Dense Graph Homomorphisms of Co spectral Rigid Circuit Graphs are Cycle Decompositions

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Abstract

Understanding the structure of graphs is essential for progress in various areas of graph theory and its applications. This paper focuses on a specific structural property across multiple classes of graphs. Graphs containing a clique of size r(i.e., Kr-cospectralchordal dense graphs) are particularly significant in external graph theory. While many results have been established regarding dense triangle-free graphs, less is known about dense Kr-cospectralchordal dense graphs for $r \geq 4$. The conditions required to decompose the entire graph of odd order into cycles of a fixed even length and the complete graph of even order minus 1-factor into cycles of a fixed odd length are also demonstrated to be sufficient. This paper highlights key findings related to cores, homomorphisms and cycle decomposition.

Keywords: Cospectral Chordal graph, dense graph, Graph homomorphism, Cograph and tree-decomposition

Introduction:

The easiest lower bound on the chromatic number can be obtained by considering the cliques of a graph. A clique of size r, denoted Kr, is a set of r vertices, every pair of which are adjacent. The clique number of a graph G, denoted ω(G), is the maximum value of r such that the graph contains a Kr. It is trivial to see that χ(G) ≥ ω (G). However, the lack of a large clique does not imply a small chromatic number. A clique K_3 of size 3 is commonly referred to as a triangle. Mycielski in [4] gave a simple construction to obtain infinitely many relatively sparse graphs for which the maximum clique size is 2, i.e., triangle-free graphs, but which have arbitrarily large chromatic number. We discuses about cospectral chordal graphs are dense homomorphism for which the maximum clique size is 3.i.e., triangle graphs.

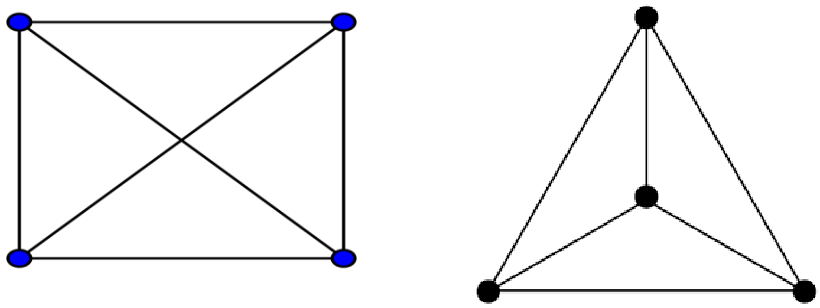

Figure 1.1.cospectral chordal dense graphs are triangle(clique)graph with chromatic number 4.

1.Cospectral Chordal dense graphs are Homomorphisms:

A homomorphism from a graph G to a graph H is a map from V(G) to V(H) which takes edges to edges.(It map a non edge to a single vertex, a non edge or an edge).
Homomorphisms are a generalization of graph colourings. A homomorphism from the cospectralchordal graph G to the complete graph K_r(with vertices numbered 1,2,…r) is exactly the same as an r-colouring of G (where the colour of a vertex is its image under the homomorphism), since adjacent vertices map to distinct vertices of the complete graph.

If a homomorphism exists between G and H, we say G$\equiv$H; otherwise we say G$\rightarrow$H and H $\rightarrow$G, Subsequently, is a preorder (named homomorphism equivalence) and the derived equivalence relation (called reflexive because the identity map is a homomorphism and transitive because the composition of homomorphism is a homomorphism). This is the homomorphism order; the equivalence classes are partially arranged by, we occasionally misuse notation when we make references to the order on specific graphs.

An image of a connected graph that is homomorphic must also be connected, as homomorphism map edges to edges.

Let ω(G) and $\phi(G)$ represent the graph G's clique and chromatic numbers, respectively. Since the image of a complete cospectralchordaldense graph under a homomorphism is a complete cospectralchordal dense graph of the same size, the clique number of G is now the biggest value of k for which $K_k \rightarrow$G. Furthermore, a graph is k-colourable if and only if it contains a homomorphism to the complete cospectralchordal dense graph K_k.

Proposition 1.1[8]If G$\rightarrow$H, then $\omega(G) \le \omega(H) \;\; and \;\; \chi(G) \le \chi(H)$.

Proof: If $\varphi : G \rightarrow H$ is a homomorphism, then composing with homomorphisms from K_k we see that

$K_k \rightarrow G \;\; implise \;\; K_k \rightarrow H$;

$H \rightarrow K_k \;\; implise \;\; G \rightarrow K_k$

Corollary 1.2 If $G \equiv H$, then $\omega(G) = \omega(H) \;\; and \;\; \chi(G) = \chi(H)$.

2 .Cores[7]:

If a cospectralchordal dense graph G and H contains the vertices of every cospectralchordal graph in its homomorphism equivalence class, then it is a core. Here, K and L are regarded as the subgraphs of the graphs G and H. Then a homomorphism G→K and H→L is a mapping $f_1 : V(G) \to V(K)$ and $f_2 : V(H) \to V(L)$, Such that $gg' \in E(G)$ implies $f_1(g) f_1(g') \in E(K)$ and $hh' \in E(H)$ implies $f_2(h) f_2(h') \in E(L)$. If K and L is a cospectralsubgraphs of G and H. Then a homomorphism $f_1 : G \to K$ and $f_2 : H \to L$. Such that f(i)=i and f(j)=j for all $i \in V(K)\ and\ j \in V(L)$ is called a retraction of G onto K and H onto L and K retract of G and also L be a retract of H. A subgraph K of G and L of G is called a core of G and H if there is a homomorphism G→K and H→L. But there is no homomorphism $G \to K'\ and$ H→L' for any proper subgraphs K' of K and L' of L. A graph which is its own core will be called simply a core. In other words, a core is a cospectral graph K and L. Which admits no homomorphism $K \to K'\ and\ L \to L'$ for any proper subgraph $K'\ of\ Kand\ L'\ of\ L$. Thus the only homomorphism of a core K and L to itself are surjectivehomomorphisms, that is, Automorphisms. A graph which admits no homomorphism to itself, expect for the identity is called cospecrtal rigid circuit graphs.

