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Novel Approach on Global Domination of Fuzzy Planar Graphs using Strong arc

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Abstract: *In this paper, we study about the Global Domination number for Fuzzy Planar Graphs using strong arcs. We present a novel approach to explore and investigate various types of Global Domination based on strong arcs configuration.*

Keywords: Fuzzy planar graph, Strong arc, non-strong arc, Domination number, Fuzzy global domination, Fuzzy global domination number.

1.Introduction:

A fuzzy graph is an extension of an ordinary graph. Hence it is normal for the fuzzy graph to inherit many characteristics similar to those of ordinary graph; however, it deviates in several aspects. A fuzzy graph G:(ρ, μ), where ρ, μ indicates the membership values of the vertices and edges in the range [0,1]. A graph is planar if its edges only collide at its vertices. Fuzzy and planar graphs are blended to form fuzzy planar graphs.

In 1962, Ore introduced the definition for domination. A. Rosenfield in 1975 introduced the notion of fuzzy graphs. Subsequent studies investigated various additional concepts of dominance. C.Y. Ponnappan and V. Senthil analysed about domination of fuzzy graphs through strong arcs. A. Somasundaram and S. Somasundaram studied about dominance in fuzzy graphs using effective arc. First, we provide some preliminary ideas and previous results that serve as the basis of our investigation.

2. Preliminaries:

Definition 2.1

An arc (u, v) of the fuzzy planar graph G:(ρ, μ) is a strong arc if μ (u, v) = μ^{∞}(u, v). If not, it is referred as non- strong. The strong neighborhood of a vertex u∈ V(G) is N_s(u)= {v ∈ V(G)\ (u, v) is strong arc}. The closed strong neighborhood of u is N_s(u)= N_s(u) ∪ {u}.

Definition 2.2

Let G:(ρ, μ) be a fuzzy planar graph and let u, v be two nodes. u dominates v if the edge (u, v) is a strong arc. D ⊆ V(G) is a dominating set if for every v ∈ V-D, their exists u ∈ D such that u dominates v. D is a minimal dominating set if no proper subset of D is a dominating set. The strong arc domination number, γ_s(G) is the minimum fuzzy cardinality and corresponding set is called minimum strong arc dominating set. The number of elements in the minimum strong arc dominating set is n[γ_s(G)]. The $\bar{G}^*$: ($\bar{\rho}^*, \bar{\mu}^*$) be the G^* complement of G and is defined as $\bar{\rho}^*$(u) = ρ(u)- [μ (u, v) ∧ μ (u, w)] and $\bar{\mu}^*$(u, v) = [ρ(u) ∧ ρ(v)]- μ (u, v) and $\bar{\mu}^*$(u, v)= 0 if μ(u, v)=0.

3. GLOBAL DOMINATION IN FUZZY PLANAR GRAPHS USING STRONG ARC

Definition 3.1

Let G be a fuzzy planar Graph where all arcs are strong arcs. D_s(G) and γ_s(G) denotes the minimal dominating set and the minimum fuzzy cardinality taken over all the dominating sets respectively. The number of elements in the minimum dominating set of G using strong arc is denoted as n [γ_s(G)].

The minimum Fuzzy cardinality taken over all the dominating sets of a graph $\bar{G}^*$, where $\bar{G}^*$ is the G^* complement of G, is called the domination number of $\bar{G}^*$ and is denoted by $\gamma_s(\bar{G}^*)$. The corresponding set is called the minimum dominating set of $\bar{G}^*$ using strong arc. The number of elements in the minimum dominating set of $\bar{G}^*$ using strong arc is denoted as n [$\gamma_s(\bar{G}^*)$].

Fuzzy Global Dominating set using strong arc is the set which is the corresponding dominating set of min{γ_s(G), $\gamma_s(\bar{G}^*)$}and fuzzy global domination number is denoted by γ_{gs}(G).

Note: Fuzzy Planar graphs with non-effective edges are examined here.

Definition 3.2

The domination is known as semi perfect global domination of fuzzy planar graph using strong arc if n [γ_s(G)] = n [$\gamma_s(\bar{G}^*)$] and have distinct dominating sets. The cardinality of the associated dominating set is represented by γ_{spgs}(G).

