ISSN 0315-3681 Volume 122, 2025

DEGREES OF PYTHAGOREAN INTUITIONISTIC FUZZY GRAPH

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Abstract:

This paper presents a comprehensive exploration of various classifications of Pythagorean Intuitionistic Fuzzy Graphs (PIFGs). Specifically, it delves into multiple types, such as regular, irregular, totally regular, totally irregular, neighbourly regular, neighbourly irregular, highly regular, highly irregular, neighbourly totally regular, neighbourly totally irregular, strongly irregular and strongly totally irregular graphs using the open and closed degrees of PIFG. Each of these categories is defined and discussed in detail to highlight their unique characteristics and implications. Furthermore, the paper provides rigorous proofs of several theorems and results that pertain to these diverse graph types, enhancing our understanding of their properties and relationships within the broader context of fuzzy graph theory.

Introduction

Krassimir Atanassov was the pioneer behind the concept of intuitionistic fuzzy sets, [3] Building on this foundational work, Yager made significant contributions to the field by introducing Pythagorean fuzzy sets, as discussed in [8]. The synthesis of these two theoretical frameworks culminates in the creation of Pythagorean intuitionistic fuzzy sets, which are detailed in [6]. This groundbreaking integration not only enhances the understanding of fuzzy concepts but also lays the groundwork for the study of Pythagorean intuitionistic fuzzy graphs (PIFGs) [6]. In this context, these graphs are rigorously defined and examined, indicating that

"$G = (\mathbb{A}, \mathbb{B})$ is a **Pythagorean intuitionistic fuzzy graph (PIFG)** of a graph $G^* = (\mathbb{V}, \mathbb{E})$ which is a crisp graph. Where $\mathbb{A} = <\mathfrak{T}_{\mathbb{A}}, \mathfrak{F}_{\mathbb{A}}>$ is an intuitionistic set on $\mathbb{V}$ and $\mathbb{B} = <\mathfrak{T}_{\mathbb{B}}, \mathfrak{F}_{\mathbb{B}}>$ is an intuitionistic relation on $\mathbb{E}$ with $\mathfrak{T}_{\mathbb{A}}$ and $\mathfrak{F}_{\mathbb{A}}$ from $\mathbb{V}$ to [0,1] signifying truth membership, falsity membership functions of $\mathbb{V}$ and $0 \leq \mathfrak{T}_{\mathbb{A}}(a_i) + \mathfrak{F}_{\mathbb{A}}(a_i) \leq 1$ for $a_i \in \mathbb{V}$ such that $\mathfrak{T}_{\mathbb{B}}(a_i a_j) \leq \mathfrak{T}_{\mathbb{A}}(a_i) \wedge \mathfrak{T}_{\mathbb{A}}(a_j)$, $\mathfrak{F}_{B}(a_i a_j) \geq \mathfrak{F}_{\mathbb{A}}(a_i) \vee \mathfrak{F}_{\mathbb{A}}(a_j)$ where $\mathfrak{T}_{\mathbb{B}}, \mathfrak{F}_{\mathbb{B}}$ from $\mathbb{V} \times \mathbb{V}$ to [0,1] stand for the truth membership, falsity membership functions of $\mathbb{E}$, with

$0 \leq \mathfrak{T}_{\mathbb{B}}^2(a_i a_j) + \mathfrak{F}_{\mathbb{B}}^2(a_i a_j) \leq 1$ and $0 \leq \mathfrak{T}_{\mathbb{B}}(a_i a_j) + \mathfrak{F}_{\mathbb{B}}(a_i a_j) \leq 1$ for all $a_i a_j \in \mathbb{E}$".

A. Nagoor Gani and K. Radha have made significant contributions to the field by introducing the concepts of regular and totally regular fuzzy graphs using degrees, as detailed in [2]. In addition to this, they also proposed the concept of irregular fuzzy graphs, which is discussed in [1]. Building upon this foundational work, S. Sivabala and N.R. Santhi Maheswari further enriched the academic discourse by presenting the concepts of neighbourly and highly irregular neutrosophic fuzzy graphs, as outlined in [7]. Moreover, N.R. Santhi Maheswari, along with C. Sekar, made important strides in graph theory by establishing the notions of neighbourly irregular graphs [5], semi-neighbourly irregular graphs [4], and by further refining the study of irregular fuzzy graphs [5]. Drawing from these foundational ideas, this paper introduces the concepts of Regular Pythagorean Intuitionistic Fuzzy Graphs (PIFGs) and Irregular PIFGs, providing a fresh perspective on these graph types.

