UtilitasMathematica

ISSN 0315-3681 Volume 122, 2025

Thermal-Diffusion Dynamics of Unsteady MHD Viscoelastic Flow in Porous Medium using the Laplace Transform Approach

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Abstract:

This research focuses on the thermal diffusion phenomenon—commonly known as the Soret effect—within the context of an unsteady magneto-hydrodynamic (MHD) viscoelastic fluid flowing through a vertical porous domain. The model incorporates the effects of chemical reactions, thermal radiation, and diffusive transport. Governing partial differential equations are non-dimensionalised and analytically solved using the Laplace transform method, with suitable boundary constraints. The influence of parameters such as Grashof number, magnetic field strength, and Schmidt number is quantitatively visualized via MATLAB to interpret the velocity, temperature, and concentration characteristics of the system.

Key Words: Soret Effect, MHD, Porous medium, Laplace Transform.

Introduction:

Newtonian fluids are known for their various applications in Aerospace, Oceanography, Geophysics, and Biomedical Engineering. The article focuses on Unsteady Magneto - hydrodynamic viscous-elastic fluid flowing into a vertical porous medium since they have a straightforward relationship between shear stress and rate. Amongst all the other effects, the Soret effect is known for validating the thermal diffusion process.

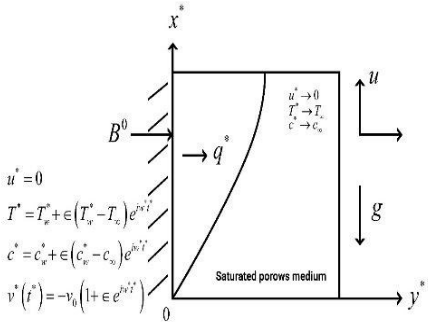
Unsteady free convection influenced by chemical reactions and radiative heat absorption near a semi-infinite, permeable surface in vertical motion—alongside internal heat generation and suction—has been previously investigated [1].

Additionally, the transient behavior of MHD flow past a vertically oscillating boundary, incorporating both thermal radiation and variable mass diffusivity, has been studied in detail [2]. A theoretical analysis of MHD-driven dissipative convective flow within a boundary layer adjacent to a permeable vertical surface, under the impact of thermal radiation, is outlined in [3]. Investigations on viscous fluid transport through heterogeneous porous media, subject to oscillatory suction and internal heating or cooling, are presented in [4]. Moreover, the convective performance of viscoelastic fluids under MHD effects near vertical porous boundaries has been explored in [5]. Magneto-hydrodynamic free convection flow in porous domains, subject to varying permeability, internal heat generation, and chemical reactions, has been previously addressed [6]. The application of Laplace transform techniques to analyze unsteady MHD Jeffrey fluid. flow over inclined porous surfaces has also been demonstrated [7]. A closed-form analytical solution for MHD flow of elastic-viscous fluids in porous structures considering radiative heat transfer is provided in [8]. Additionally, the Dufour effect under unsteady MHD conditions in a porous vertical setting with temperature ramping has been studied [9]. Recent developments include the modeling of unstable rotating MHD flows in a second-grade tetra hybrid nanofluid within a porous matrix, using both Laplace and Sumudu transform techniques [10].

In conclusion, the reviewed literature highlights various aspects of unsteady MHD flow in porous media; however, the current study is distinctly oriented toward examining the influence of the Soret effect on a viscoelastic fluid under transient conditions within a porous structure. By employing the Laplace transform technique, this work provides a novel analytical perspective on how thermal diffusion responds to temporal variations in such complex flow environments.

Mathematical Formulation:

The present model describes an unsteady free convective flow of a viscoelastic fluid moving vertically through a porous medium along an infinite flat plate. The analysis accounts for radiation absorption and the influence of an applied magnetic field, coupled with time- varying suction and permeability effects. The flow domain is aligned such that the plate lies along the x-direction, while the fluid motion occurs along the y-axis. For the initial state ($t < 0$), the fluid maintains a uniform temperature, and the species concentration is negligibly small. At time t ≥ 0, the wall temperature increases to T_s, and the concentration of the diffusing species reaches C_s, thereby introducing the thermal diffusion phenomenon, known as the Soret effect, into the system dynamics.



