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A STUDY ON SEIZURE PROPAGATION USING NETWORK

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ABSTRACT

A study on seizure propagation using connectivity aims to understand how the network of brain regions and neural pathways interacts and contributes to the spread of seizures. These studies typically focus on analyzing functional connectivity and complex neural pathways that can be represented as networks applies Menger's Theorem to an EEG-based seizure network, identifying the minimal set of nodes and edges that must be disrupted to block seizure propagation. Using graph theory techniques.

KEYWORDS: Connectivity, Mengers Theorem, Network, EEG, Seizure.

AMS CLASSIFICATION: 05Axx, 05Cxx

1. INTRODUCTION

A seizure is a sudden burst of abnormal electrical activity in the brain, disrupting normal function and leading to changes in movement, behavior, sensation, or consciousness. Seizures are classified as focal, originating in a specific brain region, or generalized, affecting both hemispheres from the onset. While a single seizure may result from temporary triggers such as head injury, fever, infections, or metabolic imbalances, recurrent, unprovoked seizures are a hallmark of epilepsy[7]. Seizure propagation refers to the process by which the abnormal electrical activity in the brain spreads from one area to others, potentially resulting in a more generalized seizure. A seizure may start in a specific part of the brain, known as the seizure focus, and if the electrical activity is not controlled, it can propagate through the brain's neural networks, leading to an expansion of the seizure[10]. This spread of activity can cause a seizure to affect larger areas of the brain, potentially involving both hemispheres, which is seen in generalized seizures. understanding how seizures propagate is critical for both diagnosis and treatment. The spread of seizure activity can be influenced by factors such as the brain's connectivity, the balance between excitatory and inhibitory signals, and changes in brain structure or function. In some cases, seizure propagation can be so extensive that it leads to severe, widespread symptoms, while in other cases, it may remain confined to a specific brain region, resulting in focal symptoms[8].

1.1 GRAPH THEORY

Graph theory is a sophisticated math tool. The network is the most advanced kind of research methodology. It has countless real-life uses in our daily lives. A graph is made up of a set of points and the lines connecting specific pairings of these points can be a convenient way to represent many real-world situations. For example, the points might be communication centers, with lines signifying communication linkages, or they could represent people, with lines connecting peers in pairs. Observe that the major focus of these diagrams is determining if two specified points are connected by a line; the specific method of connection is irrelevant.[1]

A graph G is composed of two sets: a vertex set, whose elements are known as vertices V = {v1, v2, v3, ...} (also referred to as points or nodes), and an edge set E = {e1, e2, e3, ...}, where each element is called an edge. The vertex set of G is denoted as V(G), while the edge set E(G) is represented as. Typically, a graph is expressed as G = (V, E)[2].

1.2. NETWORK

The network is a subset of graph theory, which defines networks as graphs where vertices or edges have specific attributes and analyses them through symmetric or asymmetric relationships among their discrete components, with applications across various disciplines. Euler’s solution to the Seven Bridges of Königsberg problem is widely regarded as the first formal proof in network theory[3].

1.3. CONNECTIVITY NETWORK

Connectivity Graph is a core principle in graph theory that extends the notions of cut points, bridges, and blocks. Understanding it is essential for analyzing a graph’s structural robustness and its capacity to endure disconnection. Two key invariants—connectivity and line connectivity—quantify a graph's level of connectedness, offering a way to compare different graphs in terms of cohesion and resilience [2].

A graph represents the relationships between different elements (nodes) through edges, which may be weighted, directed, or undirected. The strength of these connections is often measured using these parameters.

- Vertex Connectivity $p(G)$The minimum number of vertices required to disconnect the graph.
- Edge Connectivity $\theta(G)$: The least number of edges required to disconnect a graph.[2].

These measures are essential functions in a network, helping to assess the reliability and efficiency of various real-world systems, including transportation networks, communication infrastructures, and biological systems.

In section 2, the Materials and methods are explained. The Seizure propagation is explained in section 2.1 Network Analysis of EEG Graphs and in section 2.2. Connectivity Properties of An Epilepsy Network Derived from EEG Signals and Some Theorems. in sec 2.3 Menger theorem in EEG Graph, 3. Result and discussion.

