjacent, and the independence number β_0(G) of G is the maximum cardinality of such an independent set. Similarly, a set N of edges is independent if no two edges in N are adjacent, with the edge independence number β_1(G) being the maximum cardinality of an independent set of edges. The diameter of the graph, denoted as diam(G), is the maximum distance between any pair of vertices. For further notation details, refer to [3]. The chromatic number of a graph is denoted by χ(G) is represents the minimum number of colors needed to properly color the vertices of the graph such that no two adjacent vertices share the same colour (see [4]).

We begin by recalling some definitions from domination theory:

A set D ⊆ V (G) is a dominating set if every vertex u ∈ V (G) − D is adjacent to at least one vertex in D. The domination number γ(G) is the minimum cardinality of a dominating set of G. If every vertex in V (G) has at least one neighbor in D, then the subset D ⊆ V (G) is a total dominating set. The total domination number γ_t (G) is the cardinality of a minimum total dominating set (see [5]). A total dominating set D is total split dominating set if the induced subgraph < V − D > is disconnected. The total split domination number γ_{ts} (G) (G) of graph G is the minimum cardinality of total split dominating set. A dominating set D is non split dominating set if the induced subgraph < V − D > is connected. The split domination number γ_{ns}(G) of graph G is the minimum cardinality of nonsplit dominating set. A set F of edges in a graph G = (V, E) is an edge dominating set of G if every edge in e ∈ E(G) − F is adjacent to at least one edge in F. The edge domination number γ(G) of graph G is the minimum cardinality of edge dominating set. An edge dominating set F of G is connected edge dominating set if induced subgraph < F > is connected. The connected edge domination number $\gamma_c^{'}$ (G) of graph G is the minimum cardinality of connected edge dominating set (see [3]).

A block graph B(G) is a graph whose set of vertices is the union of set of blocks of G in which two vertices are adjacent if and only if the corresponding blocks of G are adjacent. (i.e. are share common vertex). In [6] author introduce the concept of strong block domination in graphs and studied graph theoretic properties of γ_{SB}(G) and many bounds were obtained in terms of elements of G and its relationship with other domination parameters were found. In [7] author studied endedge block domination in graphs in this work some bounds for $\gamma_{eb}^{'}$(G) are obtained in terms of elements of G. Furthermore, find the exact values of $\gamma_{eb}^{'}$ (G) for some standard graphs and relationships with other dominating parameters were obtained. In [8] studied block double domination in graphs and found many bounds on γ_{ddb}(G) in terms of G s vertices, edges and other different domination parameters.

In [9, 10] the concept of k-domination is generalized by Fink and Jacobson from domination and independence. A subset D ⊆ V (G) is k-dominating in G if for each vertex u ∈ V (G) − D, |N (u) ∩ D| ≥ k. The k-domination number γ_k (G) is the minimum cardinality of k-dominating set of G. The Bounds on k-domination number one can be refer in [11–13].



(a) Graph G (b) Block Graph B(G) of G

In this paper, we use the similar concept of k-domination on block graph B(G) of graph G, where $1 \leq k \leq \triangle(B(G))$. The block k-domination number γ_{kb} (G) is the minimum cardinality among all k-dominating set of block graph B(G)of graph G. In this work we establish upper and lower bounds on γ_{kb}(G) in terms of elements of G and also, we obtain relationship between γ_{kb}(G) and other dominating parameters of G.

2. Preliminaries

In this section we introduce some important theorems.

Theorem 2.1 ([14]) For any tree T, γ_{sslb}(T) = q.

Theorem 2.2 ([14]) For any tree T, γ_{ssbc}(T) = γ_{sslb}(T).

Theorem 2.3 ([15]) For any tree T, γ_{sstb}(T) = q + 1.

Theorem 2.4 ([14]) For any graph G, γ_{sslb}(G) = n.

Theorem 2.5 ([16]) If $G = (p, q)$ is any graph, $\gamma(G) \geq \frac{p}{1+\Delta(G)}$.

Theorem 2.6 ([3]) For any graph G, γ_{ss}(G) = α_0(G).

3. Main Results

Theorem 3.1 For any graph $T = (p, q)$, $\gamma_{kb}(T) \geq \frac{q}{\Delta'(T)+1}$.

