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Creep behavior in Metal Matrix Composite Disc with Thickness Variation

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Abstract. The objective of the current study is to examine the effect of thermal gradients on stress distributions and the resulting deformation of the revolving functionally graded material disc with hyperbolic thickness in presence of thermal gradients. It has been concluded that the thermal gradients effects the distributions of the stresses and strain rates of the anisotropic rotating disc. Thus, the existence of thermal gradients in revolving disc plays a significant role in behavior of creep.

***Keywords*:** Composites, Creep, Functionally graded material, Thermal gradients.

INTRODUCTION

Revolving disc is a fundamental component used in many engineering applications in compressors, flywheels, the jet engine, computer's discs, turbine rotors, automotive breaks, etc. In most of manufacturing applications, the revolving disc works at high thermal gradient and higher angular velocity. As a consequence of the elevated measure of pressure on the properties of disc, this analysis has increased the interest of the scholars. Observing the same, the study has carried out the study of creep behavior in metal matrix composite the disc in existence/ nonexistence of thermal gradients.

ANALYSIS

The constitutive equations for creep are described under multiaxial stress, are expressed,

$$\dot{\varepsilon}_r = \frac{\dot{\bar{\varepsilon}} \times \left[x(r) - \frac{\left(H/F \right)}{G/F + H/F} \right] \left(\frac{G}{F} + \frac{H}{F} \right)}{\left[x(r)^2 - 2 \frac{\left(H/F \right)}{G/F + H/F} x(r) + 1 \right]^{1/2}} = \frac{d\dot{u}_r}{dr} \tag{4.1}$$

$$\dot{\varepsilon}_\theta = \frac{\dot{\bar{\varepsilon}} \times \left[\left(1 + \frac{H}{F} \right) - \frac{H}{F} x(r) \right]}{\left[x^2 - 2 \frac{\left(H/F \right)}{G/F + H/F} x(r) + \frac{\left(1 + H/F \right)}{G/F + H/F} \right]^{1/2}} = \frac{\dot{u}_r}{r} \tag{4.2}$$

$$\dot{\varepsilon}_z = \frac{\dot{\bar{\varepsilon}}}{2\bar{\sigma}} \left\{ -G\sigma_r - F\sigma_\theta \right\} \tag{4.3}$$

The effective strain rate of disc is assumed, as described by following Sherby's law [1977],

$$\dot{\bar{\varepsilon}} = [M(\bar{\sigma} - \sigma_0)]^n \tag{4.4}$$

where $\dot{\bar{\varepsilon}}, M, \bar{\sigma}, \sigma_0, n$ are the effective strain rate, creep parameter, effective stress, threshold stress, the stress exponent.

Dividing Eq. (4.1) by Eq. (4.2) and integrating the resulting equation by taking limit a to r on both sides,

$$\frac{\dot{u}_r}{\dot{u}_{r_i}} = \exp\int_a^r \frac{\phi(r)}{r} dr \tag{4.5}$$

where, $x(r) = \frac{\sigma_r}{\sigma_\theta}$, is the ratio of radial and tangential stresses at any radius and $\dot{u}_r = du/dt$, is the radial deformation rate and $\dot{u}_{r_i}$, is the radial deformation rate at the inner radius.

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The equilibrium force in the radials direction is,

$$\frac{d}{dr}(r\ h\ \sigma_r) + \rho\omega^2 r^2 h = h\sigma_\theta \tag{4.6}$$

Boundary Conditions are $\sigma_r(a) = 0 = \sigma_r(b)$ (4.7)

Dividing Eq. (4.5) by r and equated to Eq. (4.2),

$$\frac{\bar{\sigma} - \sigma_0}{\psi(r)} = \frac{(\dot{u}_{r_i})^{1/n}}{M} \tag{4.8}$$

where,

$$\psi(r) = \left\{ \frac{\left[x(r)^2 - \frac{2\left(H/F\right)}{G/F + H/F} x(r) + \frac{1 + H/F}{G/F + H/F} \right]^{1/2}}{\frac{1 + H/F}{\sqrt{G/F + H/F}} - \frac{H/F}{\sqrt{G/F + H/F}}\, x(r)} \frac{\dot{u}_r}{r.\dot{u}_{r_i}} \right\}^{1n} \tag{4.9}$$