Definition 3.3

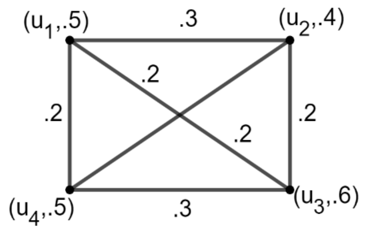
The dominating set is known as perfect global dominating set of fuzzy planar graph if the minimum strong arc dominating sets D is the dominating set of both G and $\bar{G}^*$ and its cardinality is represented as γ_{pgs}(G).

4. GLOBAL NEIGHBORHOOD CLIQUE DOMINATION IN FUZZY PLANAR GRAPHS USING STRONG ARC

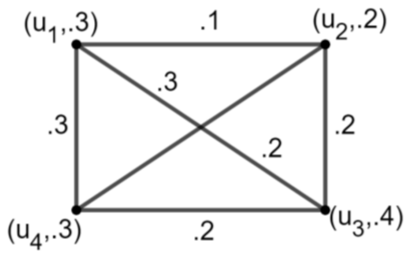
Definition 4.1

Let G be a fuzzy planar graph where all arcs are strong and without isolated vertices. If <N(D_{ncs}(G))> is complete i.e., no edge is effective and every vertex is adjacent with every other vertex, then a subset D_{ncs}(G) of V is a neighborhood clique dominating set of fuzzy planar graph using strong arc, as long as <N(D_{ncs} (G))> contains the vertices other than D_{ncs}(G). The least fuzzy cardinality over all minimal neighborhood clique dominating sets of G is the fuzzy neighborhood clique domination number or γ_{ncs}(G). Likewise, the neighborhood clique dominating set of fuzzy planar graph $\bar{G}^*$ is $D_{ncs}(\bar{G}^*)$ of V using strong arc. The neighborhood clique domination number of fuzzy planar graph using strong arc is $\gamma_{ncs}(\bar{G}^*)$. Then the minimum of γ_{ncs}(G) and $\gamma_{ncs}(\bar{G}^*)$ is the global neighborhood clique domination number of fuzzy planar graph utilizing the strong arc is γ_{gncs}(G). In other word γ_{gncs}(G)= min{γ_{ncs}(G), $\gamma_{ncs}(\bar{G}^*)$}.

Example 4.2



D_{ncs}(G)= { u_2} ∴ γ_{ncs}(G)= 0.4. <N(D_{ncs}(G))>= {u_1, u_3, u_4}



In $\bar{G}^*$, the arcs (u_1, u_2) and (u_3, u_4) are non-strong arcs.

$D_{ncs}(\bar{G}^*) = \{u_1, u_2\} \therefore \gamma_{ncs}(\bar{G}^*) = 0.5$.

<N(D_{ncs}(G))>= $\{u_3, u_4\}$ is complete.

Hence, γ_{gncs}(G) = .4

Theorem 4.3

If $\bar{G}^*$ is a strong complete fuzzy planar graph of G which is complete where all arcs are also strong and $\bar{\rho}^*(u_i)$ = c, for all u_i in V, then $\gamma_{ncs}(\bar{G}^*)$ = c

Proof:

Let $\bar{G}^*$ be a complete fuzzy planar graph with vertices {u_1, u_2, … u_n}. By definition of fuzzy complete graph, each u_i dominates the other vertices. Let D_{ncs} $(\bar{G}^*)$ be the neighborhood clique dominating set of the fuzzy planar graph $\bar{G}^*$ using strong arcs. $D_{ncs}(\bar{G}^*) = \{u_i: \bar{\rho}^*(u_i) = c\}$. Hence, N($D_{ncs}(\bar{G}^*)$) = V\ $\{u_i\}$ and N($D_{ncs}(\bar{G}^*)$) is complete. Since, $\bar{\rho}^*(u_i)$ = c ∀i, the neighborhood clique domination number of a fuzzy planar graph using strong arc is $\gamma_{ncs}(\bar{G}^*) = \bar{\rho}^*(u_i)$ = c.