2. MATERIALS AND METHODS

Electroencephalography (EEG) is a non-invasive technique for recording electrical activity in the brain using electrodes placed on the scalp. It captures neural oscillations and brainwave patterns, making it valuable for studying brain function, diagnosing neurological disorders, and developing brain-computer interfaces. it's crucial for detecting seizures, analysing brain network dynamics, and understanding seizure propagation. Graph theory and network analysis have become essential tools in EEG-based epilepsy studies, helping to model functional and structural brain networks.

Fig.1 Brain signal

The EEG electrode placements in the image suggest a spatially structured network. In this paper, connectivity metrics (PPF, SEF, MPF) can be defined using functional connectivity metrics.[8]

Table 1-Frequency Main Parameter in EEG Report

Frequency Measure	Description	Low-Value Interpretation	High-Value Interpretation
PPF (Peak Power Frequency)	Dominant frequency with the highest power	<4Hz: Slow-wave activity, post-ictal state	<4 Hz: Slow-wave activity, post-ictal state
SEF (Spectral Edge Frequency)	Frequency below which 95% of power is contained	<10Hz: Slow-wave dominance, seizure recovery	>20 Hz: Increased fast activity, common in focal seizures
MPF (Mean Power Frequency)	Average frequency weighted by power	<5Hz: Slowing of brain activity, deep seizure phases	>10Hz: Hyperexcitable state, potential seizure onset

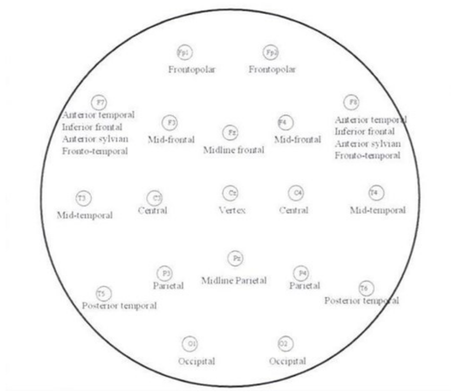

Fig.2-Electrodes position and names

2.1 NETWORK ANALYSIS OF GRAPHS

Network analysis is a powerful tool for understanding brain connectivity and neural dynamics. The brain can be represented as a graph**,** where EEG electrodes or brain regions act as nodes and functional or structural connections between them form the edges [3]. This approach helps in studying how different regions of the brain communicate and how these interactions change during cognitive tasks, sleep, or neurological disorders like epilepsy. [4]

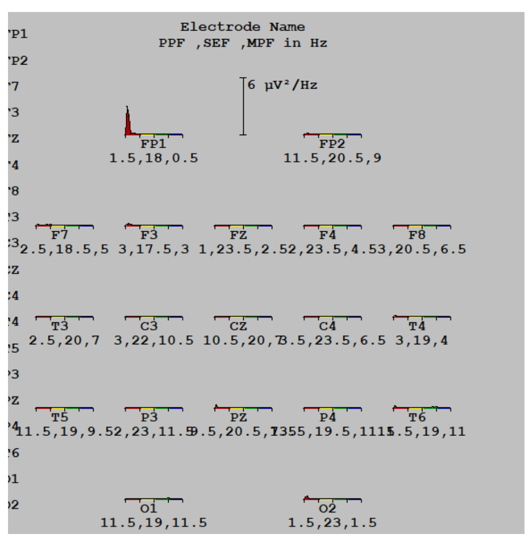

Fig.3 Frequency Spectra (SEF, PPF, MPF)

Figure 3 helps to prepare Table.2

Table 2-Frequency Parameter values in EEG Report

Channel	FP1	FP2	F3	FZ	F4	F8	C3	CZ	C4	T4	P3	PZ	P4	T6	O1	O2
PPF(HZ)	1.5	11.5	9.5	3	1	23.5	2.5	3	10.5	3	1.5	23	13.5	18.5	11.5	1.5
SEF(HZ)	18	20.5	18.5	17.5	23.5	4.5	20	22	20	19	19	11.5	19.5	19	19	23
MPF(HZ)	0.5	9	5	3	2.5	3	7	10.5	8.5	4	9.5	20.5	11	11	11.5	1.5

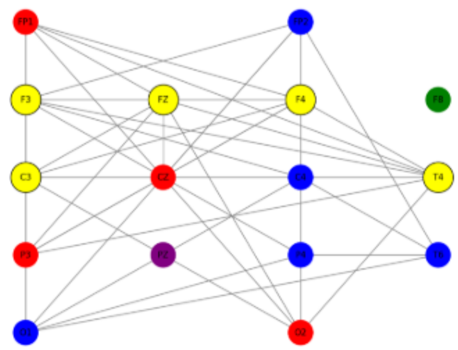

Fig.4 EEG Seizure Propagation Network

Table:3-Connectivity of EEG Seizure Propagation Network

Region Pair	VERTEX CONNECTIVITY	EDGE CONNECTIVITY	Interpretation
Frontal (FP1) → Central (C3)	5	5	Highly resilient pathway, likely strong seizure propagation.
Central (FZ) → Parietal (PZ)	0	0	Weakly connected, indicating poor seizure propagation between these areas.
Parietal (P3) → Occipital (O1)	3	5	Moderate resilience, potential seizure route.