Corollary:2.1 If K and L is a core of G and H. Then K and L is a core.

Proof: Consider any homomorphism f_1 of K to a subgraph $K'\ and\ f_2\ of\ L$ to a subgraph L'. Compose f_1 and f_2 with a homomorphism $g_1 : G \to K$ and $g_2 : H \to L$ which exits. Since K and L is a core of G and H. Their composition is a homomorphism $f_1 g_1 : G \to K'$ and $f_2 g_2 : H \to L'$. Thus K=K' and L=L' and so K and L is a core.

Corollary:2.2 Every cospectralchordal dense graph has a core.

Proof: The family of subgraphs K of G and L of H for which there is a homomorphism G→K and H→L has elements minimal with respect to inclusion.

Corollary:2.3 The core of a cospectral sub graphs are unique, upto isomorphism.

Proof: Suppose both $K_1, K_2 and\ L_1 L_2$ are cores of G and H. Let $f_1 : G \to K_1 ; f_2 : G \to K_2$ and $f_3 : H \to L_1 ; f_4 : H \to L_2$ be the homomorphism of cospectralsubgraphs. Since both $K_1, K_2 and\ L_1 L_2$ are cores, it is easy to see that the restriction of $f_1\ to\ K_2$ and the restriction of $f_2\ to\ K_1$ are the surjective homomorphism between $K_1, K_2 and\ L_1 L_2$. Therefore $K_1, K_2 and\ L_1 L_2$ are isomorphism.

Corollary 2.4 A Cospectral rigid dense graph is a core iff it admits no proper retracts.

Proof: If K and L admits a retract

Proposition 2.5[8] Every endomorphism of G is an automorphism, if G is a core.

Proof: Assume ϕ to be an endomorphism between G and H. Next, inclusion creates a homomorphism H→G and ϕ induces a homomorphism GH. Thus, G≡H. Then G is not of minimum size in its equivalence class if ϕ is not onto.

We will soon discover that the opposite is also true.

Proposition 2.6 There is a distinct core in each homomorphism of the cospectralchordaldense equivalence class, up to isomorphism.

Proof: Every homomorphism equivalence class evidently has a core. Two equivalent cores are also isomorphic. Since $G \equiv G'$ and G and G' are cores, followed by the homomorphism $\varphi : G \to G'$ and $\varphi' : G' \to G$ so that both $\varphi\varphi'$ *and* $\varphi'\varphi$ is an endomorphism between G and G $'$. Thus, they are isomorphism. Since φ is a bijective endomorphism and φ' is likewise, thus, they cannot reduce the number of edges , thus φ *and* φ' are isomorphisms.

Specifically, the homomorphism order on graph equivalency classes and the homomorphism order on core isomorphism classes are the same.

If Gis an induced subgraph of a core, then G $'$ is said to have a core.

Proposotion 2.7 A core is unique to any graph (up to isomorphism).

Proof: Let G be the center of the equivalence class of any given graph H.

There is a homomorphism $\varphi : G \to H$; G' is another core and is isomorphic to G. Since the induced subgraph G' on (VG)φ satisfies $G \to G' \to H$ and $|VG| = |VG'|$.

Now we have the promised converse:

Proposition 2.8 If and only if each endomorphism of a graph G is an automorphism, then G is a core.

Proof: Already, we have seen the future implication. On the other hand, consider that H is a core of G and that each endomorphism of G is an automorphism.

Subsequently, by definition, there is a homomorphism from G to H; this is an automorphism. Afterwards, H is embedded in G, which is an endomorphism of G. therefore G=H.

Proposition 2.9 The core of G is K_k if both the chromatic number and clique number of G equal k.

Proof: Through the chromatic number and clique characterisations we provided in the first part, we can observe that $K_k \to G$ and $G \to K_k$, hence $G \equiv K_k$, Consequently, if G is equal to a graph H with fewer than k vertices, then $\chi(G) \le \chi(K) < k$, defying the prediciton.

Proposition 2.10[14]The core of G is vertex-transitive if it is.

Proof: Assume that H is the core of G, and select the homomorphisms $\phi : H \to G$ *and* $\psi : G \to H$. So $\phi\psi$ is an automorphism of H, and ϕ *and* ψ are isomorphism between H and G, such that (VH)ϕ. Moreover, ϕ is an embedding of H as an induced subgraph of G. Hence $\phi\alpha\psi$ is an automorphism of H for any automorphism α of G. Since each vertex in (VH)ϕ can be mapped to any other by selecting α, H's automorphism group is vertex-transitive.

Proposition 2.11 The cores of classes forming an antichain in the homomorphism order are the connected parts of a core. On the other hand, a core is the disjoint union of these graphs if $\{G_1, G_2, ..., G_r\}$ is an antichain of cores.

Proof: Let the connecting components be $G_1, G_2, \ldots G_r$. Combining this with the identity on the other components results in a homomorphism that is not an automorphism, which is impossible, if $G_i \to G_j$ for $i \neq j$. Consequently, $\{G_1, G_2, ..., G_r\}$ is an antichain. The same argument would demonstrate that G would be equivalent to a smaller graph if any of these graphs, G_i, were equivalent to a smaller graph $G_i^{'}$. Hence G_1,G_2,…Gr are cores.

Corollary 2.12 A vertex-transitive graph's core is connected.

Proof: There are two distinct perspectives on this. Alternatively, we can note that a disconnected vertex-transitive network is equal to one of its connected components, whose core is obviously connected. This allows us to leverage the fact that the core is vertex-transitive, which demonstrates that it is connected.

3.Cospectralchordal graphs are Cograph[13]

The following is a recursive definition of the class of Cospectralchordalcographs

- Every Cospectral chordal cograph is a graph with a ≥ 3 vertex
- Every Cospectral chordal cograph is the disjoint union of two cographs
- Every Cospectral chordal cograph's complement is also a cograph

If and only if these rules can be applied in a finite number of ways, then a cospectralchordalcograph is a cograph.K_4 is a cograph, for instance, since its complement is a union of graphs with a single vertex.