Proof: Consider tree T, every single edge of T forms block and which corresponds to vertex

of block graph B(T). Let $A = \{b_1, b_2, ..., b_q\}$ be the vertex set of $B(T)$. Then there exists a minimal set $A_1 = \{b_1, b_2, ..., b_s\} \subseteq A$ such that every vertex in $V(B(T)) - A_1$ is adjacent to at least k vertices of A_1. Consequently, A_1 is minimal k-dominating set of $B(T)$ such that

$|A_1| = \gamma_{kb}(G)$. As every vertex in $V(B(T)) - A_1$ is adjacent to at least k vertices of A_1. So, each vertex in $V(B(T)) - A_1$ contributes at least one to the sum of degree of vertices of A_1, then we have

$$|V(B(T)) - A_1| \leq \sum_{w_i \in A_1} \deg(w_i).$$

For any tree T, there exists at least one edge $e \in E(T)$, which corresponds to a vertex in $B(T)$, such that $\deg(e) = \Delta^{'}(T)$.

We have,

$$q - \gamma_{kb}(T) = |V(B(T)) - A_1| \leq \sum_{w_i \in A_1} \deg(w_i).$$

$$\leq \Delta^{'}(G) \cdot \gamma_{kb}(G)$$

Thus, $\gamma_{kb}(G) \geq \frac{q}{\Delta^{'}(G)+1}$.

Corollary 3.2 For any graph $T = (p, q)$, $\gamma_{kb}(T) \geq \frac{p-1}{\Delta^{'}(T)+1}$.

Proof: from Theorem 3.1,

$$\gamma_{kb}(G) \geq \frac{q}{\Delta^{'}(G)+1}.$$

As we know, if T is connected tree,

$$q = p - 1.$$

Using this, we get required result.

Corollary 3.3 For any graph $T = (p, q)$, $\gamma_{kb}(T) \geq \frac{\gamma_{sslb}(T)}{\Delta^{'}(T)+1}$.

Proof: From Theorem 3.1 and Theorem 2.1, we get result.

Corollary 3.4 For any graph $T = (p, q)$, $\gamma_{kb}(T) \geq \frac{\gamma_{ssbc}(T)}{\Delta^{'}(T)+1}$.

Proof: From Theorem 2.2 and Corollary 3.3, we get result.

Corollary 3.5 For any graph $T = (p, q)$, $\gamma_{kb}(T) \geq \frac{\gamma_{ssbc}(T)}{\Delta^{'}(T)+1}$.

Proof: From Theorem 2.2 and Corollary 3.3, we get result.

Theorem 3.6 For any positive integer k, let $G = (p, q)$ be any graph with n blocks and G has one cut vertex then $\gamma_{kb}(G) = k$.

Proof: Since G has only one cut vertex then each block is adjacent to other block and these blocks correspond to the n vertices of the block graph $B(G)$. The block graph $B(G)$ of graph G is complete graph with n vertices. Thus, for every subset

D_k of $V(B(G))$ containing k vertices forms minimal k dominating set for block graph $B(G)$. This is because every vertex in $B(G)$ except those in D_k is adjacent to k vertices of D_k as $B(G)$ is complete graph.

Therefore, the set D_k forms minimal block k dominating set of G, where $|D_k|=k$. Thus, $\gamma_{kb}(G) = k$.

Theorem 3.7 If $G = (p, q)$ is block, $\gamma_{kb}(G) = 1$.

Proof: Since G is itself block then vertex set of $B(G)$ consist of a single vertex. Thus, result hold clearly.

Theorem 3.8 For any graph $G = (p, q)$ with q edges, $\gamma_{kb}(G) \leq q - 1$.

Proof: Let G be graph with p vertices and q edges and let $B(G)$ be its block graph where each vertex represents a block of G and two vertices in $B(G)$ are adjacent if the corresponding blocks share common vertex in G. At very last, relationship between blocks and edges is that each block corresponds to a single edge within the original graph G. As many blocks often contain multiple edges, especially when cycles are involved which correspond to a single vertex of $B(G)$. Therefore, there can be maximum q blocks which correspond to q vertices in the block graph $B(G)$. This gives the block k domination number cannot exceed $q-1$ edges. Thus, $\gamma_{kb}(G) \leq q-1$.

Corollary 3.9 For any tree $T = (p, q)$, $\gamma_{kb}(T) \leq p - 2$.

Proof: As we know that, if G is connected tree,

$$q = p - 1$$

Using this in Theorem 3.5, we get required result

Corollary 3.10 For any tree $T = (p, q)$, $\gamma_{kb}(T) + \chi(T) \leq p$.