Frontal (F4) → Temporal (T6)	3	4	Stable but can be weakened with proper intervention.

Seizure Propagation and Intervention

- Strongly connected hubs (FP1 → C3) are high-risk pathways for seizure spread.
- Weakly connected nodes (FZ → PZ) suggest areas less involved in seizure propagation.
- Targeting parietal and frontal-temporal regions could interrupt seizure dynamics.

2.2. CONNECTIVITY NETWORK

The connectivity $p = p(G)$ of a graph, the graph can be disconnected or reduced to a simple form by removing the fewest number of vertices, G. As a result, a disconnected graph has a connectivity of 0, while a connected graph with a cut vertex has a connectivity of 1. For a complete graph k_p, no subset of vertices can fully disconnect the graph; however, removing $p(P_{q)} = q - 1$ vertices leave a trivial structure.[2]

2.2.1 CONNECTIVITY PROPERTIES OF AN EPILEPSY NETWORK DERIVED FROM EEG SIGNALS

p(G) →node connectivity, which is the smallest number of important brain regions that must be removed in order to disconnect the seizure network.

$\theta(G)$ →edge connectivity, displays the minimum number of functional connections that must be disrupted to fragment the network.

$\mu(G)$ → Minimum degree, signifying the least-connected brain region in the network. [2].

THEOREM 2.1

Let G = (V, E) represent an epilepsy network derived from EEG signals, where nodes V correspond to brain regions and edges E represent functional connections. Then, the connectivity properties of G satisfy the inequality:

$$p(G) \leq \theta(G) \leq \mu(G) \text{----------------------} (1)$$

Proof

Given

$$p(G) \leq \theta(G) \leq \mu(G)$$

Let's take the Second part of the inequality in equation (1)

$\theta(G) \leq \mu(G)$. If G has no edges, then $\theta(G) = 0$, and since $\mu(G) = 0$, the inequality holds trivially. If G has at least one edge, consider a vertex with minimum degree $\mu(G)$Removing all edges incident to this vertex disconnects G or leaves an isolated vertex. Since at most $\mu(G)$Edges are removed in this process, it concludes that $\theta(G) \leq \mu(G)$

$$p(G) \leq \theta(G) \text{-----------------------} (2)$$

Now let's consider different cases: $p(G) = \theta(G) = 0$

If G is not connected, then the inequality holds trivially.

If G is connected and it's also a bridge a, A bridge is a single edge whose removal disconnects G, so $\theta(G) = 1$,

Since G has a cut-vertex incident to the bridge or is a complete graph on two vertices K_2,

Then, consequently, it proceeds, that $p(G) = 1$ Therefore, $p(G) \leq \theta(G)$

If G has $\theta \geq 2$edges whose removal disconnects it, if argued as follows: Removing $\theta - 1$of these edges results in a graph with a bridge $a = nm$, for every $\theta - 1$ of edges, select a vertex that is not n or m Removing these vertices also removes at least $\theta - 1$ of edges. If the graph obtained is not connected,

Then $p(G) \leq \theta(G)$, If the graph remains connected, then a is a bridge, meaning that removing n or m disconnects the graph. Thus, in all cases, $p(G) \leq \theta(G)$.

Since both inequalities hold in all cases, then conclude $p(G) \leq \theta(G) \leq \mu(G)$.

In graph theory, vertex connectivity $p(G)$ *and* edge connectivity $\theta(G)$ are fundamental measures that describe how resistant a network is to disconnection.

Fig.5 vertex and edge connectivity

Theorem 2.2 states that the maximum vertex connectivity and maximum edge connectivity of a graph are equal:

$$\theta_{max}(G) = p_{max}(G)\text{---------------}(3)$$

This means that the largest number of edges that can be removed before the graph becomes disconnected is equal to the largest number of nodes whose removal disconnects the graph. In the context of EEG-based epilepsy networks, this theorem has significant implications: it suggests that seizure propagation can be disrupted either by removing key brain regions (nodes) or by cutting critical functional connections (edges).

