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TWO BI-IDEAL IN TERNARY SEMIRING

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Abstract

This paper describes two bi-ideals in Ternary semiring. Also three 2 bi-ideals are 2 bi-ideal of Ternary semiring. It is clear that every one bi-ideal is a two bi-ideal, but the converse is not necessarily true.

Keywords:
Ternary semiring [TS],Ternary subsemiring[TSS], bi-ideal, 2 bi-ideal, Idempotent, Band

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1 Introduction

The first use of ternary algebra was in 1932, when Lehmer introduced it. Lister developed the notion of ternary rings and triplexes in 1971.Kar investigated several of their characteristics as well as the radical concept of such rings[2].The extension of quasi-ideals are bi ideals, according to biideal thinking in ternary semiring[4].The primary goal of this article is to learn more about 2 bi-ideal concepts in TS.

The study of ternary algebraic structures such as TS and ternary semigroups has gained considerable attention due to their rich algebraic properties and potential generalizations of classical binary operations. Initiated by foundational works like those of Lehmer and Lister, the exploration of ternary operations has led to the development of various structural aspects including regularity, ideal theory, and radical theory.

In particular, the notion of ideals in ternary semirings has been extensively investigated, with significant contributions by Dutta, Kar, and others who explored quasi-ideals, bi-ideals, prime and semiprime ideals, and their roles in defining the internal structure of TS.

The concepts of regularity and semiprimality, in this context, serve as vital tools in classifying and analyzing these structures. Further generalizations, such as the introduction of $(m, (p, q), n)$-quasi-ideals, and studies on right TNR and ternary N-groups, have expanded the theoretical framework, highlighting the versatility and depth of ternary algebraic systems.

This paper aims to build upon these foundational studies by 2 bi ideal in TS, thereby contributing to the ongoing development of TS theory.

2 Preliminaries

Definition 2.1. A TSS of B is known as 2 bi-ideal of f, if $Bf^2Bf^2B \subseteq B$.

Example 2.2. Let Z^- is a TS.If take subsemiring $B = 3z^-$.
Therefore B is 2 bi-ideal of f.

Definition 2.3. A TS is sub idempotent, if f satisfies the identity $a + aba = b$; $\forall a, b \in f$.

3 Main Results

In this section discussed about the some results in 2 bi-ideal in TS.

Theorem 3.1. *If* B *be a TSS, then* B *is a 2 bi-ideal of TSS.*

Proof. Let B be a TSS.
verify B is a 2 bi-ideal of TSS.
Thus

$$\begin{aligned} Bf^2Bf^2B &= B(f^2Bf^2)B \\ &= B(fBf)B \\ &\subseteq B \end{aligned}$$

Hence verified. □

Theorem 3.2. *Let* B_1, B_2, B_3 *be three 2 bi-ideal of a TSS then the product* $B_1B_2B_3$ *is also 2 bi-ideal.*

Proof. Let B_1, B_2, B_3 are three 2 bi-ideal of a TSS.
verify $(B_1B_2B_3)f^2(B_1B_2B_3)f^2(B_1B_2B_3) \subseteq B_1B_2B_3$.
Thus

$$\begin{aligned} (B_1B_2B_3)f^2(B_1B_2B_3)f^2(B_1B_2B_3) &= (B_1ff)f^2(fB_2f)f^2(ffB_3) \\ &= (B_1ff)(f^2fB_2ff^2)(ffB_3) \\ &= B_1(fB_2f)B_3 \\ &\subseteq B_1B_2B_3 \end{aligned}$$

Hence $(B_1B_2B_3)f^2(B_1B_2B_3)f^2(B_1B_2B_3) \subseteq B_1B_2B_3$. □

Theorem 3.3. *If 2 bi-ideal* B *is a idempotent then it is idempotent of 2 bi-ideal of* f.

