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MATHEMATICAL EXPRESSIONS IN FOURIER TRANSFORM BASED ANALYSIS OF BIOMEDICAL SIGNALS

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ABSTRACT

In many modern scientific issues, the Fourier transform is a commonly utilised analytical tool. The application of this mathematical technique is probably best recognised for its use in linear time-invariant system analysis. Nevertheless, the Fourier transform is emphasised as a versatile tool for tackling problems in general. Its relevance lies in the fact that any particular relationship can be viewed from an entirely different perspective. Visualising a function and its Fourier transform in your mind at the same time could lead you to the answer to a lot of difficulties. When studying the frequency domain behaviour of biological signals, Fourier transform (FT) is utilised. To obtain the signal in the frequency domain, one can use the FFT command in MATLAB. One of the most important tools for decomposing complex signals into their component frequencies and amplitudes is the Fourier transform. The k-space MR signals are frequency- and phase-encoded, and the Fourier transform resolves these signals during MR picture generation. For future diagnosis and clinical monitoring, biomedical signals such as blood pressure, electrocardiogram (ECG), and photoplethysmographic (PPG) signals must be analysed. The literature made heavy use of transforms for processing and analysis, such as the Fourier transform (FT) and the Wavelet transform (WT).

KEYWORDS: Fourier transform, biomedical signals, electrocardiogram signal, photoplethysmographic signal, wavelet transform, Fourier Transform Spectroscopy,

INTRODUCTION

The success of FT prompted the development of other transforms for use in evaluating and creating new applications. While the WT is applicable to both stationary and non-stationary data, the FT is specifically designed to analyse stationary signals. The discrete wavelet transform is widely used in many branches of engineering. It is difficult to understand and analyse pictures. The medical industry makes use of digital image processing for several purposes. Magnetic resonance imaging (MRI) studies and computed tomography (CT) scans are the most prevalent. A wide variety of fields may benefit from digital image processing and analysis, not limited to: criminology, meteorology, remote sensing, astronomy, artillery, office and industrial automation, natural resource survey and management, astronomy, and medical. The importance of visual representations in human cognition is further enhanced by the fact that vision is the most developed sense in humans. Because of its ability to convert signals from the frequency domain to the time domain, the Fourier transform (FT) is a popular analytical tool in many different areas, including medical, wireless communications, signal processing, and image analysis. The frequency domain analysis of a variety of biological data, such as electrocardiograms, photoplethysmograms, and blood pressure measurements, has been documented in several investigations using this device.

The Fourier transform spectroscopy methods covered in the previous chapter, including FTIR-VIS, FTIR-ATR, FTIR-PAS, FTIR imaging, and FTICR-MS, as well as their critical physical principles, are detailed in this chapter.

FOURIER TRANSFORM

Research in engineering and mathematics follows a two-way parallel track that coordinates and interacts towards value-added research; in other words, the examination of real-world engineering applications cannot neglect the essence of mathematical design. Specifically, one cannot even begin to fathom the use of transforms in the realm of ECE. Sustainable development of biomedical technologies for crucial monitoring and effective immunization are being considered as potential solutions to the current COVID-19 pandemic scenario. Applications in biomedical, wireless communication, signal, and image processing often make use of the Fourier transform (FT), a mathematical technique that converts signals from the time domain to the frequency domain. It has been reported in the literature that this technique has been extensively utilised by researchers to analyse biological data in the frequency domain, including ECG, PPG, and BP.

An image that a digital computer has picked up with its in-built sensors may be examined. In order to mimic human vision, digital computers use techniques that are often referred to as image processing and analysis approaches. An analogue image is one in which the amplitude values of x, y, and f are limitless. When a picture's values are negligible, we say that it's digital. Individual components with distinct functions and monetary worth comprise a digital image. You may call them what you want: pixels, pels, picture elements, or image elements. The term "pixel" is often used to describe the individual elements of a digital image. The term "digital image processing" describes the activities done on digital computers to alter images. After correctly structured visual data is input into a computer, the next step is image analysis.

To determine the size of two-dimensional arrays I_1 and I_2, where $I_1(x,y)$ and $I_2(x,y)$ represent the intensity values at position (x,y) of the corresponding images, we first define the dimensions of each image.

Let:

- M_1 and N_1 be the dimensions of image I_1 (i.e., width and height respectively).
- M_2 and N_2 be the dimensions of image I_2.