Proof: As we know, the chromatic number of connected tree T is 2. i.e. $\chi(G)$=2. Using this in Corollary 3.6, we obtain the result.

Theorem 3.11 If $G = (p, q)$ is any graph contain at least one cycle, $\gamma_{kb}(G) \leq q - 3$.

Proof: Since the graph G contains at least one cycle then there exist block B_i that contain a minimum of three edges and this block corresponds to a single vertex in the block graph $B(G)$. At last, we assuming remaining edges of graph G forms individual blocks then these blocks cor- respond to $q - 3$ vertices of $B(G)$. Therefore, there are at most $q - 2$ vertices in $B(G)$. From this, we can conclude that block k domination number cannot exceed $q - 3$. Thus, $\gamma_{kb}(G) \leq q - 3$.

Theorem 3.12 For any graph $G = (p, q)$ has n blocks, $\gamma_{kb}(G) \leq n - 1$.

Proof: If graph G has n blocks, then these corresponds to n vertices of block graph $B(G)$. There- fore $|V(B(G)| = n$. From this, we conclude the result.

Corollary 3.13 For any graph $G = (p, q)$ has n blocks, $\gamma_{kb}(G) \leq \gamma_{sslb}(G) - 1$.

Proof: From Theorem 3.12 and Theorem 2.4, we get result.

Theorem 3.14 If $T = (p, q)$ is any tree and $k \leq \delta(T)$, $\gamma_{kb}(T) + \gamma(B(T)) \leq q$.

Proof: Every single edge of T forms block and which corresponds to vertex of block graph $B(T)$. For every $k \leq \delta(T)$, there exist set $S=\{w_1, w_2, \ldots, w_r\} \subseteq V(B(T))$, be the minimal k-dominating set of $B(T)$. Let's assume that there exists vertex $w \in S$ that is not adjacent to any vertices in $V(B(T)) - S$. This would imply that w is adjacent to at least k vertices in set S. Consequently, $S - \{w\}$ is a minimal k-dominating set and $|S| - \{w\}| < |S|$. However, this contradicts our assumption S is minimal k-dominating set. Hence, every vertex in S must be adjacent to at least one vertex in $V(B(T)) - S$.

Thus, $V(B(T)) - S$ is the dominating set. Therefore, there exists a minimal set $A \subseteq V(B(T)) - S$ that covers all the vertices in $V(B(T))$. This implies that A is γ-set of $B(T)$ and which conclude that, $|A| + |S| \leq |E(T)|$. Hence, $\gamma_{kb}(T) + \gamma(B(T)) \leq q$.

Corollary 3.15 For any tree $T = (p, q)$ and if $k \leq \delta(T), \gamma_{kb}(T) \leq \frac{q \times \Delta(B(T))}{\Delta(B(T)) + 1}$.

Proof: By Theorem 2.5, for any graph G, $\gamma(G) \geq \frac{p}{1+\Delta(G)}$. It also holds for block graph B(T) of T. So, we get $\gamma(B(T)) \geq \frac{q}{1+\Delta(B(T))}$.

Using this in Theorem 3.14, we get $\gamma_{kb}(T)+\frac{q}{1+\Delta(B(T))} \leq q$

Thus, $\gamma_{kb}(T) \leq \frac{q \times \Delta(B(T))}{\Delta(B(T))+1}$.

Theorem 3.16 For any graph $G = (p, q)$ and $2 \leq k \leq \triangle(B(G))$, $\gamma_{kb}(G) \leq \gamma_k(G)$.

Proof: Case-I If G is block, then the result hold trivially.

Case-II If G is not a block.

Let S be a k dominating set of the graph G, and D be a k dominating set of block graph $B(G)$ constructed as: For each vertex $u \in S$, let the corresponding block B_u in $B(G)$ that contain u, we map u to b_u. Consider the set D in $B(G)$ is the set of blocks B_u such that at least one u from S is in B_u. By definition of a k dominating set in G, every vertex $w \notin S$ is adjacent to at least k vertices of S. Each of these vertices belongs to some block in $B(G)$, meaning that the block containing w in $B(G)$ is adjacent to at least k blocks corresponding to vertices in S.

Therefore, each block in $B(G)$ not in D adjacent to at least k blocks in D. This establishes that D is block k dominating set of G. Since the set D derived from set S, the cardinality of D is less than or equal to cardinality of set S. Specifically, each block in $B(G)$ might correspond to multiple vertices in S, but we only include one vertex per block in D. For given $k \leq \triangle(B(G))$, this mapping and construction of D shows that the D is at least as cardinality of S. Thus, $\gamma_{kb}(G) \leq \gamma_k(G)$.