THEOREM 2.2

For any (a, b) given graph G, the max line connectivity is equal to the max

connectivity of the graph. That is,

$$\theta_{max}(G) = p_{max}(G)$$

where:

$\theta_{max}(G)$ *Represent the maximum edge connectivity. and* $p_{max}(G)$ *represents the maximum vertex connectivity.*

PROOF:

Connectivity Pairs and Their Role

A connectivity pair (a, b) represents the minimum number of nodes a and b. edges that must be removed to disconnect G.

The two fundamental cases are:

- Vertex Connectivity: $(p, 0)$, where removing p vertices disconnect G. This defines the vertex connectivity $k(G)$,.
- Edge Connectivity: $(0, \theta)$ where removing θ, edges disconnect G,. This defines the edge connectivity λ(G).

We introduce a function $g(G)$ that represents the minimum number of edges required to disconnect G.after removing a nodes:

$$g(a) = \min\{b|(a,b) \text{ is a connectivity pair}\}\text{------------------------}(4)$$

where $g(0) = (G)$ and $g\big(k(G)\big) = 0$

Strictly Decreasing Property of $g(a)$

The function $g(a)$ is strictly decreasing, meaning that as more nodes are removed, fewer edges are required to disconnect G . Formally, if (a, b) is a connectivity pair with $b > 0$, then there exists another connectivity pair of $(a + 1, b - 1)$, i.e.,

$$g(a + 1) \leq g(a) - 1\text{-------------------}(5)$$

By iterating this property, removing vertices and edges follows a similar pattern.

Maximum Connectivity Relationship

Since $g(a)$ is strictly decreasing, we analyze its extreme values:

The maximum number of edges required for disconnection corresponds to the case when no vertices are removed

$$\theta_{max}(G) = g(0)\text{-----------------}(6)$$

The maximum number of vertices required for disconnection corresponds to the case when no edges are removed

$$p_{max}(G) = max\{a|g(a) > 0\}\text{------------------}(7)$$

Since $g(a)$is strictly decreasing and maps vertex removal to edge removal, it follows that

$$\theta_{max}(G) = p_{max}(G)$$

Thus, the maximum line connectivity is equal to the maximum connectivity of G.

Hence, This theorem can optimize seizure control strategies, identifying the most efficient points of intervention to prevent seizure spread.

2.3 MENGER'S THEOREM

Menger's theorem (1927) establishes a fundamental link between graph connectivity and disjoint paths. It states that the minimum number of vertices (or edges) required to separate two nodes equals the maximum number of vertex-disjoint (or edge-disjoint) paths between them. Widely applied in network analysis, optimization, and resilience studies, this theorem is particularly useful in EEG-based seizure networks. Using spectral data helps assess brain connectivity, identify critical pathways, and understand how disruptions in neural networks contribute to seizure propagation [2].

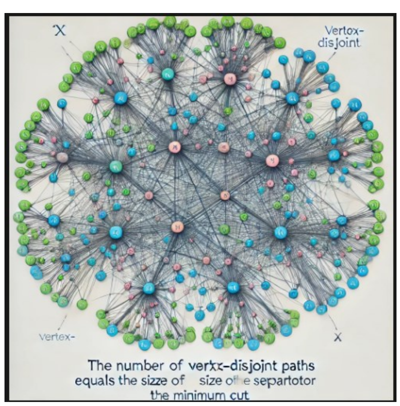

Figure:6 X and Y are two sets of nodes. The minimum number of separating vertices is equal to the number of independent directed paths between $\boldsymbol{X}$ and $\boldsymbol{Y}$

If analysing seizure propagation pathways, the smallest number of nodes or edges that need to be removed to separate seizure onset regions from other areas is essential. According to Menger's theorem, the maximum number of independent paths between two nodes corresponds to the minimum cut size, helping identify key pathways for seizure spread." [2].

EEG-derived connectivity networks, seizure propagation was modeled as a directed graph, where brain regions serve as vertices and functional connections represent edges. The following independent pathways were identified:

- Path 1: T → F → P → Y (Temporal → Frontal → Parietal → Final Region)
- Path 2: T → O → P → Y (Temporal → Occipital → Parietal → Final Region)
- Path 3: T → C → P → Y (Temporal → Central → Parietal → Final Region)[9].