The size of the two-dimensional arrays can be represented as:

$$I_1: R^{M1\times N1}, I_2: R^{M2\times N2} \quad (1)$$

Making a mapping of the spatial and intensity relationships between two images is called image registration. Determining the size of two-dimensional arrays, I_1 and I_2, where $I_1(x, y)$ maps to the intensity values of the corresponding images, is the first step in representing the mapping between two photographs.:

$$I_2(x, y) = g(I_1(f(x, y))) \quad (2)$$

where f is a spatial coordinate change in two dimensions, i.e.,

$$(x', y') = f(x, y) \quad (3)$$

where g is the symbol for the intensity or radiometric transformation. Because of its ability to convert signals from the frequency domain to the time domain, the Fourier transform (FT) is a popular analytical tool in many different areas, including medical, wireless communications, signal processing, and image analysis. The frequency domain analysis of a variety of biological data, such as electrocardiograms, photoplethysmograms, and blood pressure measurements, has been documented in several investigations using this device. The success of FT prompted the development of other transforms for use in evaluating and creating new applications. While the WT is applicable to both stationary and non-stationary data, the FT is specifically designed to analyse stationary signals. The discrete wavelet transform is widely used in many branches of engineering.

TRANSFORMS AND ITS USE IN BIOMEDICAL SIGNAL ANALYSIS

- **Transforms**

Among other transformations and significant applications, the theoretical and mathematical basis for Fourier analysis and harmonic analysis was established by the French mathematician Jean-Baptiste Joseph Fourier. For a signal x(t) in the time domain, the forward and inverse transforms of a continuous time Fourier transform (CTFT) are defined as

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\varpi t}\, \mathrm{dt} \quad (4)$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega \tag{5}$$

Here, $e^{-j\varpi t}$ is called as a basis function for CTFT.

The forward and inverse transforms of a discrete Fourier transform (DFT) for a discrete time signal x(n) are defined as; this simplifies processing by converting the continuous time signal to discrete time.

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi nk}{N}}; \ for\ k = 1,2,3......N \tag{6}$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi nk}{N}}; \ for\ n = 1,2,3......N \tag{7}$$

Here, $e^{\frac{-j2\pi nk}{N}}$ is called as a basis function for DFT.

Likewise, CWT stands for continuous wavelet transform and is known as

$$F(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} \psi\left(\frac{t-a}{b}\right) f(t)\ dt \tag{8}$$

where, f(t) is a time domain signal, ψ(t)ψt is a basis function also called as mother wavelet; $F(a,b)$ is WT of a signal f(t); ‘a’ and ‘b’ are shifting and scaling parameters respectively;

To separate the target signal into its approximation (Aj) and detailed (Dj) components,

CLASSIFICATION OF BIOMEDICAL SIGNALS

Continuous process and discrete-time or point process are two main categories into which biomedical signal sources fall. Different kinds of signals may exhibit different properties, such as being determined, stochastic, fractal, or chaotic.

A. Analysis of Biomedical signals

Separating a biological signal into its constituent spectral (or frequency) components is a typical goal of signal analysis.

B. Spectral analysis of Deterministic and Stationary random signals

Stationary signals, defined as signals whose spectral content does not change over time, are ideal candidates for analysis using the Fourier transform.

F(w) = (1)

x(t) = (2)

The sum of time-invariant, eternal sinusoids is what the Fourier transform uses to create x(t) from a linear combination of stationary signals. The signal's Fourier transform is a transformation from a time-dependent function x(t) to a frequency-dependent function F(w). A signal component's frequency w may be determined by using the function F(w) to the analysed signal. The way the signal component changes over time t is not shown. That can only be accomplished using a transform that produces a bivariate function F(t,w).

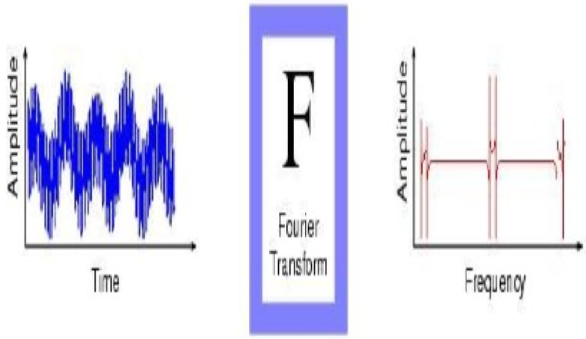

Fig.1 Fourier transform of signal

C. Fast Fourier Transform:

Any deterministic biological signal, with or without noise, may have its spectrum analysed using fast Fourier transform (FFT). One way to break down discrete time signals into their component sinusoids whose frequencies are multiples of a fundamental frequency is via the use of discrete Fourier transform (DFT). It is possible to determine the sinusoidal components' amplitude and phase using the density-functional theory (DFT).