Degrees of PIFG

Definition 2.1:

Let G = ($\mathbb{A}$, $\mathbb{B}$) be a PIFG. Degree or open neighbourhood degree of a vertex a_i in G is

$đo(a_i) = < đ_o\mathfrak{T}_{\mathbb{A}}(a_i), đ_o\mathfrak{F}_{\mathbb{A}}(a_i)>$, where $đ_o\mathfrak{T}_{\mathbb{A}}(a_i) = \sum_{a_ia_j\in\mathbb{E}} \mathfrak{T}_{\mathbb{B}}(a_ia_j)$,

$$đ_o\mathfrak{F}_{\mathbb{A}}(a_i) = \sum_{a_ia_j\in\mathbb{E}} \mathfrak{F}_{\mathbb{B}}(a_ia_j).$$

Definition 2.2:

Let *G*= ($\mathbb{A}$, $\mathbb{B}$) be a PIFG. A vertex a_i in G has a total degree or closed neighbourhood degree that is defined by $đ_c [a_i] = < đ_c\mathfrak{T}_{\mathbb{A}}[a_i], đ_c\mathfrak{F}_{\mathbb{A}}[a_i]>$,

where $đ_c\mathfrak{T}_{\mathbb{A}}[a_i] = \sum_{a_ia_j\in\mathbb{E}} \mathfrak{T}_{\mathbb{B}}(a_ia_j)+ \mathfrak{T}_{\mathbb{A}}(a_i)$, $đ_c\mathfrak{F}_{\mathbb{A}}[a_i] = \sum_{a_ia_j\in\mathbb{E}} \mathfrak{F}_{\mathbb{B}}(a_ia_j)+ \mathfrak{F}_{\mathbb{A}}(a_i)$.

Example 2.3:

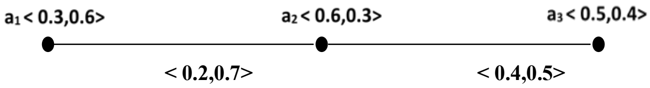


Figure 1. PIFG

$đ_o(a_1) = <0.2,0.7>$ $đ_o(a_2) = <0.6,1.2>$ $đ_o(a_3) = <0.4,0.5>$

$đ_c[a_1] = <0.5,1.3>$ $đ_c[a_2] = <1.2,1.5>$ $đ_c[a_3] = <0.9,0.9>$

Definition 2.4:

A Pythagorean Intuitionistic Fuzzy Graph (PIFG) is classified as a regular PIFG (RPIFG) when every vertex within the graph shares the same open neighbourhood degree. This uniformity in vertex degrees signifies that the connections or relationships among the vertices are consistent across the graph. Additionally, if an RPIFG has specific degrees denoted as $k_1$for the Truth membership and k_2 for the Falsity membership, it is referred to as k_1, k_2-RPIFG.

Definition 2.5:

A Pythagorean Intuitionistic Fuzzy Graph (PIFG) is termed a totally regular PIFG (TRPIFG) when every vertex in the graph possesses the same closed neighbourhood degree. This means that all vertices not only have a uniform number of connections but also share the same degree of influence when considering the vertices they are adjacent to, reflecting a consistent structural pattern throughout the graph. Furthermore, when a TRPIFG has designated degrees k_1 for Truth membership and k_2 for Falsity membership, it is specifically referred to as a k_1,k_2-TRPIFG. This classification highlights the particular membership characteristics associated with the graph's vertices.

Example 2.6:

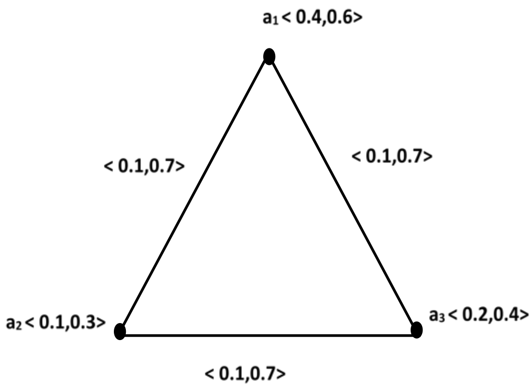


Figure 2. RPIFG

$đ_o(a_1)$ = <0.2,1.4> $đ_o(a_2)$ = <0.2,1.4> $đ_o(a_3)$ = <0.2,1.4>

This PIFG is a 0.2,0.6 - RPIFG since here open neighbourhood degrees of every vertex is same.

Definition 2.7:

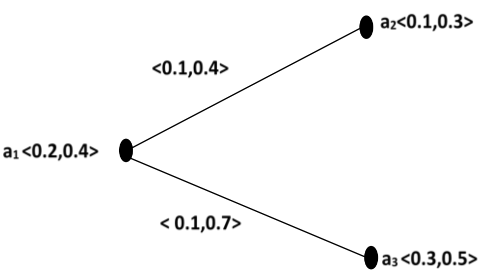
A Pythagorean Intuitionistic Fuzzy Graph (PIFG) is categorized as an irregular PIFG (IPIFG) when at least one vertex within the graph is adjacent to vertices that exhibit distinct open neighbourhood degrees. This irregularity indicates that the connections of the specific vertex led to neighbouring vertices that do not share a common degree of adjacency, reflecting a lack of uniformity in their structural relationships.

Definition 2.8:

A Pythagorean Intuitionistic Fuzzy Graph (PIFG) is designated as a totally irregular PIFG (TIPIFG) when there exists at least one vertex that is connected to other vertices with varying closed neighbourhood degrees. This situation indicates that the vertex in question is linked to neighbours that do not share the same degree of adjacency, thereby highlighting a significant lack of uniformity in the closed relationships within the graph. The presence of such a vertex introduces a level of complexity and irregularity to the graph's structure, distinguishing it from totally regular or regular types of PIFGs.