Theorem 3.17 For any tree $T = (p, q)$ except path P and $k \geq 2$, $\gamma_{kb}(T) \geq \gamma_c'(T)$.

Proof: Consider a tree T excluding path P. Within such tree, there exists at least one block comprising at least two vertices, and block consist of single edges. Let F is minimal connected edge dominating set means every edge in T is adjacent to at least one edge in the set, ensuring the induced subgraph $< F >$ formed by the edges of F is connected. For each edge in F, select corresponding block in $B(T)$ and including in the set D. To guarantee connectivity in T, then additional blocks need to be included in D. i.e. there exists vertices in $B(T)$ not covered by edges in F, thus corresponding to additional blocks of $B(T)$. So, that D forms k dominating set for $B(T)$. The constructed block k dominating set D contains at least as many blocks as the connected edge dominating set F. Therefore, $\gamma_{kb}(T) \geq \gamma'_c(T)$.

Theorem 3.18 For any tree $T = (p, q)$ except path P and $k \geq 2$, $\gamma_{kb}(T) \geq \gamma_{ts}(T)$.

Proof: Consider a tree T, except path P, then every single edge of T form block and which correspond to vertex in block graph $B(T)$. Let D be the minimal total split dominating set in T, then D is a dominating set in T such that the induced subgraph of T obtained by removing D is disconnected.

Now, we establish correspondence between the D and block 2 dominating set. We include all blocks that corresponds to vertices not covered by D in the new set D'. Additionally, include any other blocks adjacent to these blocks in D'. This ensures that the set D' forms block 2 dominating set, where $|D| \leq |D'|$. This implies that $\gamma_{2b}(T) \geq \gamma_{ts}(T)$.

We know, for every $k \geq 2$, $\gamma_{kb}(T) \geq \gamma_{2b}(T)$. Thus, $\gamma_{kb}(T) \geq \gamma_{ts}(T)$.

Theorem 3.19 For any graph $G = (p, q)$ and $k = \triangle(B(G))$, $\gamma_{kb}(G) \leq \alpha_0(G) + \gamma'(G)$.

Proof: Let D be the minimal block k dominating set of G, where $k = \triangle(B(G))$, and let F be a minimal edge dominating set of graph G. We construct new set S of edges in graph G as follows: We include all the edges from set F in S.

As we know, for each block B represented by a vertex in D of $B(G)$, choose a cut vertex u within the block B and include all edges incident to u in S. This construction ensures that S covers all vertices in G because any non-cut vertex is contained within a block, and edges from F combined with edges incident to that chosen cut vertices, guarantee this coverages. Furthermore, S dominates all edges due to their inclusion and any other edge must have at least one end point at cut vertex corresponding to block in D. Therefore, using construction of S serves as both edge cover and edge dominating set and we have, $|D| \leq |S| \leq \alpha_0(G) + \gamma'(G)$. Thus, $\gamma_{kb}(G) \leq \alpha_0(G) + \gamma'(G)$.

Corollary 3.20 For any graph $G = (p, q)$ and $k = \triangle(B(G))$, $\gamma_{kb}(G) \leq \gamma_{ss}(G) + \gamma'(G)$.

Proof: From Theorem 3.19 and Theorem 2.6, we get result.

Theorem 3.21 For any graph $G = (p, q)$ without tree and $k = \triangle(B(G))$, $\gamma_{kb}(G) + \gamma'(G) \leq q$.

Proof: Let F be a minimal edge dominating set of graph G and let D be the minimal block k dominating set of G. Now, we construct set S of edges as: Include all edges from F and for each vertex in block graph $B(G)$ that is in the minimum block k dominating set D, choose a cut vertex u from the corresponding block B and includes all edges incident on u in S. This ensures that edges in blocks not covered by D are also included in S due to adjacency to u. Each block in D contributes at most $\triangle(B(G))$ edges to S because of cut vertices can contributes up to $\triangle(B(G))$ edges. If G is not tree, the number of blocks that is correspond to vertices of $B(G)$ is strictly less than number of vertices of G. This ensures that while adding edges incident to cut vertices, we do not excessively increase the size if S compared to total number of edges in G. From this construction, yield that $\gamma_{kb}(G) + \gamma'(G) \leq q$.