The Discrete Fourier Transform (DFT) is a mathematical method for converting a sequence of time-domain discrete samples into a frequency-domain representation. The Fast Fourier Transform (FFT) is an efficient algorithm to compute the DFT of a sequence. For a discrete signal x[n], where nnn ranges from 0 to N−1, the DFT is given by:

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}, \quad k = 0, 1, 2, \ldots, N-1 \qquad (9)$$

D. **Spectral Analysis of Non- stationary signals**

When a signal's statistics fluctuate over time, we say that the signal is non-stationary. An average estimate is provided using the conventional spectrum techniques. There is another method of analysis that may be used to get time-localized spectral estimations. Algorithms such as the Short-time Fourier transform and the Wavelet transform are often used to describe spectral estimates in terms of time and frequency.

E. **Short-Time Fourier Transform (STFT)**

Multiplexing the signal by a small-time window centred on the time instant of interest is what the STFT is all about. The product's Fourier transform, which provides a rough idea of the signal's spectral composition at that precise moment. Next, we slide the small-time window along the time axis so it covers the whole signal and we can estimate the signal's spectral content at each moment. The **STFT** of a signal x(t) can be represented as:

$$X(n, \omega) = \sum_{m=-\infty}^{\infty} x[m]w[m-n]e^{-j\omega m} \tag{10}$$

The STFT thus results in spectrum that depends on the time instant to which the window is shifted.

F. Wavelet Transform

An approach to signal analysis known as the Wavelet transform relies on a collection of orthogonal basis functions that are derived from the prototype wavelet via dilations, contractions, and shifts. When dealing with transitory biological signals, wavelets have found widespread utility. The fundamental difference between WT and Fourier transform based approaches like STFT is that WT utilises often dependent windows while FT uses continuous width windows. By using tight windows for high frequency components and large windows for low frequency components, the wavelet transform enables frequency components and arbitrarily excellent frequency resolution for low frequency components. The mathematical representation of the continuous wavelet transform is generally

$$W_x(a, b) = \int_{-\infty}^{\infty} x(t)\psi^* \left(\frac{t-b}{a}\right) dt \tag{11}$$

The scale parameter a and translation parameter b allow the wavelet to be stretched or compressed and shifted in time, respectively, providing both time and frequency information about the signal.

A prototype wavelet function, also known as the mother wavelet, is scaled and shifted to produce the orthogonal basis function, shown as. One way to express the orthogonal basis functions in discrete time is as

$$\psi_{j,k}[n] = 2^{j/2}\psi(2^j n - k) \tag{12}$$

This is where the basic scale factor (a0) and the time shift (b0) come into play. Wavelet transform is most effective with long-duration signals and short-duration high-frequency components. as an example, electrical cardiac and brain wave signals.

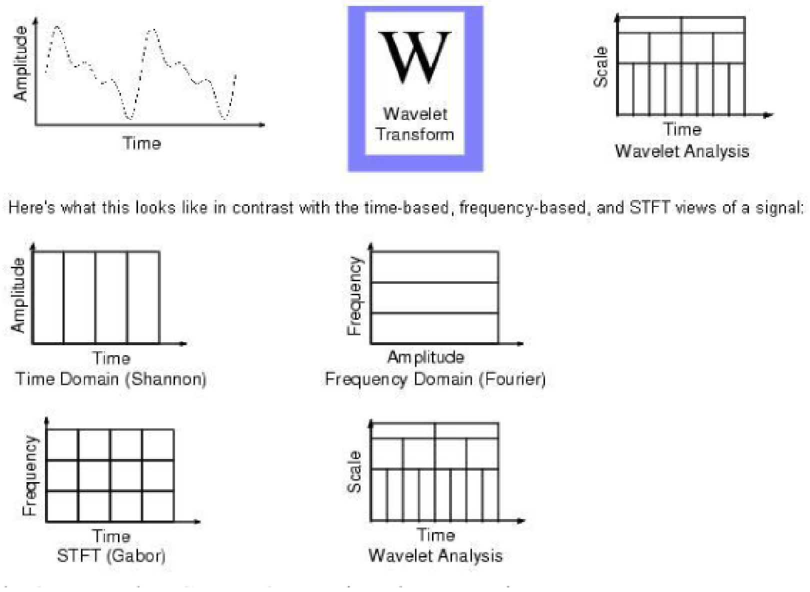

Fig.2. Wavelet, STFT & FT signal comparison

USE OF TRANSFORMS FOR ECG SIGNALS.