and

$$M(r) = e^{-35.38} P^{0.2077} T(r)^{4.98} V^{-0.622} \tag{4.10}$$

$$\sigma_0(r) = -0.03507P + 0.01057T(r) + 1.00536 - 2.11916 \tag{4.11}$$

Simplify Eq. (4.8) into Eq. (4.1) to get the tangential stress (σ_θ),

$$\sigma_\theta = \frac{\bar{\sigma} - \sigma_0}{\psi(r)} \psi_1(r) + \psi_2(r) \tag{4.12}$$

where,

$$\psi_1(r) = \frac{\psi(r)}{\left\{\left[x(r)^2 - 2\frac{H/F}{G/F + H/F}x(r) + \frac{1 + H/F}{G/F + H/F}\right]\right\}^{1/2}} \quad (4.13)$$

$$\psi_2(r) = \frac{\sigma_0}{\left\{\left[x(r)^2 - 2\frac{H/F}{G/F + H/F}x(r) + \frac{1 + H/F}{G/F + H/F}\right]\right\}^{1/2}} \quad (4.14)$$

$$\sigma_{\theta_{avg}} = \frac{1}{b-a}\int_a^b \sigma_\theta \, dr \quad (4.15)$$

where, $\sigma_{\theta_{avg}}$ is the average tangential stress.

Now radial stress can be expressed by integrating Eq. (4.6),

$$\sigma_r(r) = \frac{1}{r.h}\left[\int_a^r \sigma_\theta \, dr - \frac{\omega^2 \rho (r^3 - a^3)}{3}\right] \quad (4.16)$$

For a disc, the thermal conductivity can be calculated as,

$$K(r) = \frac{[100 - V(r)]K_m + V(r)K_d}{100} \quad (4.17)$$

Where $K_m = 247 W/m\,K$ is matrix conductivity and $K_d = 100 W/m\,K$ is diserpersoid conductivity.

$$T(r) = 619.69 + 0.6083\,r - 0.0208\,r^2 + 3.27\times10^{-4} r^3 - 1.96\times10^{-6} r^4 + 4.43\times10^{-9} r^5 \quad (4.18)$$

where $T(r)$ is the temperature taken from Gupta et al.[4].

RESULTS AND DISCUSSION

The study has been taken out for anisotropic FGM discs to analysis the consequence of thermal gradients on plastic stress distributions and strain rates and computer program has been made for this study. This study has been done for the anisotropic FGM discs containing silicon carbide whisker in a matrix of pure aluminum in presence of thermal gradients and obtained results are compared for both the discs in absence of thermal gradients to analyze the effect of thermal gradients. The distribution of tangential stresses in anisotropic whisker reinforced discs for presence and absence of thermal gradients has been shown in figure 1. An anisotropic disc with thermal gradients has little higher the tangential near the inner radius and slightly lowers near the outer radius in disc with absence of thermal gradients. The variation of radial in anisotropic whisker reinforced discs for presence and absence of thermal gradients has been shown in figure 2. The change in the magnitude of radial stress distribution is very small in the anisotropic disc with/without thermal gradients. The distribution of tangential strain rate in anisotropic rotating discs along the radius in the disc for presence and absence of thermal gradients has

Density of disc material $\rho = 2812.4\, kg/m^3$
Angular velocity, $\omega = 15{,}000\, rpm$
Inner radius of disc, $a = 31.75mm$
Outer radius of disc, $b = 152.4mm$
Particle size, $P = 1.7\, \mu m$
Particle content, $V = 20\%$
Operating temperature, $T = 623\, K$
Creep parameters for whisker reinforced disc: $m = 53.0 \times 10^{-4}\, s^{-1/8} / MPa$ and $\sigma_0 = 52.83$ MPa
The ratio of anisotropic constants, $G/F = 1.34$, $H/F = 1.64$