Theorem 3.22 For any graph $G = (p, q)$ and $k = \triangle(B(G))$, $\gamma_{kb}(G) \leq \gamma'_c(G) + \gamma_{ns}(G)$.

Proof: Let S be the non split dominating set of G and F is the connected edge dominating set of G. Using these we construct set D such that D forms k dominating set for a block graph $B(G)$, with its size does not exceed $\gamma'_c(G) + \gamma_{ns}(G)$. Since S is non split dominating set, for every vertex u in $v(G)$ is either in S or adjacent to at least one vertex in S such that induced subgraph $\langle V - S \rangle$ is connected.

For each block B of G, choose vertex u in S if it exists in B and consider the corresponding vertex in $B(G)$ as part of D. Since F is connected edge dominating set, the vertices covered by F can dominate the corresponding vertices of $B(G)$ through blocks they participate in. Addi- tionally, for each edge in F include the corresponding vertices in the relevant blocks of $B(G)$ in the set D. Thus, this construction of D from S and F ensures that D satisfies the conditions of a of block k dominating set, with $|D| \leq |S| + |F|$. Therefore, $\gamma_{kb}(G) \leq \gamma_c'(G) + \gamma_{ns}(G)$.

Theorem 3.23 For any graph $G = (p, q)$ without tree and $k = \triangle(B(G))$, $\gamma_{kb}(G) \leq \alpha_1(G) + \delta'(G)$.

Proof: Let D be the minimal block k dominating set and C be the minimal edge cover of G. Since G is not tree, it contains at least on cycle. Consider the cycle in G. Each edge in the cycle either belongs to a block or form a cut edge, which creates a new block with single edge. For block B that contain multiple edges of cycle, we exclude the cycle edges within B from our edge cover, if B corresponds to vertex in D, in this case, none of its edges are included in the initial edge cover. If B is not represented in D, we select any $k - 1$ edges incident to B (given that $\delta' \geq 1$) and include them in the constructed edge cover. To ensure all cycle edges are cover, we add uncovered cycle edges to our set noting that they can be at most $\delta'(G)$ such edges. We then construct the block k dominating set by including all vertices from D and edges selected in the previous steps.

The construction ensures that each block on the cycle either in the block k dominating set or adjacent to one and the remaining blocks are handled using the k dominating property and the cardinality of block k dominating set is at most sum of edge cover and $\delta'(G)$. Thus, $\gamma_{kb}(G) \leq \alpha_1(G) + \delta'(G)$.

Theorem 3.24 For any graph $G = (p, q)$ without tree and $k = \triangle(B(G))$, $\gamma_{kb}(G) \leq \beta_0(G) + \delta'(G)$.

Proof: Let D be the minimal block k dominating set of G. Let us construct set S as follows: for each block B in $B(G)$ represented by vertex in D, then choose one arbitrary vertex from B and it to S. Since no two vertices in S are adjacent (they come from distinct blocks and block share at most one cut vertex), S form an independent set.

Each block B that is not in D is adjacent to at least k blocks in D. For $k = \triangle(B(G))$, then knowing that S consist of vertices from D representing block and S must contain at least one vertex from every block not in D, ensuring that the vertices of S can cover those blocks. Since each block in D contributes to the block k dominating set and each block has at least $\delta'(G)$ edges, this establishes that D is bounded above by $|S|+\delta'(G)$. Therefore, $\gamma_{kb}(G) \leq \beta_0(G)+ \delta'(G)$.

EDGE COVERAGE

Theorem 3.25 A block dominating set S ensures that every edge in G is covered by at least one block in S.

Proof: Suppose S is block k dominating set in G. For every edge e in G, since e belongs to a block (by definition of block) and every block not in S is adjacent to at least k blocks in S. Thus, there exists at least one block in S that covers e.

VERTEX COVERAGE

Theorem 3.26 Block k-domination indirectly ensures vertex coverage.

Proof: Let S is block k dominating set in G. Consider any vertex v in G. v belongs to a block B_v. If $B_v \in S$, v is directly covered. If $B_v \notin S$ there are at least k blocks in S adjacent to B_v, and hence covering v.

4. CONCLUSION

In this paper, k-domination in block graph is defined. Theorems related to block k-domination are derived and the relationship between other different domination parameters are explored. Additionally, several bounds on $\gamma_{kb}(G)$ are obtained in terms of vertices, edges and other parameters of G.

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