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A Novel Approach to Sequencing Problems Using Type-2 Pentagonal Fuzzy Numbers and Centroid Defuzzification: Sequencing Problems Using Type-2 Pentagonal Fuzzy Numbers

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Abstract: Sequencing problems play a crucial role in determining the optimal order for executing a set of critical tasks, with the primary objective of minimizing the total time required to complete all the tasks. Traditionally, in classical job sequencing problems, it is assumed that the processing times for each task are known with certainty and are treated as fixed values. However, in real-world scenarios, it is often observed that the actual processing times are not precisely known in advance and tend to be uncertain or variable due to various unforeseen factors.

To address this uncertainty, fuzzy job sequencing models are employed, where the processing times are represented using fuzzy numbers to better capture the inherent vagueness and imprecision. In this study, a solution methodology is proposed for tackling fuzzy job sequencing problems, where Type-2 fuzzy numbers are specifically utilized to model the uncertain processing times more accurately.

A suitable defuzzification measure is applied to convert the fuzzy processing times into crisp (non-fuzzy) values, effectively transforming the original fuzzy sequencing problem into a standard, deterministic sequencing problem. Once defuzzified, the resulting crisp problem is solved using conventional sequencing algorithms to determine the optimal task sequence and the corresponding minimum completion time. This approach provides a more realistic and practical solution framework for job sequencing problems encountered in environments characterized by uncertainty and imprecision.

Keyword: Fuzzy sequencing problem, Fuzzy Processing time, Type-2 fuzzy number,

1. Introduction

Sequencing problems are a fundamental class of optimization problems concerned with determining the most efficient order in which a set of jobs or tasks should be performed, with the objective of minimizing total elapsed time, minimizing cost, or maximizing operational efficiency. Traditionally, sequencing models assume that the processing times of jobs are known precisely and remain constant. However, in practical scenarios such as manufacturing systems, service operations, and project management, job processing times are often uncertain due to unpredictable factors like machine breakdowns, human inefficiencies, supply chain delays, or environmental conditions. As a result, traditional deterministic models may not accurately capture the nature of real-world problems.

To effectively model such uncertainty, fuzzy set theory, first introduced by Zadeh [14], has been widely adopted. Fuzzy sets allow processing times to be described with degrees of membership rather than fixed values, better representing the vagueness and ambiguity inherent in real-life operations. Following Zadeh's foundational work, researchers like Zimmermann [15] have further expanded the application of fuzzy set theory to decision-making and optimization problems, laying the groundwork for fuzzy sequencing models.

In fuzzy sequencing problems, the job processing times are modeled using fuzzy numbers rather than crisp values. Different types of fuzzy numbers such as triangular, trapezoidal, pentagonal, hexagonal, and octagonal have been explored to represent processing times in more nuanced ways. For instance, Dr. S.U. Malini and S. Kalaivani [1] utilized octagonal fuzzy numbers to solve sequencing problems, improving flexibility in modeling uncertain processing times. Similarly, K. Selvakumari and S. Santhi [4] applied robust ranking techniques combined with octagonal fuzzy numbers to derive optimal job sequences.

Building upon this, M. Ananthanarayanan and R. Mahalakshmi [6] introduced the use of Simpson’s 1/3 Rule for defuzzification, offering a numerical integration approach to handle octagonal fuzzy processing times effectively. Helen and Uma [2] worked on pentagonal fuzzy numbers and proposed new ranking operations that further enhanced the reliability of fuzzy sequencing models. Likewise, M. Priya and Dr. P. Elumalai [10] developed an efficient ranking technique for sequencing problems modeled with pentagonal fuzzy numbers.

The characterization and application of different fuzzy numbers were systematized by Pathinathan et al. [11, 12], who outlined various properties and operations on fuzzy numbers such as pentagonal forms. Their contributions provided a stronger theoretical base for fuzzy scheduling models.

Moreover, fuzzy sequencing problems have been explored beyond simple models. Laxminarayan Sahoo [5] and M. Shanmugasundari [9] proposed novel defuzzification and ranking methods, aiming to bridge the gap between theoretical fuzzy models and real-world applications. Addressing more complex manufacturing setups, Jadhav and Jadhav [13] solved flow-shop scheduling problems using fuzzy approaches to minimize total elapsed time, demonstrating the versatility of fuzzy models in diverse operational environments.

A major development in fuzzy theory has been the introduction of Type-2 fuzzy sets, which handle higher levels of uncertainty compared to traditional (Type-1) fuzzy sets.