Theorem 4.4

If $\bar{G}^*$ is a strong complete fuzzy planar graph of a fuzzy planar graph G and $D_{ncs}(\bar{G}^*)$ is the neighborhood clique dominating set of the fuzzy planar graph $\bar{G}^*$, then V- $D_{ncs}(\bar{G}^*)$ is also a strong complete fuzzy planar graph.

Proof:

Let $\bar{G}^*$ be a strong complete fuzzy planar graph of G with vertex set V= {u_1, u_2, …u_{i-1}, u_{i+1}, … u_n}. By definition of strong complete fuzzy graph, each u_i dominates every other vertex in $\bar{G}^*$. The fuzzy neighborhood clique dominating set using strong arc $D_{ncs}(\bar{G}^*) = \{u_i/u_i$ is the vertex of minimum fuzzy cardinality}. By the definition of clique neighborhood dominating set N($D_{ncs}(\bar{G}^*)$) is complete. Hence, V- $D_{ncs}(\bar{G}^*)$ = {u_1, u_2, ...u_{i-1}, u_{i+1}, ... u_n} and <V- $D_{ncs}(\bar{G}^*)$> is complete with all arcs as strong arcs.

Theorem 4.5

If $\bar{G}^*$ is a strong complete fuzzy planar graph of G and $D_{ncs}(\bar{G}^*)$ is the neighborhood clique dominating set of $\bar{G}^*$ using strong arcs then the fuzzy neighborhood clique dominating number of $\bar{G}^*$ using strong arc is $\gamma_{ncs}(\bar{G}^*)$ = min{ $\bar{\rho}^*(u_i)/\ u_i \in$ V}.

Proof:

Let G be a strong fuzzy complete fuzzy planar graph such that its $\bar{G}^*$ is also a strong complete fuzzy planar graph with vertex set V= {u_1, u_2, ...u_{i-1}, u_i, u_{i+1}, ... u_n} have distinct fuzzy vertex cardinality. Let $D_{ncs}(\bar{G}^*)$ be the fuzzy neighborhood clique dominating set of G using strong arc. Then $D_{ncs}(\bar{G}^*)$ = {$u_i/\ u_i$ is the vertex of minimum fuzzy cardinality}. Hence, the fuzzy neighborhood clique dominating number of G using strong arc is $\gamma_{ncs}(\bar{G}^*)$ = min{ $\bar{\rho}^*(u_i)/\ u_i \in$ V}.

Theorem 4.6

If G is a fuzzy planar graph and $\bar{G}^*$ is its G^* complement then $\gamma_s(\bar{G}^*) \leq \gamma_{ncs}(\bar{G}^*)$.

Proof:

Let G be a fuzzy planar graph with all strong arcs and $\bar{G}^*$ is its G^* complement. Let $D_s(\bar{G}^*)$ and $D_{ncs}(\bar{G}^*)$ be the dominating set and neighborhood dominating set of $\bar{G}^*$ using strong arc. $\gamma_s(\bar{G}^*)$ and $\gamma_{ncs}(\bar{G}^*)$ be the domination number and neighborhood clique domination number of $\bar{G}^*$. Every neighborhood clique dominating set is also a dominating set but need not be necessarily the minimum dominating set. Hence, $\gamma_s(\bar{G}^*) \leq \gamma_{ncs}(\bar{G}^*)$.

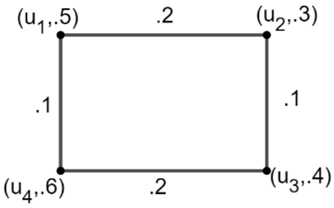
5. GLOBAL CONNECTED DOMINATION NUMBER IN FUZZY PLANAR GRAPHS USING STRONG ARC

Definition 5.1

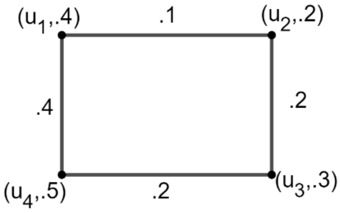
The dominating set D_{cs}(G) of a fuzzy connected planar graph is called the connected dominating set using strong arc if D_{cs}(G) is connected. The least cardinality over all minimal connected dominating sets of G is the fuzzy connected domination number of fuzzy planar graph using strong arc γ_{cs}(G).