Example 2.9:

In the example presented below, vertex a_1 is connected to vertices a_2 and a_3, each of which possesses distinct degrees of both open and closed neighbourhoods. This variance in the degrees of adjacency illustrates the lack of uniformity among the neighbouring vertices. Consequently, this graph serves as an example of both an irregular Pythagorean Intuitionistic Fuzzy Graph (IPIFG) and a totally irregular Pythagorean Intuitionistic Fuzzy Graph (TIPIFG), as it showcases the distinct characteristics that define these classifications.



$đ_o(a_1)$ = <0.2,1.1> $đ_o(a_2)$ = <0.1,1.4>

$đ_c[a_1]$ = <0.4,1.5> $đ_c[a_2]$ = <0.2,0.7>

$đ_o(a_3)$ = <0.1,0.7>

$đ_c[a_3]$ = <0.4,1.2>

Figure 3. IPIFG & TIPIFG

Definition 2.10:

A neighbourly irregular Pythagorean Intuitionistic Fuzzy Graph (NIPIFG) is defined as a graph in which the open neighbourhood degrees of any two adjacent vertices are different from one another. This means that for every pair of connected vertices, the degree of adjacency—that is, the number of direct connections to other vertices—varies, indicating a lack of uniformity among the neighbouring relationships. This characteristic distinguishes NIPIFGs from other types of PIFGs, as it highlights the diversity in the structural properties of the graph's vertices and their interconnections.

Definition 2.11:

A neighbourly totally irregular Pythagorean Intuitionistic Fuzzy Graph (NTIPIFG) is defined as a graph where the closed neighbourhood degrees of any two adjacent vertices are different. In this context, closed neighbourhood degrees refer to the total number of connections a vertex has, including itself and its direct neighbours. The distinctiveness of these degrees among adjacent vertices signifies a lack of uniformity in their relationships. This characteristic sets NTIPIFGs apart from other types of PIFGs, as it emphasizes the variability in the structural connections between neighbouring vertices, further enriching the complexity of the graph's overall configuration.

Example 2.12:

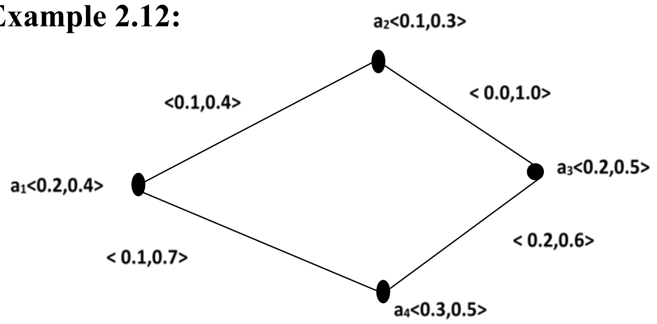


Figure 4. NIPIFG &NTIPIFG

$đ_o(a_1)$ = <0.2,1.1> $đ_o(a_2)$ = <0.1,1.4> $đ_o(a_3)$ = <0.2,1.6> $đ_o(a_4)$ = <0.3,1.3>

$đ_c[a_1]$ = <0.4,1.5> $đ_c[a_2]$ = <0.2,1.7> $đ_c[a_3]$ = <0.4,2.1> $đ_c[a_4]$ = <0.6,1.8>

In this specific example, it is evident that the open and closed neighbourhood degrees of any two adjacent vertices are not the same. This lack of uniformity clearly illustrates that the degrees vary between connected vertices. As a result, the graph exemplifies both a neighbourly irregular Pythagorean Intuitionistic Fuzzy Graph (NIPIFG) and a neighbourly totally irregular Pythagorean Intuitionistic Fuzzy Graph (NTIPIFG). The distinct neighbourhood degrees reinforce the complexities of the graph's structure and its relationships among vertices.

Definition 2.13:

A Pythagorean Intuitionistic Fuzzy Graph (PIFG) is classified as a highly irregular PIFG (HIPIFG) when every vertex in the graph is connected to neighbouring vertices that possess distinct open neighbourhood degrees. This means that for each vertex, the degrees of adjacency to its neighbours are not uniform, showcasing a significant variability in the number of direct connections among them. As a result, the highly irregular nature of the graph underscores the complexity and diversity of relationships that exist within its structure, distinguishing it from other types of PIFGs that may exhibit more regular patterns.

Definition 2.14:

A highly totally irregular Pythagorean Intuitionistic Fuzzy Graph (HTIPIFG) is characterized by the condition that every vertex is connected to at least one other vertex that has a distinct closed neighbourhood degree. This means that for each vertex, the total number of connections—considering both itself and its adjacent vertices—varies among its neighbours. This lack of uniformity in closed neighbourhood degrees highlights the complexity and diversity of the graph's structure, distinguishing highly totally irregular PIFGs from other graph types that may display more consistent relationships among their vertices.