The identification of cardiac arrhythmias is greatly assisted by electrocardiograms (ECGs). A mathematical technique that shifts focus from the temporal domain to the frequency domain is a transform. Without altering the signal's information content, the transformation modifies the signal's representation by mapping it onto a set of necessary functions. Features based on crossed wavelets, Fast Fourier transform (FFT), and the Short-Time Fourier transform (STFT) are among the many feature extraction approaches that have been available for decades. The ECG signal's low frequencies are removed via the Fast Fourier Transform. To filter out background noise, we employ the inverse fast Fourier transform. Following null-removal preprocessing, the input signal from the dataset is transformed using the Fast Fourier Transform. To improve the accuracy of peak extraction, FFT is employed to transform the time domain data into a frequency domain ECG signal. It is common practice to apply the Fast Fourier transform to get frequency coefficients from signals that are in the time domain. To further classify arrhythmias, a neural network was used after the fast Fourier transform (FFT) was utilised to detect the QRS complex peak (R) in the electrocardiogram (ECG) data.

$$X = \sum_{n=0}^{N-1} xne^{\frac{-2\pi i}{N}} nk \qquad \text{Where k = 0..., N-1 (1)} \tag{13}$$

Using the wavelet transform, one may undertake simultaneous analysis in the time domain and the frequency domain. In terms of both frequency and temporal resolution, the wavelet transform offers a variety of scales for the ECG signal. For the purpose of detecting arrhythmia and QRS complex, the discrete wavelet transform (DWT) is used [11]. This transform outperforms the others when it comes to detecting heart rate. The morphology of an electrocardiogram (ECG) may reveal cardiac arrhythmias, disorders, and other anomalies. Equation (2) defines the wavelet transform equation for the signal x(t).:

$$Wax(b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t)\psi(\frac{t-b}{a})\, dt \qquad (14)$$

Where ɑ is the scaling, b= translation parameter

REMOVAL OF ARTIFACTS FROM EEG SIGNALS

Neuroscience, cognitive science, and cognitive psychology have advanced their studies with the help of non-invasive techniques like as electroencephalography (EEG), functional near-infrared spectroscopy (fNIRS), and magnetoencephalography (EMG). Electroencephalogram (EEG) is a method for studying brain activity that involves recording data from many scalp electrodes. It is a unique and complex biological electrical signal that reflects the mental health of the individual and their functional brain state; by deducing this information, we can track their vitals, diagnose their illnesses, and find out what's wrong with their brain. Nevertheless, due to its great temporal resolution, EEG signals are susceptible to unwanted noise and artefacts. Inaccurate electrodes, line noise, and high electrode impedance are examples of measurement artefacts that can be mitigated with a more exact recording system and rigorous recording protocols; in contrast, physiological artefacts are notoriously difficult to eliminate. The electroencephalogram (EEG) data might include a wide variety of physiological artefacts, such as eye movements, blinks, heart rate, and muscle activity.

These physiological artefacts have the potential to skew cerebral information or even be falsely driven by seemingly natural processes in a real-world application like a brain-computer interface. Clinical studies on sleep disorders, Alzheimer's disease, and other similar conditions are susceptible to visual interpretation and diagnosis bias due to artefacts that mimic cognitive or pathogenic activity.

- **Types of Artifacts**

The presence of signal artefacts becomes more noticeable when EEG data is acquired from recording devices. The reliability of EEG data can be compromised by these artefacts. For this reason, effective artefact or noise removal requires in-depth familiarity with the many kinds of artefacts. Environmental noise, experimental error, and physiological artefacts are the most common sources of artefacts, which are undesired signals. In addition, physiological artefacts (such as eye blinks, muscular activity, and heart beats) might be considered intrinsic artefacts, in contrast to extrinsic artefacts (such as environmental artefacts and experimental mistake) that originate from outside the body. Three main physiological artefacts have been identified in the literature, as shown in Figure 2.