Theorem 3.1: Forb$\subseteq$(P_4)=Cospectralchordalcographs.

Proof: Observe that the cospectral chordalcograph class is induced-subgraph-closed. Furthermore, even though P_4 and its complement are connected and it has several vertices, P_4 is not a cograph.

Therefore, if graph G has P_4 as an induced subgraph, then G is not a cograph, and Forb$\subseteq$ (P_4)=Cospectral chordal cographs.

Thus, all we have to do is demonstrate that every Cospectralchordalgraph G such that $P_4 \not\subset$ G is a cograph. We assume that the claim is true for all graphs with fewer than $|V(G)|$ vertices. Since we demonstrate it by induction. We can assume $|V(G)| \geq 4$, since all graphs with a maximum of three vertices are cographs. If G is not connected, then by induction hypothesis, all the components of G are cographs, and thus G is cograph. Hence, assume that G is connected.

Let $G^{'}$=G-v and take any vertex v$\in$V(G) into consideration. The induction hypothesis states that $G^{'}$ is a cograph and $P_4 \not\subset G^{'}$.

Assume that G' is not connected. The components of G' are C_1,C_2,C_3,...$C_n$for $1 \leq i \leq n$, if v has a neighbor's of $u_i \in V(C_i)$ because G is connected. If v is adjacent to all vertices of G', then $G = \overline{\overline{G'} \cup \{v\}}$ is a cograph. Hence, we can assume that a vertex $w_1 \in V(C_1)$ is not adjacent to v. Since C_1 is connected, we can assume that u_1 is adjacent to w_1. On the other hand $w_1u_1vv_2$ is an induced path in G, which contradicts the assumption that $P_4 \not\subset$G.

Finally, consider the case that G' is connected. Since G' is a cograph with more than one vertex, its Complement is not connected. Furthermore, $\overline{P_4} = P_4$, and thus $P_4 \not\subset$G. By the same argument as in the previous paragraph, we prove that $\overline{G}$ is a cograph, and thus G is a cograph.

Note:

A graph G is cospectralchordal if no cycle of length greater than 3 is induced, that is cospectralchordal=Forb$\subseteq$(holes). There are several related characterization of cospectralchordal graphs.

When G-S is not connected, $S \subseteq V(G)$ is minimal cut for a connected graph G, however, for every $S' \subset S$ but G-S' is connected. Note that each vertex of a minimal cut set S has a neighbor in every component of G-S.

Lemma 3.2: A clique is induced by each minimal cut $S \subseteq V(G)$ if a connected graph G is cospectralchordal.

Proof: Assume u,v$\in$S are not adjacent. If C_1 and C_2 are components of G-S, then both uand v have neighbors in C_1 and C_2 as S is a minimal cut. Let P_i is the shortest path between u and v through C_i for i$\in\{1,2\}$. Then $P_1 \cup P_2$ is an induced hole in G, which contradicts the assumption that G is cospectralchordal.

Theorem 3.3: If G is cospectralchordal and not a clique, then there exist $G_1,G_2 \subset$ G such that $G=G_1 \cup G_2$ and $G_1 \cap G_2$ is clique.

Proof: We can assume that G_1 is a component of G and $G_2 = G \setminus V(G_2)$ if G not connected. Hence, assume that G is connected. There exist vertices $x, y \in V(G)$ that are not adjacent because G is not a clique. Assume that $S_0 = V(G) \setminus \{x, y\}$. In that case, G-$S_0$ is unconnected. Consequently, S is a minimal cut since $S \subseteq S_0$. Let G-S=$C_1 \cup C_2$, where C_1 and C_2 are disjoint and non-empty. Then we can set G_1=G-V(C_2) and G_2=G-V(C_1). Note that $G_1 \cap G_2$=G[s] is a clique by lemma 3.2.

Note:

If G_1 and G_2 are cospectralchordaland $G_1 \cap G_2$ is a clique, then $G_1 \cup G_2$iscospectralchordal. Hence every cospectralchordal graph can be obtained using a finite number of applications of these rules:

- A complete graph is cospectralchordal.
- If G_1 and G_2 are cospectralchordal and G is obtained from G_1 and$G_2$by gluing on a clique, then G is cospectralchordal.

A vertex v$\in$V(G) is simplicial if the neighborhood of v in G induces a clique.

Lemma 3.4: The presence of two nonadjacent simplicial vertices indicates that G is cospectralchordal and not a clique.

Proof: We assume that the claim holds for all graphs with fewer than $|V(G)|$ vertices, since we prove it by induction. There are $G_1,G_2 \subset$G, such that G=$G_1 \cup G_2$ and $G_1 \cap G_2$ is a clique, according to theorem 3.3. If G_i is not a clique for i$\in\{1,2\}$, then according to the induction hypothesis, G_i has two nonadjacent simplicial vertices, and at most one of them is a member of the clique $G_1 \cap G_2$. Consider the simplicial vertex v_i of G_i that is not a member of $G_1 \cap G_2$. Assuming G_i is a clique, let v_i be any vertex of G_i that is not a member of $G_1 \cap G_2$.

Since neither v_1 nor $v_2$belong to $G_1 \cap G_2$, Observe that v_1 and v_2 are non-adjacent simplicial vertices of G.

Theorem 3.5: If and only if simplicial vertex appears in each of a graph G is induced subgraph, then the graph is cospectralchordal.

Proof:

Lemma3.4, states that all induced subgraphs of cospectralchordal graphs are cospectralchordal and contain a simplicial vertex. Therefore, if every induced subgraph of G has a simplicial vertex, then G is cospectralchordal, and this is sufficient to demonstrate. Since we demonstrate the claim by induction on the number of vertices in G, we assume that it is true for all graphs with fewer vertices than $|V(G)|$.

Consider the simplicial vertex v of G. Every induced hole in G contains v. Since according to the induction hypothesis, G-v is chordal. However, for every hole $C=uvwz_1z_2\ldots\subseteq G$, and C is not induced because u and w are adjacent vertices. Consequently G is cospectralchordal.