Observation 2.15:

i) A neighbourly irregular Pythagorean Intuitionistic Fuzzy Graph (NIPIFG) does not automatically qualify as a neighbourly totally irregular PIFG (NTIPIFG). This distinction highlights that while both types share some characteristics, the criteria for each classification are not interchangeable.

(ii) Similarly, a neighbourly totally irregular PIFG (NTIPIFG) does not inherently qualify as a neighbourly irregular PIFG (NIPIFG). This indicates that, despite overlapping features, the specific conditions defining each type are unique and not mutually inclusive.

(iii) In the same vein, a highly irregular PIFG is not necessarily a highly totally irregular PIFG. This suggests that although both categories deal with irregularity, the nature of their irregular connections varies, preventing automatic classification from one to the other.

(iv) Finally, a highly totally irregular PIFG does not inherently imply that it is a highly irregular PIFG. This reinforces the notion that while both types address distinct degrees of irregularity, their defining characteristics differ in ways that do not guarantee one type encompasses the other.

Example 2.16:

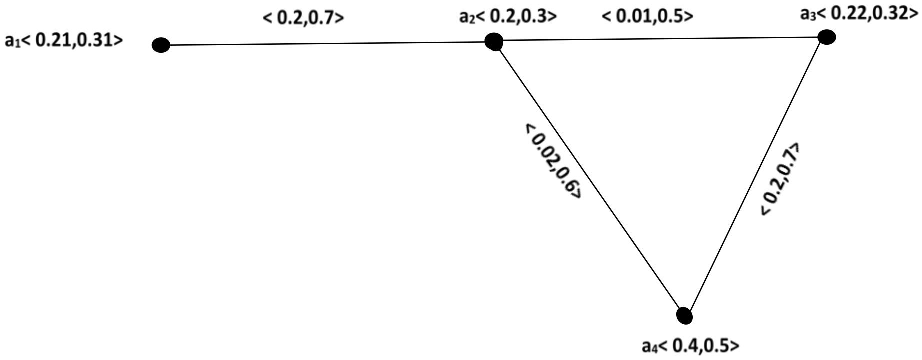


Figure 4. neighbourly totally irregular PIFG

$đ_o(a_1)$ = <0.2,0.7> $đ_o(a_2)$ = <0.21,1.2> $đ_o(a_3)$ = <0.21,1.2> $đ_o(a_4)$ = <0.22,1.3>

$đ_c[a_1]$ = <0.41,1.1> $đ_c[a_2]$ = <0.41,1.5> $đ_c[a_3]$ = <0.43,1.52> $đ_c[a_4]$ = <0.62,1.8>

Therefore, this graph should be classified not as a neighbourly irregular Pythagorean Intuitionistic Fuzzy Graph (NIPIFG), but rather as a neighbourly totally irregular PIFG (NTIPIFG). This distinction arises from the fact that the closed neighbourhood degrees of any two adjacent vertices are different from each other. However, the open neighbourhood degrees of vertices a2 and a3 are identical, indicating a lack of variability in their direct connections. This combination of characteristics clearly defines the graph as a neighbourly totally irregular PIFG, as it meets the criteria for that classification while failing to satisfy the conditions for being a neighbourly irregular PIFG.

Theorem 2.17:

Let $G = (\mathbb{A}, \mathbb{B})$ be a PIFG and $<\mathfrak{T}_{\mathbb{A}}(a_i), \mathfrak{F}_{\mathbb{A}}(a_i)>$ is a constant function $\forall a_i \in \mathbb{V}$ iff the following are equivalent:

(1) $G = (\mathbb{A}, \mathbb{B})$ is RPIFG
(2) $G = (\mathbb{A}, \mathbb{B})$ is TRPIFG.

Proof:

Necessary Part:

Given: Consider $<\mathfrak{T}_{\mathbb{A}}(a_i), \mathfrak{F}_{\mathbb{A}}(a_i)> = <c_1, c_2> \forall a_i \in \mathbb{V}$

To Prove: (1) ⇒(2) & (2) ⇒ (1)

Proof: Assume that G is a k_1, k_2 -RPIFG.

Then $đ_o(a_i) = <đ_o\, \mathfrak{T}_{\mathbb{A}}(a_i), đ_o\mathfrak{F}_{\mathbb{A}}(a_i)> = <\sum_{a_ia_j \in \mathbb{E}} \mathfrak{T}_{\mathbb{B}}(a_ia_j), \sum_{a_ia_j \in \mathbb{E}} \mathfrak{F}_{\mathbb{B}}(a_ia_j)> \forall a_i \in \mathbb{V}$.

$$= <k_1, k_2> \forall a_i \in \mathbb{V}.$$

So, $đc\,[a_i] = <đ_c\, \mathfrak{T}_{\mathbb{A}}[a_i], đ_c\mathfrak{F}_{\mathbb{A}}[a_i]> \forall a_i \in \mathbb{V}$.

$$= <\sum_{a_ia_j \in \mathbb{E}} \mathfrak{T}_{\mathbb{B}}(a_ia_j) + \mathfrak{T}_{\mathbb{A}}(a_i), \sum_{a_ia_j \in \mathbb{E}} \mathfrak{F}_{\mathbb{B}}(a_ia_j) + \mathfrak{F}_{\mathbb{A}}(a_i)> \forall a_i \in \mathbb{V}.$$

$$= <k_1+c_1, k_2+c_2> \forall a_i \in \mathbb{V}.$$

So G is a TRPIFG. Thus (1) ⇒ (2).