Flow Geometry

Using Boussinesq's approximation, the equations governed and it's the needed boundary condition for the problem are as follows,

$$\frac{\partial u}{\partial t} = \vartheta \frac{\partial^2 u}{\partial y^2} + g\beta_T (T - T_0) + g\beta_C (C - C_0) - \sigma B_0^2 \frac{u}{\rho} - \vartheta \frac{u}{k} - \frac{k_0}{\rho} \left(\frac{\partial^3 u}{\partial t \partial y^2} + v \frac{\partial^3 u}{\partial y^3}\right) \qquad (1)$$

$$\frac{\partial T}{\partial t} \rho C_p = K \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} - Q_T (T - T_0) + Q_C (C - C_0) \qquad (2)$$

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial y^2} - K_C (C - C_0) + \frac{DK_T}{T_m} \frac{\partial^2 T}{\partial y^2} \qquad (3)$$

Initial-Boundary's Condition:

u = 0, T = T + $\varepsilon (T - T_0)e^{i\omega t}$, C = C + $\varepsilon (C - C_0)e^{i\omega t}$ ∀ y = 0.

u → 0, T → T∞, C → C∞ as y → ∞.

Dimensionless Quantities

y. = $\frac{v_0 t}{\vartheta}$, y. = $\frac{v_0 y}{\vartheta}$, $t_. = \frac{v_0 t}{4\vartheta}$, $\omega_. = \frac{4\vartheta\omega}{v_0}$, u. = $\frac{u}{u_0}$, T = $\frac{T_. - T_0}{T - T_0}$,

C = $\frac{C_. - C_0}{C - C_0}$, S = $\frac{\vartheta_S}{v_0^2}$, $K_p = \frac{v_0^2 K_\rho^2}{\vartheta^2}$, $M^2 = \frac{\sigma B_0^2 \vartheta}{v_0^2 \rho}$, Pr = $\frac{\vartheta}{k}$, $S_c = \frac{\vartheta}{D}$,

Rc = $\frac{v_0^2 k_0}{\vartheta^2 \rho}$, $G_c = \frac{vg\beta(C - C_0)}{v_0^3}$, $G_r = \frac{vg\beta(T - T_0)}{v_0^3}$, F = $\frac{4I\vartheta}{v_0^2 \rho C_p}$, S = $\frac{Q_T \vartheta}{v_0^2 \rho C_p}$,

R = $\frac{Q_C \vartheta (C - C_0)}{v_0^2 \rho (T - T_0)}$, $K_c = \frac{K_r \vartheta}{v_0^2}$, $S_r = \frac{D_{KT}}{T_m} \frac{(T - T_0)}{(C - C_0)}$

Non-Dimensional Equations

Using the above Dimensionless quantities, we get the below given dimensionless equations,

$$\frac{1}{4}\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + G_rT + G_cC - (M^2 + \frac{1}{K_p})u - \frac{1}{4} Rc \{\frac{\partial^3 u}{\partial t \partial y^2} - 4\left(1 + \varepsilon e^{i\omega t}\right)\frac{\partial^3 u}{\partial y^3}\} \quad (4)$$

$$\frac{1}{4}\frac{\partial T}{\partial y} = \frac{1}{P_r}\frac{\partial^2 T}{\partial y^2} - HT + Rc \quad (5)$$

$$\frac{1}{4}\frac{\partial C}{\partial y} = \frac{1}{S_C}\frac{\partial^2 C}{\partial y^2} - K_rC + S_r\frac{\partial^2 T}{\partial y^2} \quad (6)$$

Method of Solution:

Laplace Equations

$$L[\frac{\partial u}{\partial t}] = 4L[\frac{\partial^2 u}{\partial y^2}] + 4G_rL[T] + 4G_cL[C] - 4\left(m^2 + \frac{1}{k_p}\right)L[u] - R_c\left\{L[\frac{\partial^3 u}{\partial t \partial y^2}] - 4\left(1 + \varepsilon e^{iwt}\right)L[\frac{\partial^3 u}{\partial y^3}]\right\} \quad (7)$$

$$L[\frac{\partial T}{\partial t}] = \frac{4}{P_r}L[\frac{\partial^2 T}{\partial y^2}] - 4HL[T] + 4L[R_c] \quad (8)$$

$$L[\frac{\partial c}{\partial t}] = \frac{4}{S_c}L[\frac{\partial^2 c}{\partial y^2}] - 4k_rL[C] + 4S_r[\frac{\partial^2 T}{\partial y^2}] \quad (9)$$

Boundary's Condition

$t < 0$, $u = 0$, $T = T + \varepsilon\,(T - T_0)e^{i\omega t}$, $C = C + \varepsilon\,(C - C_0)e^{i\omega t}$ $\forall$ $y = 0$.

$t \geq 0$, $u \to 0$, $T \to T\infty$, $C \to C\infty$ when $y \to \infty$.