4.A tree decomposition:

A tree decomposition of a graph G is a pair (T, β) where each vertex of T is assigned a bag β(n) by the formula $\beta:V(T)\rightarrow 2^{V(G)}$.

- Every vertex is in some bag for every $v\in V(G)$ because there are $n\in V(T)$ such that $v\in \beta(n)$
- Every edge us un some bag and for every $uv\in E(G)$, there exist $n\in V(T)$ such that $u,v\in \beta(n)$
- Every vertex appears in a connected sub tree of the decomposition, for every $V\in V(G)$, the set$\{n\in V(T):v\in \beta(n)\}$ induces a connected sub tree of T.

Lemma 4.1: [9] Complete cospectralchordalgraphs:

Proof: Let G=(V={a,b,c,d},E) be the complete graph with 4 vertices. To obtain a treewidth smaller than n-1 we need to build sets with less than n elements. Without limitation of generality we assume the sets {a,b,c},{b,c,d} to be taken. To cover all edges we need to add the {a,d}. We can now easily see that the path condition of tree decomposition can't be fulfilled anymore. Because $\{a,b,c\}\cap\{a,d\}=\{a\}\not\subset\{b,c,d\}$, $\{b,c,d\}\cap\{a,d\}=\{d\}\not\subset\{a,b,c\}$, $\{a,b,c\}\cap\{b,c,d\}=\{b,c\}\not\subset\{a,d\}$.

Lemma 4.2: There is a reduced tree decomposition (T', β') of G for each tree decomposition (T, β) of G such that each bag of (T', β') must equal to some bag of (T, β).

Proof: Let $\beta(n_1)\subset\beta(n_2$ be satisfied $n_1n_2\in E(T)$. Suppose that T_1 is tree that is obtained from T by contracting the edge n_1n_2, and that the vertex that is obtained from n_1 and n_2 by this contraction is $n\in V(T_1)$. Since $\beta(n_1)=\beta(n_2)$ and $\beta_1(n')=\beta(n')$. Let $\beta_1:V(T_1)\rightarrow 2^{V(G)}$be defined for each n' $\in V(T_1)\backslash\{n\}$. Consequently, a tree decomposition of G with fewer vertices is (T_1, β_1). A reduced tree decomposition of G is ultimately obtained by repeating this operation as many times as feasible.

Corollary 4.3: The minimum degree of G is at most k if its tree decomposition has a width of atmost k.

Proof: Let n represent a leaf of T and (T, β) be a reduced tree decomposition of G with a width of no more than k. The reduction of (T, β) means that there is a $v\in V(G)$ such that v only occurs in the bag of n. Therefore, deg(v)$\leq|\beta(n)|-1\leq k$ so that all of v's neighbors are members of β(n).

Lemma 4.4: A tree with one vertex and a bag equal to V(G) is the only reduced tree decomposition of a complete graph G.

Proof: Given that there is an edge n_1n_2, let (T, β) be a reduced tree decomposition of G.

If $x\in\beta(n_1)\backslash\beta(n_2)$ and $y\in\beta(n_2)\backslash\beta(n_1)$ exist because (T, β) is reduced. Let $T_x$and T_y be the subtrees of T that are induced by the vertices whose bags contain x and y, respectively. The edge n_1n_2 does not belong to either $T_x$or T_y since $n_1\notin V(T_x)\, and\, n_2\notin V(T_y)$. If $T_x$and $T_y$are disjoint. Since they are subgraphs of distinct components of T-n_1n_2. Nevertheless, it is contradictory to assume that some bag contains both x and y since$xy\in E(G)$. Consequently, there are no edges in the reduced tree decomposition of G.

Corollary 4.5: If n$\in$V(T) such that S$\subseteq \beta(n)$ exist, then S$\subseteq$V(G) induces a clique in G and (T, β) is a tree decomposition of G.

Proof: For each n$\in$V(T), let $\beta_1 : V(T) \to 2^S$ be defined as follows: $\beta_1(n) = \beta(n) \cap S.$ A tree decomposition of the clique G[S] is (T, β_1). Every bag in the reduced tree decomposition (T$_2$, β_2) of G[S] is equal to some bag of (T, β_1) according to Lemma 4.2. Let n be a vertex of T such that $\beta_2(n_2) = \beta_1(n)$. According to lemma 4.4, T$_2$ has only one vertex n$_2$, and we obtain $\beta_2(n_2) = S \;\; for\, S = \beta_2(n_2) = \beta_1(n) \subseteq \beta(n).$

Theorem 10: If and only if a graph G has a tree decomposition (T, β) that causes each bag to induce a clique, then the graph is cospectralchordalchordal.

Proof: Let G iscospectralchordalchordal. We can assume that the claim is true for all graphs with fewer than $|V(G)|$ vertices since we produced by induction. V(T)={n} and $\beta(n) = V(G)$ can be set if G is a clique. Assume that G is not clique, according to theorem 3.3, there exist $G_1 G_2 \subset G$ such that $G = G_1 \cup G_2$ and $G_1 \cap G_2$ is clique. The induction hypothesis suggests that there is a tree decomposition (T_i, β_i) of G$_i$ such that each bag induces a clique for i$\in${1,2}. According to corollary 4.5, n$_i \in$V(T$_i$) such that $V(G_1 \cap G_2) \subseteq \beta_i(n_i).$ Let T represent the tree that is created by adding the edges n$_1$n$_2$ from T$_1$ and T$_2$. Assume that β is defined as follows: $\beta(n) = \beta_1(n)$ if n is in V(T$_1$) and $\beta(n) = \beta_2(n)$ if n is in V(T$_2$). After that (T, β) is a tree decomposition of G in which each bag creates a clique.