Assume G is a k_3, k_4 – TRPIFG (i.e.) $đ_c\,[a_i] = <k_3, k_4> \forall a_i \in \mathbb{V}$.

$$đ_o(a_i) + <\mathfrak{T}_{\mathbb{A}}(a_i), \mathfrak{F}_{\mathbb{A}}(a_i)> = <k_3, k_4> \forall a_i \in \mathbb{V}.$$

$$đ_o(a_i) + <c_1, c_2> = <k_3, k_4> \forall a_i \in \mathbb{V}.$$

$$đ_o(a_i) = <k_3, k_4> - <c_1, c_2> \forall a_i \in \mathbb{V}.$$

$$= <k_3 - c_1, k_4 - c_2> \forall a_i \in \mathbb{V}.$$

So G is a RPIFG. Thus (2) ⇒ (1).

Sufficient Part:

Assumption: (1) $\Leftrightarrow$ (2)

To Prove: $<\mathfrak{T}_{\mathbb{A}}(a_i), \mathfrak{F}_{\mathbb{A}}(a_i)> \forall a_i \in \mathbb{V}$ is a constant function.

Proof: Assume $<\mathfrak{T}_{\mathbb{A}}(a_i), \mathfrak{F}_{\mathbb{A}}(a_i)>$ is not a constant function.

Then $<\mathfrak{T}_{\mathbb{A}}(a_i), \mathfrak{F}_{\mathbb{A}}(a_i)> \neq <\mathfrak{T}_{\mathbb{A}}(a_j), \mathfrak{F}_{\mathbb{A}}(a_j)>$ for some $a_i, a_j \in \mathbb{V}$.

Let G be a k_1, k_2 -RPIFG then $đ_o(a_i) = đ_o(a_j) = < k_1, k_2>$

So $đ_c[a_i] = đ_o(a_i) + <\mathfrak{T}_{\mathbb{A}}(a_i), \mathfrak{F}_{\mathbb{A}}(a_i)> = < k_1, k_2> + <\mathfrak{T}_{\mathbb{A}}(a_i), \mathfrak{F}_{\mathbb{A}}(a_i)>$

$đ_c[a_j] = đ_o(a_j) + <\mathfrak{T}_{\mathbb{A}}(a_j), \mathfrak{F}_{\mathbb{A}}(a_j)> = < k_1, k_2> + <\mathfrak{T}_{\mathbb{A}}(a_j), \mathfrak{F}_{\mathbb{A}}(a_j)>$

Since $<\mathfrak{T}_{\mathbb{A}}(a_i), \mathfrak{F}_{\mathbb{A}}(a_i)> \neq <\mathfrak{T}_{\mathbb{A}}(a_j), \mathfrak{F}_{\mathbb{A}}(a_j) \Rightarrow đ_c[a_i] \neq đ_c[a_j]$

$\Rightarrow$It is not a TRPIFG which is a contradictory.

Assume G is a k_3, k_4 -TRPIFG. Then $đ_c(a_i) = đ_c(a_j)$

$\Rightarrow đ_o(a_i) + <\mathfrak{T}_{\mathbb{A}}(a_i), \mathfrak{F}_{\mathbb{A}}(a_i)> = đ_o(a_j) + <\mathfrak{T}_{\mathbb{A}}(a_j), \mathfrak{F}_{\mathbb{A}}(a_j)>$

$\Rightarrow đ_o(a_i) - đ_o(a_j) = <\mathfrak{T}_{\mathbb{A}}(a_i), \mathfrak{F}_{\mathbb{A}}(a_i)> - <\mathfrak{T}_{\mathbb{A}}(a_j), \mathfrak{F}_{\mathbb{A}}(a_j)> \neq 0$

$\Rightarrow$It is not a RPIFG, which is a contradictory.

Therefore, $<\mathfrak{T}_{\mathbb{A}}(a_i), \mathfrak{F}_{\mathbb{A}}(a_i)> \forall a_i \in \mathbb{V}$ is a constant function.

Theorem 2.18:

Let $G = (\mathbb{A}, \mathbb{B})$ be a Pythagorean Intuitionistic Fuzzy Graph (PIFG) in which all vertices have distinct open neighbourhood degrees. Then, G is classified as both a highly irregular PIFG and a neighbourly irregular PIFG.

Proof:

Since the provided PIFG features distinct open neighbourhood degrees for each vertex

(i.e.) $đ_o(a_i) \neq đ_o(a_i)$ for all $a_i, a_j \in \mathbb{V}$

(i.e.) All adjacent vertices exhibit distinct open neighbourhood degrees, meaning that each pair of connected vertices has a different number of direct connections to other vertices in the graph. This lack of uniformity in their degrees underscores the variability in the relationships among these vertices.

Consequently, the given Pythagorean Intuitionistic Fuzzy Graph G qualifies as both a highly irregular PIFG and a neighbourly irregular PIFG.