Table 1

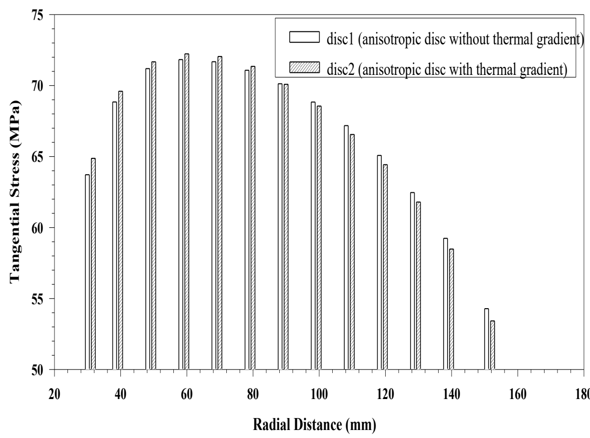

FIGURE 1. Variation of tangential stresses in anisotropic disc with/without thermal gradients.

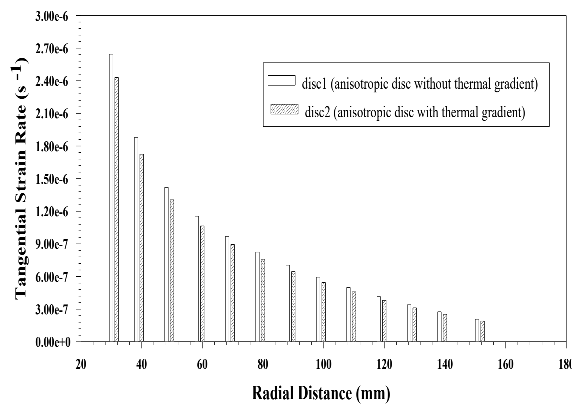

FIGURE 3. Variation of tangential strain rates in anisotropic disc with/without thermal gradients.

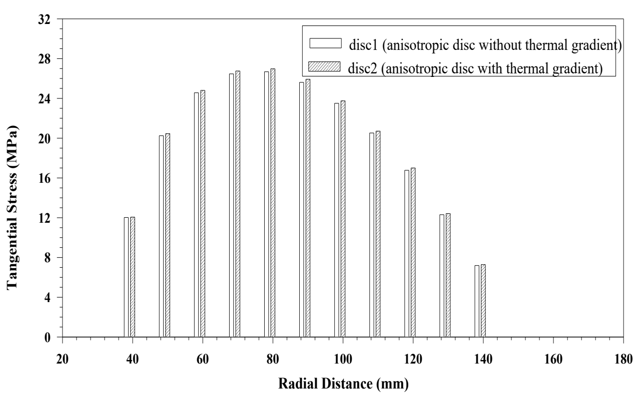

FIGURE 2. Variation of radial stresses in anisotropic disc with/without thermal gradients.

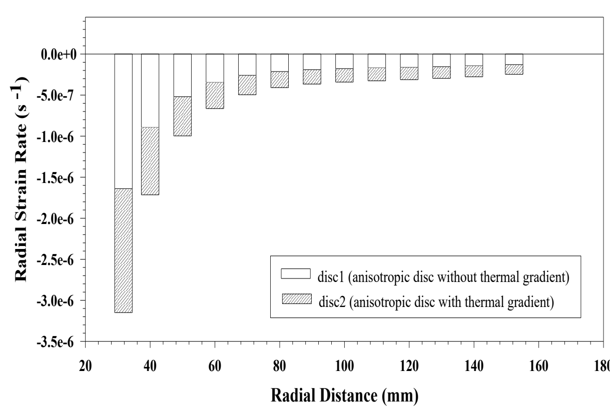

FIGURE 4. Variation of radial strain rates in anisotropic disc with/without thermal gradients.

been shown in figure 3. The magnitude due to thermal gradients is smaller compared to disc without thermal gradients. But, the trend of variation of tensile strain rate in tangential direction remains the same in the anisotropic discs with/without thermal gradients. In figure 4, the radial strain rate has been obtained in the anisotropic whisker reinforced discs with thermal gradients and the results have been compared with those obtained for an anisotropic whisker reinforced discs without thermal gradients. By employing thermal gradients in anisotropic discs, the magnitude of radial strain rate can be reduced as compared to anisotropic discs without thermal gradients.

CONCLUSION

The effect of thermal gradients significantly affects the creep behavior in an anisotropic whisker reinforced rotating disc although the effect of thermal gradients on stresses is relatively small. So, this aspect which is requiring for safe designing of a rotating disc should be taken care of.

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