Proof. Let $e \in B$ be an idempotent.
Now

$$\begin{aligned} ef^2ef^2e &= e(f^2ef^2)e \\ &= eee \\ &\subseteq e \end{aligned}$$

Hence $ef^2ef^2e \subseteq e$ □

Theorem 3.4. *Suppose TSS of* B *is an 2 bi ideal of regular iff it is 2 bi-ideal.*

Proof. consider B is an 2 bi-ideal of regular in f.
Let $b \in$ B, $\exists\, t \in$ f such that $bt^2bt^2b = b$. $\Longrightarrow$ $b \in$ $\mathrm{Bf^2Bf^2B}$.
clearly $\mathrm{B \subseteq Bf^2Bf^2B}$. Now

$$\begin{aligned} \mathrm{Bf^2Bf^2B} &= \mathrm{Bf^2Bf^2B} \\ &= \mathrm{B(ffBff)B} \\ &= \mathrm{B(f(fBf)f)B} \\ &= \mathrm{BfB} \\ &\subseteq \mathrm{B} \end{aligned}$$

Conversely, assume that B regular.

$$\begin{aligned} \mathrm{B} &= \mathrm{BfB} \\ &= \mathrm{BfBfB} \\ &= \mathrm{BfBfBfBfB} \\ &= \mathrm{BffBffB} \\ &\subseteq \mathrm{Bf^2Bf^2B} \end{aligned}$$

Hence $\mathrm{B \subseteq Bf^2Bf^2B}$ □

Theorem 3.5. *Suppose TSS of* B *a regular is an m bi-ideal of* f *iff* $\mathrm{B}^m = \mathrm{B}^m\mathrm{f}\,\mathrm{B}^m$.

Proof. Let B is a m bi-ideal of f.
verify $\mathrm{B}^m\mathrm{f}\,\mathrm{B}^m \subseteq \mathrm{B}$.

$$\begin{aligned} \mathrm{Bf}^m\mathrm{B} &= \mathrm{B}^m\mathrm{f}\mathrm{B}^m \\ &= \mathrm{(BB....B)f(BB...B)} \\ &= \mathrm{BfB} \\ &\subseteq \mathrm{B} \end{aligned}$$

Conversely

$$\begin{aligned} \mathrm{B} &= \mathrm{BfB} \\ &= \mathrm{(BfB)f(BfB)} \\ &= \mathrm{(BfBfBfB)f(BfBfBfB)} \\ &= \mathrm{B}^m\mathrm{f}\mathrm{B}^m \end{aligned}$$

∴ B is a m bi-ideal of f. □

Definition 3.6. An element $e \in$ f is an idempotent if $e^3 = (eee) = e$. In particular, An element $i \in C$ is said to be an idempotent, if $i^3 = (iii) = -i$, then every 1- bi ideal TS is an idempotent but 2- bi ideal of TSS not an idempotent.

Theorem 3.7. *A TS of C is bi ideal. Then it is 1-bi ideal of idempotent in* f *not in 2 bi ideal.*

Proof. Let C be a TS. Since B is an idempotent of C.
Then

$$\begin{aligned} \mathrm{B}C\mathrm{B}C\mathrm{B} &= ecece \\ &\subseteq e(cec)e \\ &\subseteq ece \\ &\subseteq e \end{aligned}$$

Similarly,

$$\begin{aligned} \mathrm{B}C^m\mathrm{B}C^m\mathrm{B} &= \mathrm{B}(C^m\mathrm{B}C^m)\mathrm{B} \\ &\subseteq \mathrm{B}C^m\mathrm{B} \\ &\subseteq \mathrm{B} \end{aligned}$$

Where m is an odd complex number. When $m = 2$,

$$\begin{aligned} \mathrm{B}C^2\mathrm{B}C^2\mathrm{B} &= e(C^2eC^2)e \\ &= eC^2e \\ &= ec \\ &= c \\ &\not\subseteq \mathrm{B} \\ &\not\subseteq e \end{aligned}$$

Hence it is satisfy only for an idempotent in odd values of m. □

Theorem 3.8. *Let* f *be a 2 bi ideal TS. If* f *is a mono TS then* $(\mathrm{f}, \cdot)$ *is a band.*