Theorem 2.19:

Let $G = (\mathbb{A}, \mathbb{B})$ be a NIPIFG and $<\mathfrak{T}_{\mathbb{A}}(a_i), \mathfrak{F}_{\mathbb{A}}(a_i)>$ is a constant function $\forall a_i \in \mathbb{V}$ then G is

NTIPIFG.

Proof:

Given that G is a NIPIFG, the open neighbourhood degrees of any two adjacent vertices are distinct.

(i.e.) $đ_o(a_i) \neq đ_o(a_j)$ for all $a_i, a_j \in \mathbb{V}$.

Also, it is given that $<\mathfrak{T}_{\mathbb{A}}(a_i), \mathfrak{F}_{\mathbb{A}}(a_i)>$ is a constant function $\forall a_i \in \mathbb{V}$.

So $<\mathfrak{T}_{\mathbb{A}}(a_i), \mathfrak{F}_{\mathbb{A}}(a_i)> = < c_1, c_2>$

Then $đ_c [a_i] = đ_o(a_i) + <\mathfrak{T}_{\mathbb{A}}(a_i), \mathfrak{F}_{\mathbb{A}}(a_i)>$

$= đ_o(a_i) + < c_1, c_2>$

and $đ_c [a_j] = đ_o(a_j) + <\mathfrak{T}_{\mathbb{A}}(a_j), \mathfrak{F}_{\mathbb{A}}(a_j)>$

$= đ_o(a_j) + < c_1, c_2>$

To prove G is a NTIPIFG, we need to show $đ_c[a_i] \neq đ_c[a_j]$

Assume by contradiction that is $đ_c[a_i] = đ_c[a_j]$

$\Rightarrow đ_o(a_i) + < c_1, c_2> = đ_o(a_j) + < c_1, c_2>$

$\Rightarrow đ_o(a_i) = đ_o(a_j)$

Which is a contradiction to the fact that $đ_o(a_i) \neq đ_o(a_j)$.

Hence G is a NTIPIFG.

Theorem 2.20:

Let $G = (\mathbb{A}, \mathbb{B})$ be a NTIPIFG and $<\mathfrak{T}_{\mathbb{A}}(a_i), \mathfrak{F}_{\mathbb{A}}(a_i)>$ is a constant function $\forall a_i \in \mathbb{V}$ then G is NIPIFG.

Proof:

Since G is a NTIPIFG, the closed neighbourhood degrees of any two adjacent vertices are different.

(i.e.) $đ_c[a_i] \neq đ_c[a_j]$ for all $a_i, a_j \in \mathbb{V}$.

Also, it is given that $<\mathfrak{T}_{\mathbb{A}}(a_i), \mathfrak{F}_{\mathbb{A}}(a_i)>$ is a constant function $\forall a_i \in \mathbb{V}$.

So $<\mathfrak{T}_{\mathbb{A}}(a_i), \mathfrak{F}_{\mathbb{A}}(a_i)> = < c_1, c_2>$

$đ_c [a_i] = đ_o(a_i) + <\mathfrak{T}_{\mathbb{A}}(a_i), \mathfrak{F}_{\mathbb{A}}(a_i)> = đ_o(a_i) + < c_1, c_2>$

and $đ_c [a_j] = đ_o(a_j) + <\mathfrak{T}_{\mathbb{A}}(a_j), \mathfrak{F}_{\mathbb{A}}(a_j)> = đ_o(a_j) + < c_1, c_2>$

To prove G is a NIPIFG, we need to show $đ_o(a_i) \neq đ_o(a_j)$

Let us suppose, for the sake of contradiction, that $đ_o(a_i) = đ_o(a_j)$

$\Rightarrow đ_o(a_i) + < c_1, c_2> = đ_o(a_j) + < c_1, c_2>$

$\Rightarrow đ_c[a_i] = đ_c[a_j]$

This stands in opposition to the fact that $đ_c[a_i] \neq đ_c[a_j]$. Hence G is a NIPIFG.

Definition 2.21:

A Strongly irregular PIFG (SIPIFG) is characterized by the condition that every pair of vertices have unique open neighbourhood degrees.

Example 2.22:

Consider PIFG

Figure 4. strongly irregular PIFG

$đ_o(a_1)$ = <0.04,1.14> $đ_o(a_2)$ = <0.03,1.05> $đ_o(a_3)$ = <0.06,1.18> $đ_o(a_4)$ = <0.07,1.27>
In the above figure all the open degrees are unique. Hence it is classified as Strongly Irregular PIFG.

Definition 2.23:

A Strongly totally irregular PIFG (STIPIFG) is characterized by the condition that every pair of vertices have unique closed neighbourhood degrees.