Conversely, suppose that G has a tree decomposition (T, β) such that every bag induces a clique. Consider a cycle C$\subseteq$ G, and let $\beta^{'} : V(T) \to 2^{V(C)}$ be defined by $\beta^{'}(n) = \beta(n) \cap V(C)$ for every n$\in$V(T). Then $(T, \beta^{'})$ is a tree decomposition of C such that every bag induces a clique. By corollary 4.3, any tree decomposition of a cycle must contain a bag of size at least three, and thus C contains a clique of size three. Therefore, C is a triangle. It follows that G contains no induced hole, and thus G is cospectralchordal.

5. Cycle Decompositions of CospectralChordal Graphs and Fixed Length Cycles

In this work, the circumstances in which a cospectralchordal (complete) graphs admits a decomposition in to cycles of a fixed length are investigated. Since all vertices must have even degrees for such a decomposition to exist, the entire graph must have an odd number of vertices. However, this question can be extended to graphs where each vertex has an even degree and the number of vertices is even. A cospectralchordal (complete) graph with an even number of vertices can be naturally modified by removing a 1-factor to produce such graphs that are extremely close to complete graphs.

In addition to the requirement that n be parity, there are two other clear-cut requirements that $3 \le m \le n$ and that the cycle of length m divides the number of edges in either K$_n$, that is

$\frac{n(n-1)}{2}$ or K_n-I that is $\frac{n(n-2)}{2}$ This raises the question of when K_n or K_n-I, whichever is appropriate, admit a decomposition into cycles of a fixed length m.

5.1 Definition and Terminology

We start by defining some fundamental terms. This paper use only simple cospectralchordal graphs. To represent the complete graph on n vertices, we use K_n . For n even, we use K_n –I which represents K_n with a 1-factor I eliminated. A complete g-partite graph with m vertices in each part is denoted by $K_{g(m)}$, while $\overline{K_n}$ represents the complement of K_n, $K_{m,n}$ represents the complete bipartite graph with the bipartition sets of sizes m and n. A cycle of length m or an m-cycle is presented be C_m. Hamilton cycle is the term for an n-cycle in a graph with n-vertices.

Definition 5.1.1 The decomposition of a cospectralchordalgraphs G into its subgraphs H_1 and H_2 is represented by G=$H_1 \oplus H_2$, provided that G is the edge disjoint union of H_1 and H_2. If G=$H_1 \oplus$ …. $\oplus H_k$, whereH_k is isomorphic to H, then G is considered H-decomposable, and $\{H_1,\ldots,H_k\}$ is an H decomposition of G. In particular G is C_m-decomposable if it can be decomposed into subgraphsisomorphic to an m-cycle.

Definition 5.1.2 The join G ∇ H of graphs G and H is the graph ithvetex set V(G) $\cup$ V(H)and edge set E(G) $\cup$ E(H) $\cup$ {uv: u $\in$ V(G), v $\in$ V(H)}. If K_n is isomorphic to $K_{n-1} \nabla K_1$ and for n even, K_n-I is isomorphic to (K_{n-2}-I) $\nabla \overline{K_2}$. In this representations the vertices of K_1 and $\overline{K_2}$, respectively are called the central vertices.

Definition 5.1.3 Let G be a graph with vertex set V(G)= $\{x_0, x_1 \ldots x_{k-1}\}$. We define G(2) to be the graph with vertex set $V(G(2)) = \{x_i^j : x_i \in V(G), j \in Z_2\}$ and edge set $E(G(2)) = \{x_{i1}^{j1}\, x_{i2}^{j2} : x_{i1}x_{i2} \in E(G), j_1, j_2 \in Z_2\}$. It is observe that $K_k(2)$ is isomorphic to the join K_k ∇ K_k with a 1-factor removed that is to K_{2k}-I. For a set $S \subseteq Z$ we write –S={-s : s $\in S$} and, for any x $\in Z$, $S + x = \{s + x : s \in S\}$.

Definition 5.1.4 Let k be a positive integer and L a subset of $\left\{1,2,\ldots\left[\frac{k}{2}\right]\right\}$. A circulant X=X(k;L) is a graph with vertex set V(X)= $\{u_0, u_1 \ldots u_{k-1}\}$ and edge set E(X)= $\{u_i u_{i+l} : i \in Z_k, l \in L\}$ The edge $u_i u_{i+l}$ where l $\in$ L is said to be of length l, and L is called the edge length set of the circulant X. when k is even, the edge length $\frac{k}{2}$ is called the diameter length and edges $u_i u_{i+l}$ *and* $u_{i+\frac{k}{2}} u_{i+\frac{k}{2}+l}$ of length l are called diametrically opposed.

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Note: Rather than using its edge length set L, a circulant is more frequently described in the symbol S, where S=L $\cup$ (-L), so that S $\subseteq$ {1,…k-1} and –S=S. But we find that the description via the edge length set is more practical.

Observe that for n odd, K_n is isomorphic to the circulant $X\left(n;\left\{1,\ldots,\frac{n-1}{2}\right\}\right)$, and for n even, K_n-I is isomorphic to the circulant $X\left(n;\left\{1,\ldots,\frac{n}{2}-1\right\}\right)$.

The subgroup of the automorphism group Aut(G) produced by γ, a graph G is automorphism γ is represented by $<\gamma>$. In a graph G, for every vertex v and $\Gamma(v)$ is the orbit of Γ containing v in the subgroup Γ of Aut(G). It is possible to naturally extend the action of Aut(G) to the edges of G and to the subgraphs of G.

Definition: 5.1.5 Consider a circulant with vertex set $\{u_0,\ldots,u_{k-1}\}$, where X=X(k;L). We refer to the cyclic permutation $(u_0,\ldots,u_{k-1})$ as the rotation ρ.

Observe that the set $\{u_i u_{i+l} : i \in Z_k\}$ for all l $\in$ L ; that is, the sets containing all edges of the same length are the edge orbits of the subgroup $\langle\rho\rangle = \{\rho^i : i = 0,\ldots,k-1\}$ of Aut(X). The Permutation $(u_0u_2\ldots u_{k-2})(u_1u_3\ldots u_{k-1})$ for k even means thatwith edge orbits $\langle\rho\rangle$ is a subgroup of $\langle\rho^2\rangle \aleph$ L. Hence $\in$ for each l $\{u_i u_{i+l} : i \in Z_k, i\, odd\} = \langle\rho^2\rangle(u_1u_{1+l})$ and $\{u_i u_{i+l} : i \in Z_k, i\, even\} = \langle\rho^2\rangle(u_0u_l)$ every set of edges of the same length divides into an "even" and an "odd" orbit of $\langle\rho^2\rangle$. For our decomposition into m-cycles $\langle\rho\rangle$ and $\langle\rho^2\rangle$ are both crucial.