Mendel and John [7], followed by Mendel, John, and Liu [8], simplified and popularized Type-2 fuzzy logic systems, paving the way for their application in complex sequencing problems with deep-rooted uncertainties.

In addition to these advancements, recent studies have explored new fuzzy number types and hybrid solution approaches to further improve sequencing models under uncertainty. R. Karthikeyan and M. Sundar [16] examined job sequencing problems using triangular fuzzy numbers, while V. Kumari and P. Meenakshi [24] employed trapezoidal fuzzy numbers to capture processing time variability more flexibly.

Similarly, S. Rajeswari and P. Saranya [17] introduced the use of hexagonal fuzzy numbers, suggesting that the additional vertices of the hexagon enable a richer and more detailed representation of uncertainty. V. Shobana and R. Dhanasekaran [19] applied pentagonal fuzzy numbers with new defuzzification strategies to obtain more accurate sequencing solutions.

Effective ranking and defuzzification techniques remain crucial in transforming fuzzy sequencing problems into solvable crisp forms. P. Kalyani and J. Deepa [18] proposed an optimization of sequencing problems using a ranking index method, while M. Bharathi and G. Shunmugapriya [25] developed new ranking techniques specifically designed for fuzzy processing times.

Advanced approaches like the application of Interval Type-2 fuzzy sets, as demonstrated by G. Kannan and P. Mathivanan [22], and the integration of fuzzy methods with computational intelligence techniques such as genetic algorithms (as proposed by T. Deepak and R. Devi [23]) have expanded the methodological landscape for solving fuzzy sequencing problems.

Furthermore, multi-criteria decision-making frameworks under fuzzy environments, discussed by D. Pradeep and M. Arthi [21], have provided methods to optimize not just a single objective but multiple goals simultaneously. Comparative studies such as the one conducted by C. Manikandan and S. Manonmani [20] have demonstrated that the choice of fuzzy environment, ranking method, and defuzzification technique significantly influences the sequencing outcomes.

In summary, the integration of fuzzy set theory into sequencing and scheduling problems has significantly enhanced the modeling of uncertainty in real-world applications. Various fuzzy numbers — triangular, trapezoidal, pentagonal, hexagonal, and octagonal — along with sophisticated defuzzification measures, ranking methods, and hybrid optimization approaches have enriched the field. The evolution from Type-1 to Type-2 fuzzy models further extends the ability to manage deeper levels of uncertainty.

The present study aims to contribute to this rich body of work by proposing a novel solution methodology for fuzzy job sequencing problems. Specifically, the study employs **Type-2 fuzzy numbers** to represent processing times, applies a **defuzzification measure** to transform the fuzzy model into a crisp one, and solves the resulting crisp problem using **standard sequencing algorithms**. This integrated approach is intended to derive an optimal sequence and minimize the total completion time, offering a robust solution framework for uncertain scheduling environments.

2 Basic Concepts

2.1 Membership Function

X is a collection of objects denoted generally by X , then a fuzzy set A in X is defined as a set of ordered pairs $\widetilde{A} = \{x, \mu_{\widetilde{A}}(x); x \in X\}$ where $\mu_{\widetilde{A}}(x)$ is called the membership function maps each elements of X to a membership grades(or membership value) between 0 and 1.

2.2 Fuzzy Number

A fuzzy number $\widetilde{A}$ is a fuzzy set on the real line R, must satisfy the following conditions.

(i) $\mu_{\widetilde{A}}(x_0)$ is piecewise continuous.

(ii) There exist at least one $x_0 \in R$ with $\mu_{\widetilde{A}}(x_0) = 1$

(iii) A must be normal and convex

2.3 Pentagonal Fuzzy Number

A fuzzy set $\widetilde{A} = \{a, b, c, d, e\}$ where a , b ,c ,d , e are real numbers is said to be a pentagonal Fuzzy Number ifits membership function is given below

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0 & for\ x \prec a \\ \frac{1}{2}\left(\frac{x-a}{b-a}\right) & for\ a \le x \le b \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-b}{c-b}\right) & for\ b \le x \le c \\ 1 & for\ x = c \\ \frac{1}{2} + \frac{1}{2}\left(\frac{d-x}{d-c}\right) & for\ c \le x \le d \\ \frac{1}{2}\left(\frac{e-x}{e-d}\right) & for\ d \le x \le e \\ 0 & for\ x \succ e \end{cases}$$

2.4 Sequencing Problem

The total effectiveness, which may be the time or cost that is to be minimized is the function of the order of sequence. Such type of problem is known as Sequencing Problem.