Proof. Let $(\mathrm{f}, +, \cdot)$ be a mono TS, $a + b = a^2b$. Since f is a 2 bi ideal TS, if $bt^2b^2b \subseteq b$.

$$a + (bt^2bt^2b) =$$
$$a^2(bt^2b^2b)\ a + (bt^2b) =$$
$$a^2(bt^2b)$$

Taking $a = b$

$$\begin{aligned} b + (bt^2b) &= b^2bt^2b \\ b + b &= b^3 \\ b &= b^3 \end{aligned}$$

∴ $(\mathrm{f}, \cdot)$ is a band. □

Theorem 3.9. *Let* f *be a sub idempotent and zero square of TS.If* $(\mathrm{f}, \cdot)$ *is band then* $a^2 = a$. *Proof.*

Let $(\mathrm{f}, +, \cdot)$ be a sub idempotent of TS, $a + aba = b$
∵ f is a 2 bi ideal of TS, $bt^2b^2b \subseteq b$.

$$a + aba = b$$
$$a + a(bt^2b^2b)a = bt^2b^2b$$
$$a + a(bt^2b)a = bt^2b$$
$$a + a(btbt)a =$$
$$bt^2b\ a + (abta)$$
$$= bt^2b\ a +$$
$$(aba)t = bt^2b\ a$$
$$+ at = bt^2b$$

Multiplying with a^2 bothsides,

$$a^2(a + at) = a^2(bt^2b)$$
$$a^3 + a^2 = a^2b$$
$$0 + a^2 = aba$$
$$a^2 = a$$

Hence $a^2 = a$.

□

Theorem 3.10. *Let* f *be a sub idempotent and zero square of TS and* $(f, \cdot)$ *is commutative then* f *is band.*

Proof. W.K.T f is a 2 bi ideal of TS.

$$a + aba = b$$
$$a + a(bt^2bt^2b)a = (bt^2bt^2b)$$
$$a + a(bt^2b)a = bt^2b$$

Multiplying with b^2 on bothsides, we get

$$b^2(a + a(bt^2b)a) =$$
$$b^2(bt^2b)\ bab + b^2a$$
$$= b^3$$
$$b + b = b^3$$
$$b = b^3$$

□

Theorem 3.11. *Let* f *be a sub idempotent and 2 bi ideal of TS. If* $(f, \cdot)$ *is a band, then* $(f, +)$ *is a band.*

Proof. Since f is a 2 bi ideal of TS.
We have

$$a + aba = b$$
$$a + a(bt^2bt^2b)a = (bt^2bt^2b)$$
$$a + a(bt^2b)a = bt^2b$$

Taking $b = a$

$$(a + a(bt^2b)a) = (bt^2b)$$
$$a + aaa =$$
$$a\,a + a^3 =$$
$$a\,a + a =$$
$$a$$

Hence (f, +) is a band.

□

Theorem 3.12. *Let* f *be a sub idempotent and 2 bi ideal of TS. If* (f, +) *is a band,then* (f, +) *is right singular.*

Proof. Given Let f be a sub iempotent of TS. If (f, +) is a band, (i.e) $a + a = a$; $\forall a \in$ f. We have $a + aba = b$; $\forall a, b \in$ f. Since f is 2 bi ideal of TS s $bt^2b^2b \subseteq b$.
Now

$$a + aba = b$$
$$a + a(bt^2bt^2b)a = (bt^2bt^2b)$$
$$a + a(bt^2b)a = bt^2b$$

Adding a on bothsides, we get

$$a + a + a(bt^2b)a = a + (bt^2b)$$
$$a + aba = a + b$$
$$b = a + b$$

Hence (f, +) is right singular. □

4 Conclusion

In this direction this work will investigate the structure of 2 bi-ideals in TS . It is also proved that all bi-ideals are 2 bi-ideals but the converse is not always true. These results open the way to future studies on more general higher-order bi-ideals and their algebraic properties, as well as contribute to the development of our understanding of ideal theory in TS.

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