Since K_n and K_n-I are represented as the join $K_{n-1} \nabla K_1$ and (K_{n-2}–I) $\nabla \overline{K_2}$ respectively, we define ρ as the permutation that rotates the graph's circulant portion (K_{n-1} or K_{n-2} –I) and leaves the central vertices fixed.

A path of length p or one with p edges and p+1 vertices is called a path. The vertices encountered first, third…(that is $x_0, x_1,\cdots$) in a path $P = x_0x_1\ldots,x_p$ are referred to as the odd vertices, and the vertices encountered second, fourth,…(that is $x_1, x_3,\cdots$) are even vertices. The vertices x_0 and x_p are called the end points of P, x_0 being the initial vertex and x_p being the terminal vertex. The internal vertices are $x_1,\ldots x_{p-1}$.

If PQ will represent the (p+q)-path $x_0, x_{1,\dots} x_p y_1 \dots y_q$, the concatenation of P and Q, then $P = x_0 x_1 \dots x_p$ is a p-path and $Q = y_0 y_1 \dots y_q$ is a q-path and $x_p = y_0$ is the only vertex the two paths share. Additionally, $\overleftarrow{P}$ representation the path $x_p x_{p-1} \dots x_1 x_0$, which is P in reverse. Obviously P and $\overleftarrow{P}$ represent the same cospectralchordal graph but when concatenating vertices, the order in which they are listed matters.

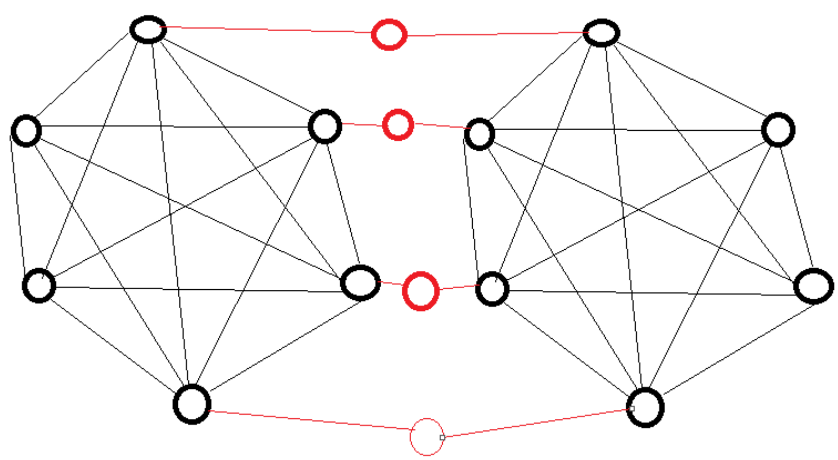


Figure 5.1.cospectralchordal graph concatenating vertices

Definition: 5.1.6 Azig-zag path is defined as a path P in a circulant X(k;L) that has the characteristic that none of its edges are the same length. The edge length set of P is the collection of all edge lengths represented in P, denoted by L(P). A zig –zag path of the form $\rho^i(P)\, for\ i \in \{0,1,\dots,k-1\}$ is a copy of a zig-zag path P in X(k;L).

The way such a path can be built from its edge length set is where the term " zigzag path" originates. The fact that $\{a_1, a_2, \dots a_p\} \subseteq L$ is readily apparent, where $1 \le a_1 < \dots < a_p \le \left[\frac{k}{2}\right]$. Each of the zigzag paths $\{u_0 u_{a_1} u_{a_1-a_2} \dots u_{a_1-a_2+\dots+(-1)^{p-1} a_p}\}$ and $\{u_0 u_{-a_1} u_{-a_1+a_2} \dots u_{-a_1+a_2-\dots+(-1)^p a_p}\}$ in X(k;L). A copy of a zigzag path of one of these forms will be referred to as basic. Observe that a basic path P and its diametrically opposed path $\rho^{\frac{k}{2}}(P)$ are vertex disjoint. If k is even and $a_p < \frac{k}{2}$, in our decompositions, nearly every cycle is built using zigzag paths.

5.2 An overview of the Methods Employed in the Constructions

The three fundamental m-cycle types that we employ in our constructions are introduced. In this section, Let's first clarify what we mean when we use the term "Peripheral cycle".

Note: A strangulated graph is a cospectralchordal graph in which deleting the edges of any induced cycle of length greater than three would disconnect the remaining graph. That is, there are the graphs in which every peripheral cycle is a triangle. (Every chordal graph is strangulated, because the only induced cycles in chordal graphs are triangles, so there are no longer cycles to delete)

- Generalizing chordalgraphs define a strangulated graph to be a graph in which every peripheral cycle is a triangle. They characterize these graphs as being the clique sums of chordal graphs and maximal planar graphs.

Definition 5.2.1: The circulantcospectralchordal graph G=X(k;L) has the vertex set $\{u_0, u_1, \ldots, u_{k-1}\}$. Assume d=gcd(k,m), $n' = \frac{k}{d}$ and $m' = \frac{m}{d}$. We are going to assume that $d \geq 3$. A peripheral cycles is an m-cycle C of G that first into one of the two categories listed below:

1. C is the concatenation of m'-paths P, $\rho^{n'}(P), \rho^{2n'}(P), \ldots, \rho^{(d-1)n'}(P)$ (or)
2. C is composed of $(m'-1)$-paths P, $\rho^{n'}(P), \rho^{2n'}(P), \ldots, \rho^{(d-1)n'}(P)$ along with d connecting edges.

In both situations , the peripheral cycle C is said to be generated by path P.