2.5 Processing Order

Processing order: It refers to the order (sequence) in which the machines are required for completing the job.

2.6 Processing Time

Processing time: It indicates the time required by a job on each machine.

2.7Total Elapsed Time

Total elapsed time: It is the time interval between starting the first job and completing the last Job including the idle time (if any) in a particular order by the given set of machines.

2.8 Idle Time

ldle time on a machine: It is the time for which a machine does not have a job to process. i.e., idle time from the end of job (i - 1) to the start of job i.

2.9 No Passing Rule

No passing rule : It refers to the rule of maintaining the order in which jobs are to be processed on given machines.

2.10 Centroid Measure

Let A be a **pentagonal fuzzy number** represented by five distinct parameters corresponding to its membership function.

The **centroid measure** of a pentagonal fuzzy number A is defined as the arithmetic average of its defining points.

If $A=(a_1,a_2,a_3,a_4,a_5)$ represents the pentagonal fuzzy number, then its centroid measure C(A) is given by:

$$\text{Centroid measure}=\left(\frac{a+b+5c+d+e}{9}\right)$$

3Algorithm:

Step 1: Consider the fuzzy sequencing problem with the Processing time as type-2 pentagonal fuzzy number

Step 2: Using centroid measure the type-2 pentagonal fuzzy processing time is now converted into pentagonal fuzzy number problem .

Step 3: Againusing centroid measure the pentagonal fuzzy processing time is now converted into a crisp valued problem.

Step 4: The optimal sequence for the crisp sequence problem is determined using crisp sequencing problem.

Step 5: After finding the optimal sequence,determine the total elapsed fuzzy time and also the fuzzy ideal time on each machine.

4 Numerical Example

Example 4.1.

Consider the fuzzy sequencing problem. Here the processing time $\widetilde{t}_{ij}$ of five jobs is given whose elements are fuzzy quantifiers. Then the problem is solved by processing n jobs through two machines

Step 1

Table 1. Processing time in Type-2 pentagonal Fuzzy Numbers

Job sequence	Machine I	Machine II
A	((0,1,5,6,7, (1,3,4,7,9), (3,5,8,10,11), (7,9,11,12,15),(13,18,21,25,27))	((1,3,7,9,10),(2,3,6,11,15),(13,14,19,20,21),(19,22,28,31,32),(20,23,27,30,38))
B	((1,3,4,8,10),(3,6,8,11,13),(8,9,10,11,13),(12,14,15,16,17),(15,16,18,20,21))	((1,3,5,7,9),(2,4,5,6,10),(4,6,7,10,12),(3,7,8,11,15),(16,18,19,23,24))
C	((1,5,7,14,16),(11,12,14,17,19),(20,21,22,24,26),(27,28,29,30,31),(29,32,34,35,37))	((5,7,9,11,13),(12,14,15,20,21),(18,20,22,23,25),(20,22,26,28,30),(24,28,30,31,32))
D	((5,6,7,8,9),(3,4,8,9,10),(8,10,12,14,15),(7,11,12,15,17),(16,17,18,20,21))	((1,2,4,6,8),(7,8,11,15,16),(12,14,15,19,20),(20,22,23,25,27),(21,23,26,28,30))
E	((0,3,5,8,10),(5,8,10,13,15),(14,15,17,19,20),(18,22,23,25,26),(27,28,31,32,34))	((3,4,8,10,12),(7,10,15,16,20),(18,22,24,27,30),(28,30,33,35,36),(30,34,37,39,40))

Step 2

Using Centroid measure=$\left(\frac{a+b+5c+d+e}{9}\right)$ the type-2 pentagonal fuzzy number is now converted into pentagonal fuzzy number.

Table 2. Processing time in pentagonal Fuzzy Numbers

Jobs	Machine I	Machine II
A	(4.3,4.4,7.7,10.8,20.8)	(6.4,6.8,18.1,27.1,27.3)
B	(4.7,8.1,10.1,14.9,18)	(5.0,5.2,7.4,8.4,19.6)
C	(7.8,14.3,22.3,29.0,33.7)	(9.0,15.8,21.8,25.6,29.4)
D	(7.0,7.3,11.9,12.2,18.2)	(4.1,11.2,15.6,23.2,25.8)
E	(5.1,10.1,17.0,22.9,30.7)	(7.7,14.2,24.1,32.7,36.4)

Step 3

Again using Centroid measure= $\left(\dfrac{a+b+5c+d+e}{9}\right)$ the pentagonal fuzzy number is now converted into crisp value.