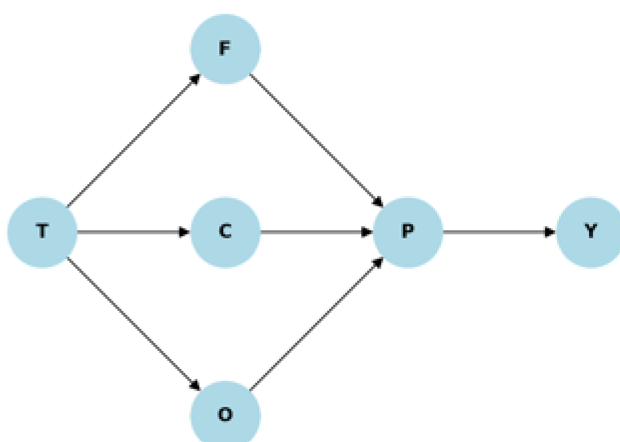

Fig.7 Graph of Menger's theorem

Menger's Theorem states that the minimum number of nodes required to separate two sets of vertices (X, Y) is equal to the maximum number of independent paths connecting them.If

T (Temporal Lobe) is the seizure onset zone and Y (Final Region) is the seizure spread area, then the minimum number of brain regions required to block seizure propagation is equal to the number of independent paths, which is three.

THEOREM 2.3

Let G be a finite directed graph, and X and Y be two sets of vertices in G.If suppose the smallest number of vertices that form an XY-separator is k. Then, there exists an XY -connector Z : $|Z \cap X| = k$ [9].

PROOF:

If G has no edges,

let simply define $Z = X \cap Y$. Thus, we assume that G contains at least one edge and proceeds by induction.

Consider an edge e directed from u to v, and

let G_0 be the graph obtained by removing e. By the inductive hypothesis, the theorem holds for G_0 , meaning there exists an XY -separator T in G_0 with $|T| < k$.

Now, define two new separators:

$P = T \cup \{u\}$, which acts as an XY -separator in G. (8)

$Q = T \cup \{v\}$, which is also an XY -separator in G (9)

Since both $P$and Q separate X from Y, their sizes must be exactly k:

$$|P| = |Q| = |T| + 1 = k$$

Since G_0 satisfies the theorem, it has:

An XP-connector W , which includes P and links X to P.

A QY-connector Z, which contains Q and connects Q to Y.

Since W and Z both include T, their intersection is precisely T, i.e.,

$$W \cap Z = T \text{ --------------------(10)}$$

Finally, we construct the required XY -connector by combining these sets and restoring the removed edge:

$$Z = (WUZ) + e \text{ -----------------(11)}$$

This Z satisfies the conditions of an XY -connector and ensures that exactly k elements from X are included, completing the proof [13].

Using graph theory and Menger's Theorem with EEG-based seizure networks gives a mathematical framework for identifying crucial intervention targets. Blocking significant brain regions along many independent routes can effectively impair seizure propagation, hence assisting in the creation of tailored epilepsy treatments [14].

3. RESULTS AND DISCUSSION

Network Analysis in Seizure

In Section 2.1 I. Strongly Connected Hubs (FP1 → C3):

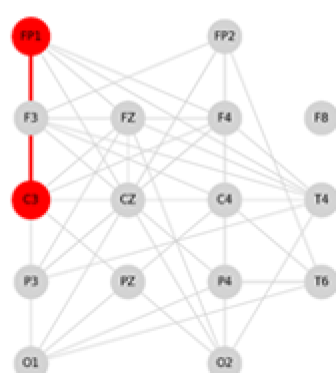

Fig.8 High-Risk Pathways for Seizure Spread

Highly interconnected regions, like the one connecting the FP1 to the C3, are essential for the spread of seizures because they permit irregular electrical activity to be quickly transmitted between different parts of the brain.

II. Weakly Connected Nodes (FZ → PZ):

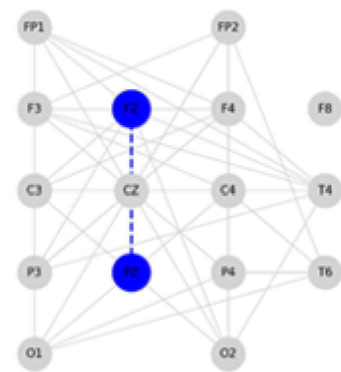

Fig 7: Areas Less Involved in Seizure Propagation:

Weakly connected nodes, like the one from FZ to PZ, represent regions that are less crucial for seizure spread and may have a small role in its propagation.