The path P that creates a peripheral cycle in practically all of our constructions is zigzag path. The following circumstances are adequate for a path P to produce a peripheral cycle, in case (1) of the definition, if the terminal vertex of P coincides with the initial vertex of $P^{n'}(P),$ and if the sets of internal vertices of paths $P^{jn'}(P), j = 0,1,\ldots,d-1$, are mutually disjoint. In case (2), if the paths $P^{jn'}(P)$ are pairwise vertex-disjoint and if there are appropriate linked edges.

If C is a peripheral cycle in G=X(k;L), then so are $\rho(C), \rho^2(C), \ldots \rho^{n'-1}(C)$. These cycles are said to be generated by the same $(m'-1)$ or m'-path P as C. It is not difficult to see that if C is generated by a zig-zag m'-path P with edge length set L_p, *then* $\{\rho^i(C) : i = 0,1,\ldots,n'-1\}$ is a C_m – decomposition of $X(k;L_p)$.

In certain of our constructions C in $\rho(C), \rho^2(C), \ldots \rho^{n'-1}(C)$ are also used for each peripheral cycle C in the C_m-decomposition. When $m \leq n < 2m$, however, n' is even and for each peripheral cycle C, there are two possibilities: either C, $\rho(C), \rho^2(C), \ldots \rho^{n'-1}(C)$ are all used in the C_m –decomposition, in which case C is called coupled or only C, $\rho^2(C), \ldots \rho^{n'-2}(C)$ are used as such, while the copies of the generating path that appear in C, $\rho(C), \rho^3(C), \ldots \rho^{n'-1}(C)$ are incorporated into other cycles, in which case C is called solitary.

Joining $(m^{'}-1)$-paths into a peripheral m-cycle with the aid of an auxiliary circulant:

Alspachand Gavlas are creted with this method [1]. The zigzag $(m^{'}-1)$-paths $p_{i,0}, i=1,2,...,c,$ are assumed to have the same initial vertex u_0 and terminal vertex u_t (in our constructions $t \in \{-1, n^{'}-1, n^{'}+1\}$), with pairwise disjoint edge length sets L_i. Furthermore, each of the families $\mathrm{P}_i = \{P_{i,j} = \rho^{jn^{'}}(P_{i,0}) : j = 0,1,...,d-1\}$ has pairwise vertex disjoint $(m^{'}-1)$-paths. Hamilton cycles in an auxiliary circulant X with vertex set $\{v_0, v_1, .., v_{d-1}\}$ are used to select the connecting edges.

Assume that we are presented with a Hamilton cycle C in X. To create a Hamilton-directed cycle $\vec{C}$ orient the cycle arbitrarily. Let $v_{j1}v_{j2}$ be an edge in C of length $j_2 - j_1$. If $\vec{C}$ uses the arc from $v_{j1}\ to\, v_{j2}$.Connect the terminal vertex of $p_{i,j1}\,(that\, is\, u_{j1n^{'}+t})$ with the initial vertex $p_{i,j2}\,(that\, is\, u_{j2n^{'}})$ in G using an edge of length $(j_2 - j_1)n^{'} - t$ (or in this case $(j_1 - j_2)n^{'} + t$ if $(j_2 - j_1) = 1\ and\ t = n^{'} + 1$). In contrast, if $\vec{C}$ contains the arc from $v_{j2}\ to\, v_{j1}$, use an edge of length $(j_2 - j_1)n^{'} + t$ in G to connect the initial vertex of $p_{i,j1}$ (that is $u_{j1n^{'}}$) with the terminal vertex of $p_{i,j2}$ (that is $u_{j2n^{'}+t}$). Consequently, an edge with $(j_2 - j_1)n^{'} + t$ in G is used. Either way , an m-cycle is formed by the linking edges that result from a directed cycle $\vec{C}$ in X and the family P_i of $(m^{'}-1)$-paths. Ecah Hamilton cycle in X can be used to connect two families of $(m^{'}-1)$-paths, one orientation for each family, since the two arcs that correspond to a fixed edge of the auxiliary circulant X produce two connecting edges os different lengths in G.

Next, we describe the decompose of the auxiliary circulant X into Hamilton cycles. Let the set of X's edge length be $\{l_1, l_2, ..., l_b\}$.this is done by first breaking down X into circulants of degrees 2 and 4, specifically circulants of the forms X(d; $\{l_i\}$) where gcd (d,l_i)=1 and X(d; $\{l_i, l_j\}$) where the edge length l_i and l_j are selectedso that the circulant is connected. A circulantof the first form is obviously a d-cycle, and the following theorm by Bermond [19] states that a circulant of the second form can be broken down into Hamilton cycles.

Theorem 5.2.2 [19]: It is possible to break down any connected circulant of degree 4 into Hamilton cycles.

Proof: Circulants of the forms $X(d;\{l, l+1\})$, $X(d;\{2l-1, 2l+1\})$, and if d is odd, $X(d;\{2l, 2l+2\})$. Thus, the auxiliary circulant X has a Hamilton decomposition. In this decomposition, cycle C employs either two different edge lengths l_i and l_j or a single edge length l_i. The connecting edges that originate from a single orientation of C will all have the same length in the first scenario, which is either $l_{in'} - t \ or \ l_{in'} + t$. The resulting m-cycles in G will have all of its edges covered after applying $\rho, \rho^2, \ldots, \rho^{n'-1}$. Therefore, we can link up one or two families of $(m' - 1)$-paths by using both or just one of the two orientations of C. A single orientation of C, however, may result in connecting edges of four different edge lengths; $l_{in'} - t, l_{in'} + t, l_{jn'} - t \ and \ l_{jn'} + t$. If C uses two different edge lengths l_i and l_j. We must thus use both orientations of C in order to account for all of the edges of these lengths in G. This means that four families of $(m' - 1)$-paths are connected by a circulant $X(d; \{l_i, l_j\})$. It is now clear that any number of families of $(m' - 1)$-paths can be connected into m-cycles, as long as a sufficient number of appropriate edge lengths $l_1, l_2, \ldots, l_b$ can be found. When necessary, we will explain how to select the edge lengths in X and how to decompose X down in to circulants of degree 2 and 4 for each construction that employs this technique.

