SKEW- SEMI DERIVATION ON SEMI-PRIME RINGS

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Abstract: For this study, Π represents a semi-prime ring and we introduce a new class of derivations that generalizes skew derivation and semi - derivation, we call it " skew-semi derivation". By assuming that Ώ is a skew-semi derivation with an automorphism α:Π→Π associated with it. In this paper, we prove some results on skew - semi derivation for semi-prime ring.

Keywords: skew-derivation, semi-derivation, semi-prime rings, automorphism.

1. Introduction.

In 1957, Posner [4] introduced the notion for a ring Π . An additive mapping h: Π→ Π is said to be derivation if h(ă ƅ) = h(ă) ƅ + ă h(ƅ) for all ă, ƅ ϵ Π .

In 1983, Bergen [5] introduced the concept of semi-derivations as an extension of derivations of a ring Π. Recall that an additive map d:Π→ Π is said to be a semi-derivation if d(ă ƅ)=d(ă)g(ƅ)+ ă d(ƅ)=d(ă) ƅ + g(ă)d(ƅ) and d(g(ă)) = g(d(ă)) where ă, ƅ ϵ Π and g is an arbitrary map on Π .

In 1985, Leory [6] introduced the notion of skew-derivation over a ring and further studied it over skew fields and prime rings. An additive map δ:Π→ Π is said to be a skew-derivation if δ (ă ƅ)= δ (ă) ƅ + α(ă) δ (ƅ) for all ă, ƅ ϵ Π , where α is an automorphism on Π .

In 2023, Funmilola et al. [11] proved the results for skew-derivation with semi-prime rings by taking the following conditions.

(i) δ (ă) δ (ƅ) + ă ƅ = 0, (ii) δ (ă) δ (ƅ) - ă ƅ = 0, for all ă, ƅ ϵ Π.

we generalized these results for skew semi - derivations.

2. Preliminaries.

We shall use the following basic commutator identities.

The symbol [ă,ƅ] and ă o ƅ , represents the Lie product ăƅ - ƅă and the Jordan product ăƅ + ƅă, respectively, where ă,ƅ are elements of a ring Π .

$$[ă,ƅc] = ƅ[ă,c] + [ă,ƅ]c ,$$
$$[ăƅ,c] = [ă,c]ƅ + ă[ƅ,c] ,$$
$$ă \circ (ƅc) = (ă \circ ƅ)c - ƅ [ă,c] = ƅ(ă \circ c) + [ă,ƅ] c ,$$
$$(ă ƅ) \circ c = ă(ƅ \circ c) - [ă,c]ƅ = (ă \circ c)ƅ + ă[ƅ,c].$$

Definition 2.1., If Ώ(ă ƅ) = Ώ(ă)g(ƅ) + α(ă)Ώ(ƅ) = Ώ(ă)α(ƅ) + g(ă)Ώ(ƅ) and Ώ(g(ă)) = g(Ώ(ă)), where ă, ƅ ϵ Π, α is an automorphism and g is a map on Π, then an additive map Ώ:Π → Π is a skew-semi derivation.

3. Main Results

Theorem 3.1. Suppose Π is a semi-prime ring and Ώ is a skew-semi derivation and g is an identity map. If Ώ(ă)Ώ(ƅ) + ăƅ = 0, or Ώ(ă)Ώ(ƅ) - ăƅ = 0, for all ă, ƅ ϵ Π, then the skew - semi derivation is zero.

Proof. we have,

$$Ώ(ă)Ώ(ƅ) + ăƅ = 0, \text{ for all } ă,ƅ \in Π. \quad (3.1)$$

Replacing ƅ by ƅc in equation (3.1), we get

$$Ώ(ă)Ώ(ƅc) + ăƅc = 0, \text{ for all } ă,ƅ,c \in Π.$$
$$Ώ(ă)(Ώ(ƅ)g(c) + α(ƅ)Ώ(c)) + ăƅc = 0, \text{ for all } ă,ƅ,c \in Π.$$
$$Ώ(ă)Ώ(ƅ)g(c) + Ώ(ă)α(ƅ)Ώ(c)) + ăƅc = 0, \text{ for all } ă,ƅ,c \in Π. \quad (3.2)$$

Using equation (3.1), becomes

$$- ăƅg(c) + Ώ(ă)α(ƅ)Ώ(c) + ăƅc = 0, \text{ for all } ă,ƅ,c \in Π. \quad (3.3)$$

Replacing g(c) by c in equation (3.3), we get

$$- ăƅc + Ώ(ă)α(ƅ)Ώ(c) + ăƅc = 0, \text{ for all } ă,ƅ,c \in Π.$$
$$Ώ(ă)α(ƅ)Ώ(c) = 0, \text{ for all } ă,ƅ,c \in Π. \quad (3.4)$$

Replacing ă by c in equation (3.4), we get

$$Ώ(c)α(ƅ)Ώ(c) = 0, \text{ for all } ƅ, c \in Π.$$

Since α is an automorphism, we have α^{-1}(Ώ(c))Ѣ α^{-1}(Ώ(c)) = 0, for all Ѣ,c ∊ Π.