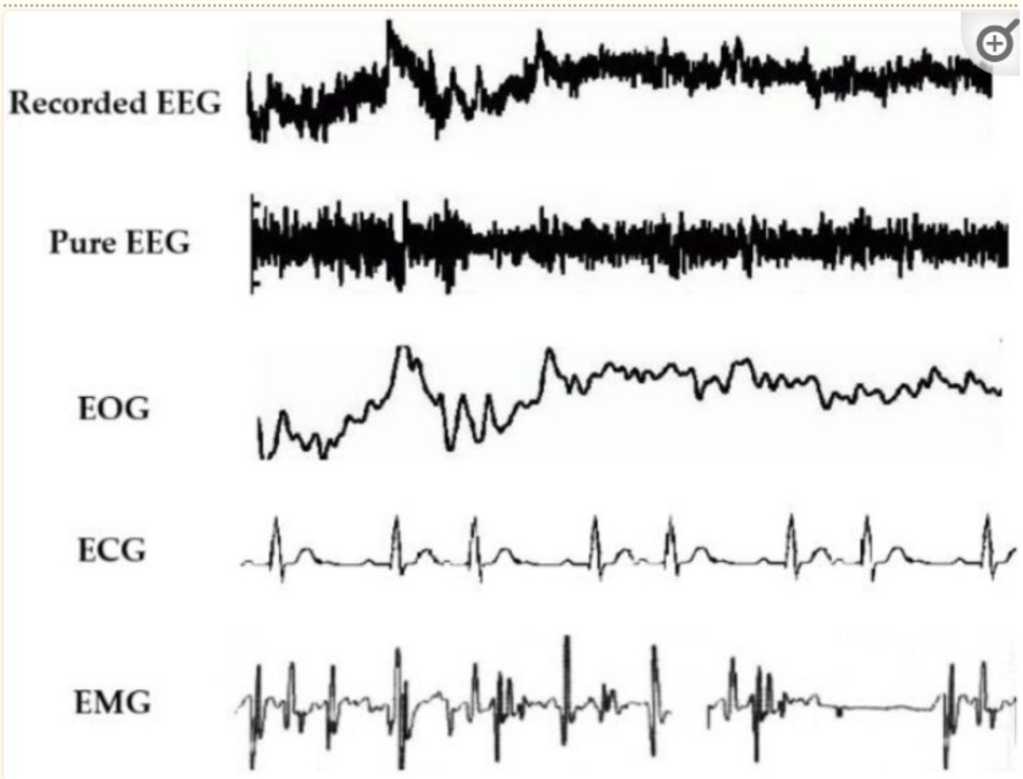

Figure 3_Physiological artifacts present in EEG signals.

- **Ocular Artifacts**

Significant artefacts in EEG recordings can be caused by ocular artefacts. Because eye movements and blinks can travel across the scalp and be picked up by EEG activity, these phenomena are known as ocular artefacts. Blink artefacts generated by variations in ocular conductance as a result of changes in contact between the cornea and the eyelid, and eye movement artefacts caused by changes in the orientation of the retina and the cornea dipole are two examples. Furthermore, the volume conduction effect caused the ocular artefact and the EEG activity to go to the skull surface and be recorded by the electrodes. Electrooculograms (EOGs) are able to record these kinds of visual impulses.

- **Muscle Artifacts**

Because it can originate in many different kinds of muscles, the problem of muscular activity contaminating EEG data is well-known and difficult to solve. These artefacts can be created by the subject talking, sniffing, swallowing, or by any muscle contraction or stretch near the signal recording sites. Theoretically, electromyogram (EMG) measuring artefacts in muscles have a wide frequency range, from 0 Hz to >200 Hz. The amplitude and waveform of artefacts are affected by the degree of muscular contraction and stretch. In comparison to EOG and eye-tracking, obtaining the activity from a single channel measurement is incredibly challenging.

- **Cardiac artifacts**

Electrodes, when placed on or near a blood vessel, can introduce cardiac artefacts caused by the heart's expansion and contraction movements. Pulse artefacts, which cause EEG to look like a similar waveform and have a frequency of about 1.2 Hz, are notoriously difficult to eliminate. An electrocardiogram (ECG) is a method of monitoring the heart's electrical signal. Since electrocardiograms (ECGs) may be recorded independently of brain activity and have a characteristic regular pattern, it may be easier to eliminate these artefacts by simply comparing them to a reference waveform.

- **Extrinsic Artifacts**

In addition to the aforementioned imperfections, EEG measurements can be skewed by artefacts from outside sources. The loss of electrodes or the rerouting of cables are the two main sources of instrument artefacts, which are considered to be external artefacts. The correct method and preparation can eliminate these artefacts. Another kind of external artefact that could impact EEG recordings is electromagnetic interference from the environment. The widely identifiable frequency spectrum of such environmental artefacts makes them amenable to degradation by a basic filter.

CONCLUSION

A deep comprehension of the complex relationship between mathematics and medical diagnostics can be gained by investigating mathematical expressions in biomedical signal processing based on the Fourier transform. As we have seen on this study's path, mathematical concepts are crucial for improving healthcare and understanding the human body by revealing the intricacies concealed inside biological signals.

By following mathematical equations, the Fourier transform has revealed a surprising way to convert signals from the time domain to the frequency domain. In addition to illuminating the individual frequency components, this transformation sheds light on the underlying physiological phenomena of the signals. The importance of peaks, troughs, and patterns in the frequency domain representation has been discovered through careful study, paving the way for the interpretation of clinical insights.

Numerous advantages have resulted from the complementary work of mathematics and the biological and medical sciences.

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