The other two categories of cycles are now presented.

Definition 5.2.3 Let G =X(k;L) be a circulant with $\frac{k}{2} \in L$ and let m and k be even positive integers. A diameter cycle C_D is an m-cycle in G that has two edges of diameter length $\frac{k}{2}$ and two zigzag $\left(\frac{m}{2} - 1\right)$-paths P and $\rho^{\frac{k}{2}}(P)$. The diameter cycle C_D is said to be generated by the zigzag $\left(\frac{m}{2} - 1\right)$-paths.

Definition 5.2.4 A central m-cycle in $K_n \ or \ K_n - I$ is neither a diameter cycle nor a peripheral cycle. The construction of central cycles varies significantly from case to case, hence the open-ended definition.

6.APPLICATIONS OF COSPECTRAL CHORDAL GRAPH ISOMORPHISM

Images are represented by graphs, which are highly helpful in the field of image processing because they provide a structural description of the image. In this type vertices are represented by the region of the image, while linkages known as edges reflect the relationships between regions [27, 28, 29].

Image processing: 2D, 3D, and 4D objects can be used to represent images [30]. Images' information can be saved in a database, and when it expands, we must retrieve the data from the database. To turn a color image into a black and white one, we must first preprocess it. Optical

Character Recognition (OCR) will be utilized to extract the data from the image. The Principal Component Analysis (PCA) algorithm will be used to extract the image's features. Graph theory can be used to enhance image processing.

One way to extract information from a picture is through image analysis. Since an image may be digital, digital image techniques can be used to process it. Segmentation, or breaking an image up into several important portions, is done for medical images [31].

Protein structure: Protein structure is represented by nodes, while edges show how nodes interact with one another. The representation of proteins is a network [32]. Numerous techniques for comparing the structures of two proteins are discussed in [33][34], and biological networks heavily rely on them.

Computer Design System: The subgraph Isomorphism applications includes Computer Aided Design or Computer Aided Mechanical Drawings [35].

Regarding the graph Applications of isomorphism include determining if two states are symmetric [36].
Database design, software engineering, computer networks, and other related fields are examples of computer science [37].

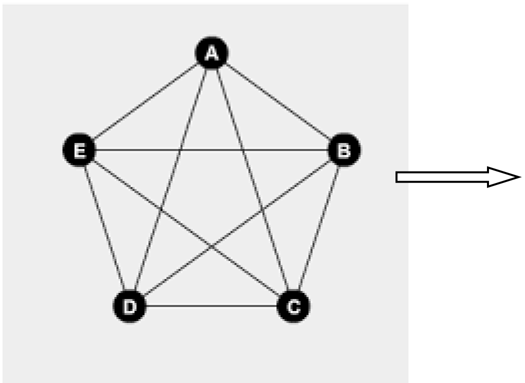
Chemical Structures: In this structures atoms are represented by nodes in this structure, while covalent bonds are represented by edges. Chemical information is displayed via a structural formula, which should be specifically identified and indexed [38]. Graph theory studies molecular structures.

Social networks: In these networks, people are represented by nodes, and their relationships are represented by edges. Finding positions and their placements can be accomplished with the help of pattern matching. Social networks include sites like Facebook and Twitter [39]. Sociology includes social networks. It is a method of recognizing structure in social networks. It is vital to determine the shortest path between two individuals in these kinds of networks based on their relationship; from this assessment, one can also determine the individual's location or psychology. Try to build a cluster that will hold related data from several people because of this.

Singular Value Decomposition (SVD): SVD decomposes an image matrix into singular values and singular vectors. The singular values can be considered the "cospectrum" of the image. Two images with the same set of singular values would be related in terms of their energy content and rank.

COMPARATIVE DISCUSSION OF COSPECTRAL CHORDAL GRAPH APPLICATIONS

In Image Processing , two distinct images are matched using cospectralchordal graph . Convert the input image to a graph first in this kind of application. An algorithm is required to convert input images to graphs, so attempt to develop a conversion algorithm utilizing a few features. But the process takes a lot of time. Once the graph has been obtained, use the graph isomorphism algorithm to confirm if the input graph matches one of the data sets graphs. Subgraph cospectralchordal graph follows the same steps; the only difference is it takes a lot longer because only a portion of the image needs to be searched rather than the entire image. The matching of an input image from a full image is shown in figure 2.



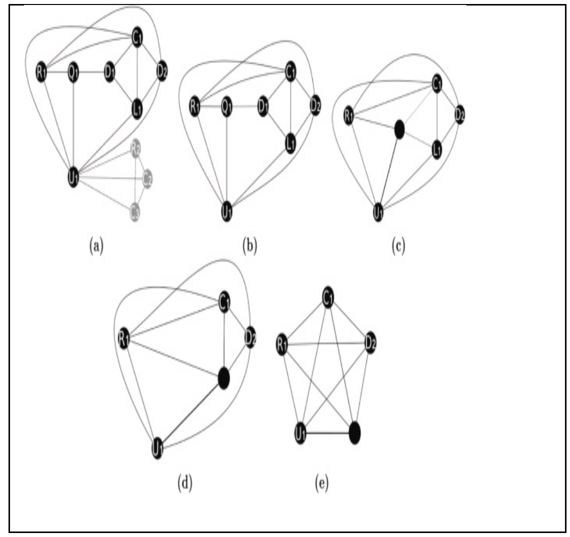


Fig 6.1. Image based subgraph cospectralchordal graph

Conclusion: The classes of Kr- cospectralchordal dense graphs and cycle decomposition were considered and results were obtained which describe the structure of dense graphs in these classes. We were able to extend results on the colorings and homomorphisms of dense triangle graph graphs to Kr- cospectralchordal dense graphs. Additionally, we looked at the core cospectralchordal dense graph, cograph and tree decompositon and we were able to improve some bounds on the binding number of Kr- cospectralchordal dense graphs.

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