Since Π is a semi-prime ring we have α^{-1}(Ώ(c)) = 0, for all c ∊ Π.

i.e., Ώ(c) = 0, Hence Proved the result.

Theorem 3.2. Suppose Π is a semi-prime ring and Ώ is a skew - semi derivation and g is an identity map. If Ώ(Ѣ)Ώ(ă) + Ѣă = 0 , or Ώ(Ѣ)Ώ(ă) - Ѣă = 0, for all ă,Ѣ ∊ Π then the skew - semi derivation is zero.

Proof. we have,

Ώ(Ѣ)Ώ(ă) + Ѣă = 0, for all ă,Ѣ ∊ Π. (3.5)

Replacing ă by ăc in equation (3.5), we get

Ώ(Ѣ)Ώ(ăc) + Ѣăc = 0, for all ă, Ѣ, c ∊ Π.

Ώ(Ѣ)(Ώ(ă)g(c) + α(ă)Ώ(c)) + Ѣăc = 0, for all ă, Ѣ, c ∊ Π.

Ώ(Ѣ)Ώ(ă)g(c) + Ώ(Ѣ)α(ă)Ώ(c)) + Ѣăc = 0, for all ă, Ѣ, c ∊ Π. (3.6)

Using equation (3.5), becomes

- Ѣăg(c) + Ώ(Ѣ)α(ă)Ώ(c) + Ѣăc = 0, for all ă,Ѣ,c ∊ Π. (3.7)

Replacing g(c) by c in equation (3.7), we get

- Ѣăc + Ώ(Ѣ)α(ă)Ώ(c) + Ѣăc = 0, for all ă,Ѣ,c ∊ Π.

Ώ(Ѣ)α(ă)Ώ(c) = 0, for all ă,Ѣ,c ∊ Π. (3.8)

Replacing Ѣ Ѣy c in equation (3.8), we get

Ώ(c)α(ă)Ώ(c) = 0, for all ă, c ∊ Π.

Since α is an automorphism, we have α^{-1}(Ώ(c))Π α^{-1}(Ώ(c)) = 0, for all c ∊ Π.

Since Π is a semi-prime ring we have α^{-1}(Ώ(c) = 0, for all c ∊ Π.

i.e., Ώ(c) =0, Hence proved the result.

Theorem 3.3. Suppose Π is a semi prime ring and Ώ is a skew - semi derivation and g is an identity map. If Ώ(ăѢ) + ă o Ѣ = 0 , or Ώ(ăѢ) - ă o Ѣ = 0, for all ă,Ѣ ∊ Π, then the skew - semi derivation is zero.

Proof. we have,

Ώ(ăѢ) + a o Ѣ = 0, for all ă,Ѣ ∊ Π. (3.9)

Replacing ă by ăѢ in equation (3.9), we get

Ώ(ăѢ)Ѣ + ăѢ o Ѣ = 0, for all ă,Ѣ ∊ Π.

Ώ(ăѢ)g(Ѣ) + α(ăѢ)Ώ(Ѣ) + (ă o Ѣ)Ѣ + ă[Ѣ,Ѣ] = 0, for all ă, Ѣ ∊ Π.

Ώ(ăѢ)g(Ѣ) + α(ăѢ)Ώ(Ѣ)) + (ă o Ѣ)Ѣ = 0, for all ă, Ѣ ∊ Π. (3.10)

Replacing $g(ƅ)$ by $ƅ$ in equation (3.10), we get

$$\acute{\Omega}(ăƅ)ƅ + \alpha(ăƅ)\acute{\Omega}(ƅ)) + (ă \circ ƅ)ƅ = 0, \text{ for all } ă,ƅ \in \Pi.$$

$$(\acute{\Omega}(ăƅ) + (ă \circ ƅ))ƅ + \alpha(ăƅ)\acute{\Omega}(ƅ) = 0, \text{ for all } ă,ƅ \in \Pi. \quad (3.11)$$

Using equation (3.9) in equation (3.11), we get

$$\alpha(ăƅ)\acute{\Omega}(ƅ)) = 0, \text{ for all } ă,ƅ \in \Pi.$$

$$\alpha(ă)\alpha(ƅ)\acute{\Omega}(ƅ) = 0, \text{ for all } ă,ƅ \in \Pi. \quad (3.12)$$

Left Multiplication $\alpha(ƅ)\acute{\Omega}(ƅ)$ in equation (3.12), we get

$$\alpha(ƅ)\acute{\Omega}(ƅ)\alpha(ă)\alpha(ƅ)\acute{\Omega}(ƅ) = 0, \text{ for all } ƅ \in \Pi.$$

Since α is an automorphism, we have

$$\alpha^{-1}(\alpha(ƅ)\acute{\Omega}(ƅ))ă\, \alpha^{-1}(\alpha(ƅ)\acute{\Omega}(ƅ)), \text{ for all } ƅ \in \Pi.$$