Solution:

$$u(y,t) = ae^{\frac{t}{4}}\{(\frac{t}{2} + \frac{4x}{t})e^{x\sqrt{4t}}erfc(\frac{x\sqrt{x}}{2\sqrt{t}} + 2\sqrt{x}) + (\frac{t}{2} - \frac{xPr}{8H})e^{-x\sqrt{4x}}erfc(\frac{x\sqrt{x}}{2\sqrt{t}} + 2\frac{\sqrt{xt}}{\sqrt{t}}) + \frac{u_0 + \frac{G_rT + G_cC}{S + \frac{\sigma B_0^2}{\rho} + \frac{\vartheta}{K}}}{1+\lambda}[\frac{y}{2\sqrt{\frac{1+\lambda}{\vartheta}}t}\exp(-\frac{y^2}{4\sqrt{\frac{1+\lambda}{\vartheta}}t}) + (\frac{G_rT}{S + \frac{K}{\rho C_p}})\frac{y}{2\sqrt{\frac{K}{\rho C_p}}t}\exp(-\frac{y}{4\frac{K}{\rho C_p}t}) + + (\frac{G_CC}{S+K+Dm})\frac{y}{2\sqrt{\frac{Sr}{S+K}}t}\exp(-\frac{y^2}{4Srt}) - [\frac{y}{2\sqrt{\frac{1+\lambda}{\vartheta}}t}\text{erfc}(\frac{y^2}{4\sqrt{\frac{1+\lambda}{\vartheta}}t}) - (\frac{G_rT}{S + \frac{K}{\rho C_p}})\frac{y}{2\sqrt{\frac{K}{\rho C_p}}t}\text{erfc}(\frac{y}{4\frac{K}{\rho C_p}t}) - (\frac{G_CC}{S+K+Dm})\frac{y}{2\sqrt{\frac{Sr}{S+K}}t}\text{erfc}(\frac{y^2}{4Dmt})]] + e^{-4Krt}\{e^{-KcSc}.erfc(\frac{x\sqrt{Sc}}{2\sqrt{t}} + \sqrt{-4Krt}) + e^{\sqrt{-KcSc}}.erfc(\frac{x\sqrt{Sc}}{2\sqrt{t}} - \sqrt{-4Krt})\}$$

$$T(y,t) = ae^{\frac{t}{Pr}}\{(\frac{t}{2} + \frac{xPr}{8H})e^{x\sqrt{4H}}erfc(\frac{x\sqrt{Pr}}{2\sqrt{t}} + 2\frac{\sqrt{Ht}}{\sqrt{Pr}}) + (\frac{t}{2} - \frac{xPr}{8H})e^{-x\sqrt{4H}}erfc(\frac{x\sqrt{Pr}}{2\sqrt{t}} + 2\frac{\sqrt{Ht}}{\sqrt{Pr}}) - R_ct$$

$$C(y,t) = e^{-4Krt}\{e^{-KcSc}.erfc(\frac{x\sqrt{Sc}}{2\sqrt{t}} + \sqrt{-4Krt}) + e^{\sqrt{-KcSc}}.erfc(\frac{x\sqrt{Sc}}{2\sqrt{t}} - \sqrt{-4Krt})\} + \frac{Sc(4H)}{(Pr - \frac{Sc}{4})(a1)}\{e^{\frac{4Ht}{Pr}}[e^{y\sqrt{4H}}.erfc(\frac{y\sqrt{Pr}}{2\sqrt{t}} + \frac{\sqrt{4H}}{\sqrt{Pr}}t) + e^{-y\sqrt{4H}}.erfc(\frac{y\sqrt{Pr}}{2\sqrt{t}} - \frac{\sqrt{4H}}{\sqrt{Pr}}t)]\} + \frac{4ScSrKr}{(Sc - 4\,Pr)a_2}\left\{e^{-4Krt}\left[e^{-x\sqrt{-KrSc}}.erfc\left(\frac{x\sqrt{Sc}}{\sqrt{t}} - \sqrt{-4Krt}\right) + e^{x\sqrt{KrSc}}.erfc\left(\frac{x\sqrt{Sc}}{\sqrt{t}} + \right.\right.\right.$$

$$\sqrt{-4Krt}\Big)\Big]\Big\} + \frac{Sr(Sc\, a_2 - 4KrSc)}{(Sc - 4\mathrm{Pr})a_2}\Big\{e^{(a_2-4Kr)t}\Big[e^{-x\sqrt{4\frac{(a_2-4Kr)}{Sc}}}.\, erfc\left(\frac{x\sqrt{Sc}}{\sqrt{t}} - \sqrt{(a_2-4Kr)t}\right) +$$

$$e^{x\sqrt{4\frac{(a_2-4Kr)}{Sc}}}.\, erfc\left(\frac{x\sqrt{Sc}}{\sqrt{t}} + \sqrt{(a_2-4Kr)t}\right)\Big]\Big\}$$

Results:

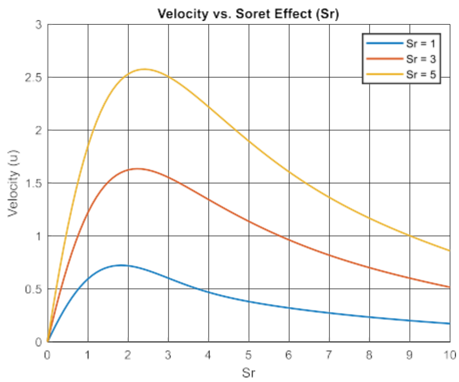


Fig 1.