Table 3. Processing time in crisp Numbers

Jobs	Machine I	Machine II
A	8.8	17.6
B	10.7	8.4
C	21.8	20.1
D	11.6	15.8
E	17.1	23.5

Step 4 The optimum job sequence is obtained as follows

Table 6. Optimum job sequence

A	D	E	C	B

Step 5

Total elapsed time, Idle time for each machines are calculated and given below

Table 7. Total Elapsed time and Idle time for Type-2 pentagonal Fuzzy Numbers

	Machine I		Machine II	
	Time In	Time Out	Time In	Time Out
A	0	8.8	8.8	26.4
B	8.8	20.4	26.4	42.2
C	20.4	37.5	42.2	65.7
D	37.5	59.3	65.7	85.8
E	59.3	70.0	85.8	94.2

Total Elapsed Time = 94.2 Hrs Idle Time for Machine I = 24.2 Hrs

Idle Time for Machine II = 8.8 Hrs

Thus the type -2 pentagonal job sequencing problem is solved after defuzzification and an optimum order is reached, and the total elapsed time and ideal time for each machine is calculated.

4. Conclusion

In this study, a novel solution methodology was proposed for addressing fuzzy job sequencing problems under uncertainty. Specifically, a new measure technique was introduced to defuzzify Type-2 pentagonal fuzzy numbers, which were used to model the uncertain processing times of the jobs. Recognizing that real-world environments are often characterized by vagueness and ambiguity, the proposed approach offers a more realistic and practical modeling framework compared to traditional deterministic or Type-1 fuzzy models.

The methodology first involved formulating the sequencing problem wherein the processing times were represented as Type-2 pentagonal fuzzy numbers, capable of capturing a broader range of uncertainties. To transition from the fuzzy domain to a solvable crisp form, a centroid-based defuzzification measure was applied. This step systematically transformed the fuzzy processing times into precise numerical values, resulting in a crisp sequencing problem that retained the essential characteristics of the original uncertainty model.

Subsequently, standard sequencing algorithms were employed to solve the crisp problem, thereby determining the optimal job sequence and calculating the idle times for each machine. This two-stage approach — defuzzification followed by classical optimization — enabled the derivation of practical, implementable schedules even in highly uncertain operating environments.

To validate the effectiveness and applicability of the proposed method, a numerical example was thoroughly solved and the results were presented. The example demonstrated how the developed centroid measure could accurately translate fuzzy processing information into actionable scheduling decisions. The final results showed that the proposed method is capable of producing optimal sequences that minimize total elapsed time while efficiently managing machine idle times.

The contributions of this work are significant in several respects. First, by utilizing Type-2 fuzzy numbers, it acknowledges and effectively manages higher levels of uncertainty, a feature that is often overlooked in conventional scheduling models. Second, the newly developed centroid measure provides a robust and computationally efficient means of defuzzification. Third, the method is both simple enough for practical application and powerful enough to handle complex fuzzy sequencing problems.

In conclusion, this research bridges the gap between theoretical fuzzy set advancements and real-world scheduling needs. The proposed framework offers a flexible and reliable tool for decision-makers operating in environments fraught with uncertainty. Future research could extend this work by incorporating multi-machine, multi-objective sequencing environments, or by integrating heuristic or metaheuristic algorithms to address even larger and more complex fuzzy scheduling problems.

5. References

[1] Dr. S.U. Malini and S. Kalaivani, "Solving fuzzy sequencing problem using octagonal fuzzy number," *International Journal of Mathematical Archive*, vol. 9, no. 9, pp. 109–114, 2018.

[2] R. Helen and G. Uma, "A New Operation and Ranking on Pentagon Fuzzy Number," *International Journal of Mathematical Sciences and Applications*, vol. 5, no. 2, pp. 341–346, 2015.

[4] K. Selvakumari and S. Santhi, "An Approach for Solving Fuzzy Sequencing Problems with Octagonal Fuzzy Numbers using Robust Ranking Techniques," *International Journal of Mathematics Trends and Technology (IJMIT)*, vol. 56, no. 3, 2018.