Since Π is semi-prime ring we have $\alpha^{-1}(\alpha(ƅ)\acute{\Omega}(ƅ)) = 0$, for all $ƅ \in \Pi$.

$$\alpha(ƅ)\acute{\Omega}(ƅ) = 0, \text{ for all } ƅ \in \Pi. \quad (3.13)$$

Again Left multiplication $\acute{\Omega}(ƅ)$ in equation (3.13), we get

$$\acute{\Omega}(ƅ)\alpha(ƅ)\acute{\Omega}(ƅ) = 0, \text{ for all } ƅ \in \Pi.$$

Since α is an automorphism, we have

$$\alpha^{-1}(\acute{\Omega}(ƅ))ƅ\, \alpha^{-1}(\acute{\Omega}(ƅ)) = 0, \text{ for all } ƅ \in \Pi.$$

$$\alpha^{-1}(\acute{\Omega}(ƅ))\Pi\, \alpha^{-1}(\acute{\Omega}(ƅ)) = 0, \text{ for all } ƅ \in \Pi.$$

Since Π is semi-prime ring we have, $\alpha^{-1}(\acute{\Omega}(ƅ)) = 0$, for all $ƅ \in \Pi$.

i.e., $\acute{\Omega}(ƅ) = 0$. Hence proved the result.

Theorem 3.4. Suppose Π is a semi-prime ring and $\acute{\Omega}$ is a skew-semi derivation and g is an identity map. If $\acute{\Omega}(ăƅ) + [ă, ƅ] = 0$, or $\acute{\Omega}(ăƅ) - [ă,ƅ] = 0$, for all $ă, ƅ \in \Pi$, then the skew - semi derivation is zero.

Proof. we have,

$$\acute{\Omega}(ăƅ) + [ă,ƅ] = 0, \text{ for all } ă, ƅ \in \Pi. \quad (3.14)$$

Replacing $ă$ by $ăƅ$ in equation (3.14), we get

$$\acute{\Omega}(ăƅ)ƅ + [ăƅ,ƅ] = 0, \text{ for all } ă, ƅ \in \Pi.$$

$$\acute{\Omega}(ăƅ)g(ƅ) + \alpha(ăƅ)\acute{\Omega}(ƅ) + [ă,ƅ]ƅ + ă[ƅ,ƅ] = 0, \text{ for all } ă, ƅ \in \Pi.$$

$$\acute{\Omega}(ăƅ)g(ƅ) + \alpha(ăƅ)\acute{\Omega}(ƅ)) + [ă,ƅ]ƅ = 0, \text{ for all } ă, ƅ \in \Pi. \quad (3.15)$$

Replacing $g(ƅ)$ by $ƅ$ in equation (3.15), we get

$$\acute{\Omega}(ăƅ)ƅ + \alpha(ăƅ)\acute{\Omega}(ƅ)) + [ă,ƅ]ƅ = 0, \text{ for all } ă,ƅ \in \Pi.$$

$('\Omega(ăƅ) + [ă,ƅ])ƅ + \alpha(ăƅ)'\Omega(ƅ) = 0$, for all $ă,ƅ \in \Pi$. (3.16)

Using equation (3.14) in equation (3.16), we get

$\alpha(ăƅ)'\Omega(ƅ) = 0$, for all $ă,ƅ \in \Pi$.

$\alpha(ă)\alpha(ƅ)'\Omega(ƅ) = 0$, for all $ă,ƅ \in \Pi$. (3.17)

Left Multiplication $\alpha(ƅ)'\Omega(ƅ)$ in equation (3.17), we get

$\alpha(ƅ)'\Omega(ƅ)\alpha(ă)\alpha(ƅ)'\Omega(ƅ) = 0$, for all $ă,ƅ \in \Pi$.

Since α is an automorphism, we have

$\alpha^{-1}(\alpha(ƅ)'\Omega(ƅ))ă\,\alpha^{-1}(\alpha(ƅ)'\Omega(ƅ))$, for all $ă,ƅ \in \Pi$.

Since Π is semi-prime ring we have $\alpha^{-1}(\alpha(ƅ)'\Omega(ƅ)) = 0$, for all $ƅ \in \Pi$.

$\alpha(ƅ)'\Omega(ƅ) = 0$, for all $ƅ \in \Pi$. (3.18)

Again Left multiplication $'\Omega(ƅ)$ in equation (3.18), we get

$'\Omega(ƅ)\alpha(ƅ)'\Omega(ƅ) = 0$, for all $ƅ \in \Pi$.

Since α is an automorphism, we have

$\alpha^{-1}('\Omega(ƅ))ƅ\,\alpha^{-1}('\Omega(ƅ)) = 0$, for all $ƅ \in \Pi$.

$\alpha^{-1}('\Omega(ƅ))\Pi\,\alpha^{-1}('\Omega(ƅ)) = 0$, for all $ƅ \in \Pi$.

Since Π is semi-prime ring we have $\alpha^{-1}('\Omega(ƅ)) = 0$, for all $ƅ \in \Pi$.

i.e., $'\Omega(ƅ) = 0$. Hence proved the result.

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