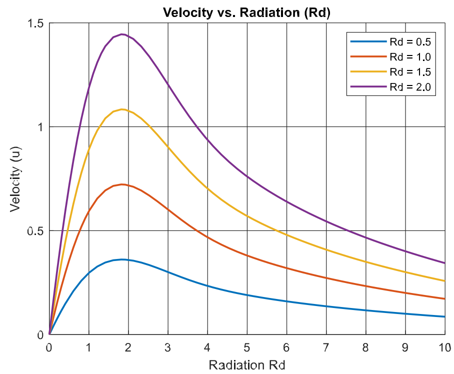


Fig 2.

Figures 1 and 2 show the increase in the Soret effect (Sr = 1, 3, 5) and the Radiation (Rd = 0.5, 1, 1.5, 2) increases the velocity of the fluid flowing through the vertical porous medium

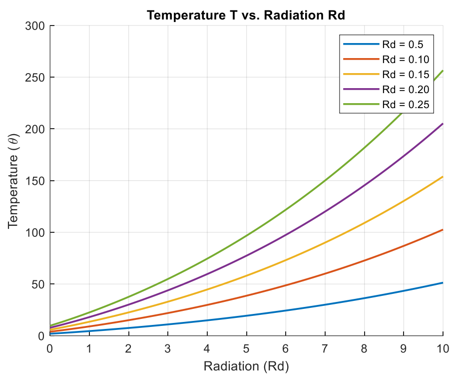


Fig 3.

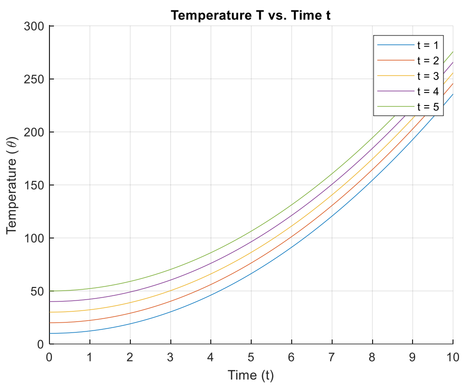


Fig 4.

Figures 3 and 4 explain that when radiation (Rd = 0.5, 0.10, 0.15, 0.20, 0.25) and the time (t = 1, 2, 3, 4, 5) increases the temperature of the particular fluid is also seen to rise to a certain level due to the effect of systematic chemical reaction that takes place with time t > 0.

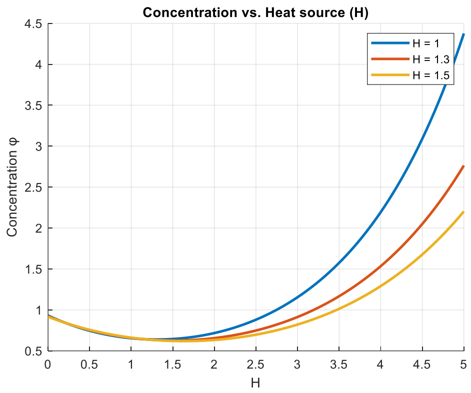


Fig 5.

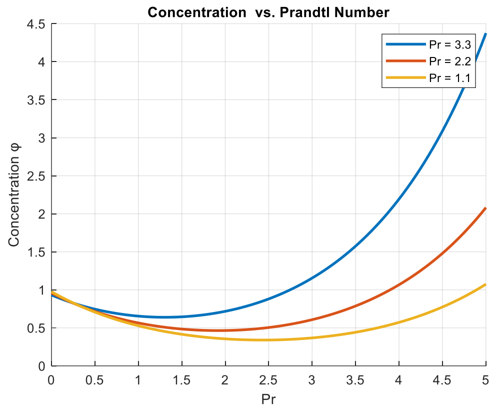


Fig 6.

Figures 5 and 6 depict that increase in the Heat source (H = 1, 1.3, 1.5) decreases the concentration whereas the increase in the Prandtl number (Pr = 1.1, 2.2, 3.3) increases the fluid's concentration.

Conclusion:

The Laplace solution for the effect of Soret on an unstable MHD viscoelastic fluid flow through a porous vertical medium is found. The Velocity, Temperature and concentration profiles effect for multiple parameters are obtained graphically, in the notable results are listed below,

- The magnetic parameter increases the velocity.
- The velocity profile rises due to the presence of the Soret effect.
- The temperature rise is due to increase as the radiation increases.
- The heat source increases the concentration profile.

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