Example 2.24:

Consider the figure 4,

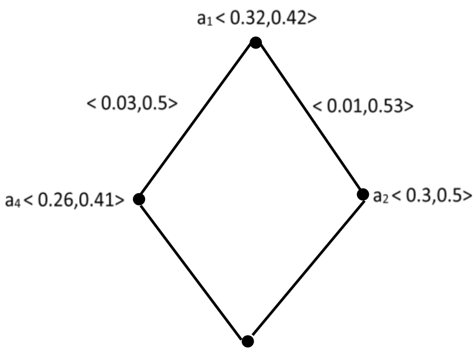
$đ_c[a_1]$ = <0.24,1.54> $đ_c[a_2]$ = <0.33,1.55> $đ_c[a_3]$ = <0.36,1.68> $đ_c[a_4]$ = <0.47,1.87>
All the pairs of vertices in figures 4 have unique closed neighbourhood degrees.

So, It is a Strongly totally irregular PIFG.

Observation 2.25:

(i) A strongly irregular PIFG does not necessarily possess the characteristic required to be classified as a strongly totally irregular PIFG.
(ii) Similarly, a strongly totally irregular PIFG does not necessarily have to be a strongly irregular PIFG.
(iii) A PIFG is categorized as totally irregular PIFG does not guarantee that it also meets the conditions required for being strongly totally irregular PIFG.
(iv) A highly irregular PIFG is not necessarily strongly irregular PIFG.

Example 2.26:



< 0.04,0.66>   < 0.02,0.54>

a3 < 0.3,0.5>

Figure 5. SIPIFG but not STIPIFG

đ$_o(a_1)$ = <0.04,1.03>   đ$_o(a_2)$ = <0.03,1.07>   đ$_o(a_3)$ = <0.06,1.20>   đ$_o(a_4)$ = <0.07,1.16>

đ$_c[a_1]$ = <0.36,1.45>   đ$_c[a_2]$ = <0.33,1.57>   đ$_c[a_3]$ = <0.36,1.70>   đ$_c[a_4]$ = <0.33,1.57>

In this figure, each vertex has a distinct open neighbourhood degree. However, the vertices a_2 and a_4 share the same closed neighbourhood degree.

As a result, the PIFG qualifies as strongly irregular but does not meet the criteria for being strongly totally irregular.

Also, the vertex a_2 and a_4 have distinct closed neighbourhood degrees for the adjacent vertices.

So, this is totally irregular PIFG but not strongly totally irregular PIFG.

Example 2.27:

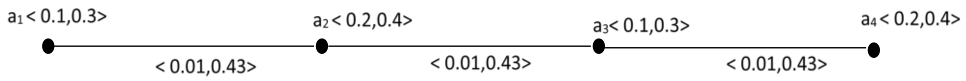


Figure 6. HIPIFG but not SIPIFG

đ$_o(a_1)$ = <0.01,0.43>   đ$_o(a_2)$= <0.02,0.86>   đ$_o(a_3)$ = <0.02,0.86>   đ$_o(a_4)$ = <0.01,0.43>

In the above figure, each vertex is connected to vertices with distinct open neighbourhood degrees. However, the vertices a2 and a3 share the same open neighbourhood degree. Therefore, the given graph qualifies as a highly irregular PIFG but does not satisfy the conditions for being strongly irregular PIFG.

Theorem 2.28:

Let $G = (\mathbb{A}, \mathbb{B})$ be a PIFG and $\langle \mathfrak{T}_{\mathbb{A}}(a_i), \mathfrak{F}_{\mathbb{A}}(a_i) \rangle$ is a constant function $\forall a_i \in \mathbb{V}$ then G is SIPIFG iff G is STIPIFG.

Proof:

Given that $\langle \mathfrak{T}_{\mathbb{A}}(a_i), \mathfrak{F}_{\mathbb{A}}(a_i) \rangle$ is a constant function $\forall a_i \in \mathbb{V}$.

So $\langle \mathfrak{T}_{\mathbb{A}}(a_i), \mathfrak{F}_{\mathbb{A}}(a_i) \rangle = \langle c_1, c_2 \rangle$

Suppose G is a SIPIFG, then all vertices have distinct open neighbourhood degrees.

(i.e.) đ$_o(a_i) \neq$ đ$_o(a_j)$ for all $a_i, a_j \in \mathbb{V}$.

Then đ$_c[a_i]$ = đ$_o(a_i)$ + $\langle \mathfrak{T}_{\mathbb{A}}(a_i), \mathfrak{F}_{\mathbb{A}}(a_i) \rangle$

= đ$_o(a_i)$ + $\langle c_1, c_2 \rangle$

and đ$_c[a_j]$ = đ$_o(a_j)$ + $\langle \mathfrak{T}_{\mathbb{A}}(a_j), \mathfrak{F}_{\mathbb{A}}(a_j) \rangle$

= đ$_o(a_j)$ + $\langle c_1, c_2 \rangle$

To prove G is a STIPIFG, we need to show $đ_c[a_i] \neq đ_c[a_j]$

Since $đ_o(a_i) \neq đ_o(a_j)$

$đ_o(a_i) + < c_1, c_2> \neq đ_o(a_j) + < c_1, c_2>$

$đ_o(a_i) + <\mathfrak{T}_{\mathbb{A}}(a_i), \mathfrak{F}_{\mathbb{A}}(a_i)> \neq đ_o(a_j) + <\mathfrak{T}_{\mathbb{A}}(a_j), \mathfrak{F}_{\mathbb{A}}(a_j)>$

$đ_c [a_i] \neq đ_c [a_j]$ for all $a_i, a_j \in \mathbb{V}$

Hence G is a STIPIFG.