Likewise, the fuzzy connected domination number of $\bar{G}^*$ using strong arc is $\gamma_{cs}(\bar{G}^*)$. Next, the fuzzy planar graph's connected global domination number is the minimum of γ_{cs}(G) and $\gamma_{cs}(\bar{G}^*)$. i.e., γ_{gcs}(G)= min{γ_{cs}(G), $\gamma_{cs}(\bar{G}^*)$}.

Example 5.2



D_{cs}(G) = {u2, u3}, γ_{cs}(G) = 0.7



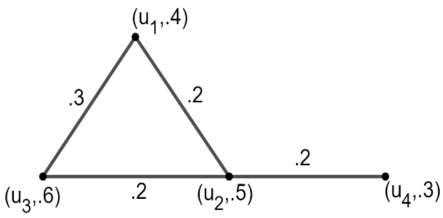
$D_{cs}(\bar{G}^*)$ = {u1, u2}, $\gamma_{cs}(\bar{G}^*)$ = 0.6

γ_{gcs}(G) = 0.6

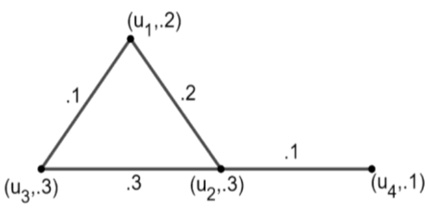
Remark 5.3

If D_{gcs}(G) is a global connected dominating set of G of fuzzy planar graph using strong arcs, then <V-D_{gcs}(G)> need not be connected.

Example 5.4



D_{cs}(G)= {u2}, γ_{cs}(G) = 0.5



$D_{cs}(\bar{G}^*)$ = {u2}, $\gamma_{cs}(\bar{G}^*)$ = 0.3, γ_{gcs}(G) = 0.3

Theorem 5.5

For any connected fuzzy planar graph G, γ_{gcs}(G)≤ γ_{cs}(G)

Proof:

By definition, global connected domination number of fuzzy planar graph using strong arc is the minimum of γ_{cs}(G) and $\gamma_{cs}(\bar{G}^*)$.

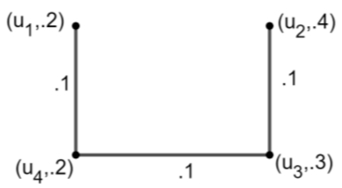
6. GLOBAL CONNECTED SPLIT DOMINATION IN FUZZY PLANAR GRAPHS USING STRONG ARC

Definition 6.1

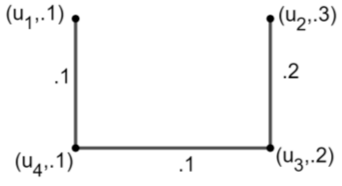
If D_{css}(G) is connected, then the dominating set D_{css}(G) of a fuzzy planar connected graph G is a connected split dominating set of fuzzy planar graph employing strong arc. The least cardinality over all minimal connected split dominating set of G is the connected domination number of fuzzy planar graph using strong arc γ_{css}(G).

Similarly, $\gamma_{css}(\bar{G}^*)$ is the connected split domination number of fuzzy planar graph using strong arc of $(\bar{G}^*)$. Then, global connected split domination number of fuzzy planar graph γ_{gcss}(G) of a connected fuzzy planar graph G is the minimum of γ_{css}(G) and $\gamma_{css}(\bar{G}^*)$. i.e., γ_{gcss}(G) = min{γ_{css}(G), $\gamma_{css}(\bar{G}^*)$}

Example 6.2



D_{css}(G)= {u3, u4}, γ_{css}(G)= .5



D_{css}(G)= {u3, u4},$\gamma_{css}(\bar{G}^*)$ = .3

Remark 6.3

Every connected split dominating set in a fuzzy planar graph using strong arc is a connected dominating set of fuzzy planar graph using strong arc. However, the vice versa is not true.

Conclusion:

In this paper, using strong arcs we have introduced a new method for global domination in fuzzy planar graphs. In order to study the structural characteristics of fuzzy planar graphs, we investigated few domination concepts. Our results greatly advance the study of fuzzy graph theory and can be used in social network analysis, transportation systems etc.

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