III. Targeting Parietal and Frontal-Temporal Regions:

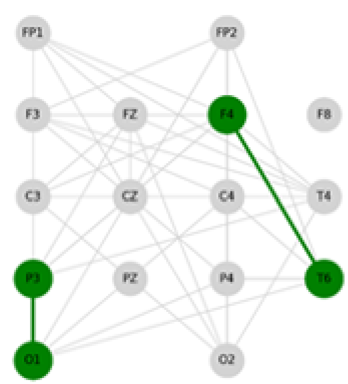

Fig. 8. Interrupting Seizure Dynamics

The parietal and frontal-temporal areas are often involved in seizure networks. By focusing on specific regions, therapeutic approaches like neurostimulation, surgery, or medication may help break up the network and lower the chance of seizures spreading.

This network technique encourages precision medicine by improving seizure control and quality of life for individuals who have not responded to traditional medications.

In Section 2.2.

I. Vertex Connectivity $p(G)$= 2

Vertex Connectivity indicates that interrupting only two essential nodes, P4 and T6, may successfully cease seizure propagation, suggesting them as important targets for intervention.

II. Edge Connectivity $\theta(G)$=2

Removing the crucial edges (F8–PZ) and (F8–T6) can considerably limit seizure propagation, according to edge connectivity, suggesting possible modulation techniques using neurostimulation or connectivity-targeted therapies

The Key nodes P4 and T6, as well as crucial edges F8–PZ and F8–T6, are identified in the study as important targets to stop the spread of seizures in drug-resistant epilepsy. This makes it possible to apply individualized approaches to therapy for better results and precision-based therapies like neurostimulation or surgery. This strategy promotes precision medicine by emphasizing network connectivity, providing hope for improved seizure control.

In sec2.3

In our EEG-based seizure network:

Menger's Theorem, the minimum number of nodes - brain regions - that must be eliminated to disconnect two groups of vertices, notably seizure onset and seizure spread regions, is equal to the maximum number of independent paths connecting them. It detected three independent paths through the Seizure Onset Zone (T) to the Terminal Seizure Spread Region (Y).

Potential intervention methods include:

- Surgical Resection: Removing or disconnecting key propagation regions.
- Neuromodulation (e.g., Deep Brain Stimulation): Disrupting functional connectivity at critical nodes.

- Pharmacological Approaches: Applying targeted drug delivery to interrupt seizure pathways.

Predicting Seizure Dynamics in EEG Data:

By analysing EEG functional connectivity and applying Menger's Theorem, clinicians can identify which brain regions act as critical connectors in seizure propagation also this allows for personalized treatment plans, focusing on the most influential regions based on patient-specific EEG networks to improving seizure control in epilepsy patients.

4. CONCLUSION

EEG frequency analysis using Peak Power Frequency (PPF), Spectral Edge Frequency (SEF), and Mean Edge Frequency (MEF) provides valuable insights into seizure propagation. Changes in PPF indicate the transition from seizure onset to rapid spread, while SEF variations highlight regions involved in seizure generalization. MEF fluctuations reveal overall network excitability, helping to differentiate seizure and non-seizure states. These parameters allow for precise identification of seizure-prone areas, aiding in targeted intervention strategies, and Graph-theoretic analysis using Menger's Theorem and Network analysis to confirm that seizure propagation can be disrupted by removing two critical nodes (P4, T6) or severing two key edges (F8–PZ, F8–T6)**.** The theorem also helps identify minimum cut-sets, optimizing connectivity-based intervention approaches. The findings highlight the frontal-temporal and parietal regions as essential seizure hubs, supporting neurostimulation and connectivity-guided resection as effective treatment strategies.

Authors' Contributions

K. Suspritha conceptualized the research, developed the methodology, and contributed to the theoretical analysis of indeterminacy in seizure networks and Menger's theorem.

G. Jayalalitha: Provided supervision, guidance on network analysis, graph connectivity and critical revisions of the manuscript.

Conflict of Interest
The author declares no conflict of interest.

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Ethical Conduct
This research does not involve any animal biological material. As such, ethical approval and informed consent are not applicable.

Data Availability Statement
The data supporting this study's findings are available from the corresponding author upon reasonable request. Some new data were created or analyzed in this study. All references and theoretical frameworks are based on publicly available data and literature.

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