Conversely, assume that G is a STIPIFG, then all vertices have distinct closed neighbourhood degrees

$đ_c [a_i] \neq đ_c [a_j]$ for all $a_i, a_j \in \mathbb{V}$

$đ_o(a_i) + <\mathfrak{T}_{\mathbb{A}}(a_i), \mathfrak{F}_{\mathbb{A}}(a_i)> \neq đ_o(a_j) + <\mathfrak{T}_{\mathbb{A}}(a_j), \mathfrak{F}_{\mathbb{A}}(a_j)>$

$đ_o(a_i) + < c_1, c_2> \neq đ_o(a_j) + < c_1, c_2>$

$đ_o(a_i) \neq đ_o(a_j)$ for all $a_i, a_j \in \mathbb{V}$.Hence G is a SIPIFG.

Application of Regular and Irregular PIFG

Hospital Patient Monitoring System:

PIFG is used to model patient monitoring in a hospital.

The model elements are patients in the form of vertices and edges denote how close their medical conditions are under uncertainty.

The most critical goal of regular PIFG is to observe the patient cluster which is uniformly and diagnostically similiar in equal proportions to one another.

Consider 3 patients V = {(P1,0.4,0.6), (P2,0.1,0.3), (P3,0.2,0.4)} and each edge (Pi,Pj) is associated with

Truth $\mathfrak{T}_{\mathbb{B}}$: Degree of medical similarity

Falsity $\mathfrak{F}_{\mathbb{B}}$: Degree of dissimilarity

Define a Regular PIFG where each patient is equally connected to others

Edge	$\mathfrak{T}_{\mathbb{B}}$	$\mathfrak{F}_{\mathbb{B}}$
(P1P2)	0.1	0.7
(P2P3)	0.1	0.7
(P3P1)	0.1	0.7

đo(P1) = <0.2,1.4>, đo(P2) = <0.2,1.4>, đo(P3) = <0.2,1.4>

Every patient has the same open degree, implying it is Regular PIFG.

With this, if one patient alerts, like patients can be flagged as well due to uniform resemblance. This assists in implementing uniform monitoring protocols or resource distribution for homogeneous patient cohorts.

Now model an unsimilar patient group where patients have different extent of similarity in the medical condition.

Consider another 3 patients V = {(P1,0.5,0.5), (P2,0.4,0.4), (P3,0.3,0.5)} and each edge (Pi,Pj) is associated with

Truth $\mathfrak{T}_{\mathbb{B}}$: Degree of medical similarity

Falsity $\mathfrak{F}_{\mathbb{B}}$: Degree of dissimilarity

Define a Regular PIFG where each patient is equally connected to others

(P1P2)	0.4	0.6	
(P2P3)	0.2	0.7	
(P3P1)	0	0.6	

đo(P1) = <0.4,1.2>, đo(P2) = <0.6,1.3> , đo(P3) = <0.2,1.3>

Here the open degrees of all the 3 patients are different so it is a Irregular PIFG.

In this data P1 is highly similar to P2 but not to P3. Therefore, medical alerts for P1 should prioritize P2. So Tailored care plans can be created based on individual condition similarity.

From the above datas, the following are concluded

FEATURE	REGULAR PIFG	IRREGULAR PIFG
Node Degrees	Equal for all patients	Varying degrees across patients
Use case	Uniform monitoring protocols	Personalized or prioritized patient care
Structure	Balanced, symmetrical connections	Diverse, asymmetrical connections.

Fraud Detection in Financial Systems:

Case Study: Detecting anomalous Transactions

Mathematical Model

Let A = {(C1,0.6,0.3),(C2,0.4,0.3),(C3,0.7,0.1),(B,0.7,0.3)} be customers and a bank, with edges representing transactions.

And B = {(C1B,0.59,0.4), (C2B,0.2,0.3), (C3B,0.5,0.4), (C2C3,0.3,0.5),(C3C1,0.4,0.6)}

C1 and Bank B have high degrees while C2 and C3 have irregular connectivity, indicating possible fraud.

Anomalies occur where a customer C3 transacts more with another customer C1 rather than the bank, which is unusual.

This method can be used in Visa/Mastercard fraud detection systems which helps in identifying money laundering activities.

Consider a real-world dataset for fraud detection in financial transactions. The dataset will be processed into a PIFG by assigning truth and falsity membership values based on transaction frequency and risk scores.

Conclusion:

The main contribution of this manuscript is to introduce the idea of regularity and irregularity in Pythagorean Intuitionistic Fuzzy Graph. In this paper, we have described the notion of $đ_o$ degree and $đ_c$ degree of vertices in a PIFG. Different types of PIFG such as regular, totally regular, irregular, totally irregular, highly irregular, highly totally irregular, Neighbourly irregular, Neighbourly totally irregular, strongly irregular, strongly totally irregular were introduced here. In the future, we will focus on the study of Pythagorean Interval-Valued Intuitionistic Fuzzy graphs.

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