[5] Laxminarayan Sahoo, "Solving Job Sequencing Problems with Fuzzy Processing Times," *IJARIIE*, vol. 3, issue 4, 2017.

[6] M. Ananthanarayanan and R. Mahalakshmi, "An Application of Solving Fuzzy Sequencing Problem with Octagonal Fuzzy Numbers using Simpson’s 1/3 Rule," *Advances and Applications in Mathematical Sciences*, vol. 21, issue 8, pp. 4399–4407, 2022.

[7] J.M. Mendel and R.I.B. John, "Type-2 Fuzzy Sets Made Simple," *IEEE Transactions on Fuzzy Systems*, vol. 10, no. 2, pp. 117–127, 2002.

[8] J.M. Mendel, R.I.B. John, and F. Liu, "Interval Type-2 Fuzzy Logic Systems Made Simple," *IEEE Transactions on Fuzzy Systems*, vol. 14, no. 6, pp. 808–821, 2006.

[9] M. Shanmugasundari, "Fuzzy Sequencing Problems: A Novel Approach," *International Journal of Pure and Applied Mathematics*, vol. 113, no. 13, pp. 56–64, 2017.

[10] M. Priya and Dr. P. Elumalai, "Fuzzy Sequencing Problems with Pentagonal Fuzzy Numbers Using Ranking Technique," *Journal of Emerging Technologies and Innovative Research (JETIR)*, vol. 8, issue 8, pp. 545–551, 2021.

[13] V.S. Jadhav and O.S. Jadhav, "Solving flow-shop scheduling problem to minimize total elapsed time using fuzzy approach," *International Journal of Statistics and Applied Mathematics*, vol. 4, no. 5, pp. 130–133, 2019.

[14] L.A. Zadeh, "Fuzzy Sets," *Information and Control*, vol. 8, pp. 338–353, 1965.

[15] H.J. Zimmermann, *Fuzzy Set Theory and Its Applications*, 4th ed., Kluwer Academic Publishers, Dordrecht, 1996.

[16] R. Karthikeyan and M. Sundar, "A Study on Job Sequencing Problem with Triangular Fuzzy Numbers," *International Journal of Engineering and Advanced Technology (IJEAT)*, vol. 8, no. 6, pp. 102–107, 2019.

[17] S. Rajeswari and P. Saranya, "Solving Fuzzy Sequencing Problems Using Hexagonal Fuzzy Numbers," *International Journal of Scientific Research in Mathematical and Statistical Sciences*, vol. 7, issue 2, pp. 45–51, 2020.

[18] P. Kalyani and J. Deepa, "Optimization of Fuzzy Job Sequencing Problems Using Ranking Index Method," *Mathematical Modelling and Engineering Problems*, vol. 9, no. 1, pp. 27–34, 2021.

[19] V. Shobana and R. Dhanasekaran, "Fuzzy Scheduling with Pentagonal Numbers Using New Defuzzification Measures," *International Journal of Applied Engineering Research*, vol. 12, no. 19, pp. 8659–8665, 2017.

[20] C. Manikandan and S. Manonmani, "A Comparative Analysis of Job Sequencing Problems under Fuzzy Environment," *Journal of Computational and Theoretical Nanoscience*, vol. 14, no. 6, pp. 2790–2795, 2017.

[21] D. Pradeep and M. Arthi, "Multi-Criteria Decision Making in Fuzzy Job Scheduling Problems," *International Journal of Research and Analytical Reviews*, vol. 6, no. 2, pp. 230–238, 2019.

[22] G. Kannan and P. Mathivanan, "Application of Interval Type-2 Fuzzy Sets in Sequencing Problems," *International Journal of Mathematical Archive*, vol. 10, no. 11, pp. 1–5, 2019.

[23] T. Deepak and R. Devi, "Hybrid Approach for Solving Fuzzy Sequencing Problems Using Genetic Algorithms," *Journal of Applied Mathematics and Physics*, vol. 8, no. 7, pp. 1236–1245, 2020.

[24] V. Kumari and P. Meenakshi, "Solving Job Sequencing Problem with Trapezoidal Fuzzy Numbers," *International Journal of Innovative Technology and Exploring Engineering*, vol. 9, no. 2, pp. 110–115, 2020.

[25] M. Bharathi and G. Shunmugapriya, "New Ranking Techniques for Fuzzy Processing Time in Sequencing Problems," *Asian Journal of Research in Mathematics*, vol. 14, no. 4, pp. 34–41, 2021.