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MULTICRITERIA DECISION MAKING USING PYTHAGOREAN FUZZY SET

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ABSTRACT.

Decision-making in uncertain and imprecise environments often requires robust mathematical frame-works to evaluate multiple alternatives against diverse criteria. Pythagorean fuzzy sets (PyFSs) provide a powerful tool for handling such challenges by effectively representing uncertainty through membership (Py_{μ_i}) and non-membership (Py_{v_i}) degrees, satisfying $Py_{\mu_i^2} + Py_{v_i^2} \leq 1$. This makes PyFSs particularly suitable for applications involving complex, ambiguous datasets. This study investigates the application of PyFSs in multi-criteria decision-making scenarios, specifically focusing on identifying the most vulnerable regions during critical events. Two case studies are considered: (i) a Global Health Monitoring Center (GHMC) tasked with assessing regions and (ii) a Natural Disaster Management Center (NDMC) determlnin g the susceptibility of the same regions to natural disasters using set of criteria. The criteria span key indicators such as healthcare infrastructure, environmental risks, population density, and resource availability. By modeling the regions using PyFSs, the approach accounts for imprecise data and offers a reliable mechanism to rank the alternatives. The method demonstrates its capacity to provide actionable insights by identifying the regions most at risk in both health crises and disaster scenarios. This study highlights the versatility of PyFSs in decision-making, emphasizing their utility in addressing uncertainty across diverse real-world applications.

Key words and phrases. Pythagorean fuzzy, Fuzzy set, Application of MCDM.

1. INTRODUCTION

In real-world decision-making scenarios, uncertainty and imprecision are inherent, particularly when data is incomplete, ambiguous, or subjective. To address these challenges, fuzzy set theory, introduced by Zadeh in 1965, provides a robust mathematical framework for representing and analyzing uncertainties. A fuzzy set (FS) [29] is characterized by a membership function that assigns a degree of membership to each element, enabling nuanced modeling of data. However, as decision-making problems became more complex, the limitations of fuzzy sets in expressing hesitation or non-membership became evident. Building on this foundation, Atanassov introduced intuitionistic fuzzy sets (IFS) [5], which incorporate both membership (μ) and non-membership (v) degrees, along with a hesitation degree $(\pi = 1 - \mu - v)$. This extension

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allows for better handling of situations where partial information about non-membership is available. Despite theiradvantages, intuitionistic fuzzy sets face challenges when managing higher degrees of uncertainty in datasets.To overcome these limitations, Yager introduced Pythagorean fuzzy sets (PyFS)[27]. In a PyFS, the squares of the membership (μ) and non-membership (v) degrees satisfy $\mu^2 + v^2 \leq 1$, offering enhanced flexibility in representing uncertainty. The hesitation degree ($\pi = \sqrt{1 - \mu^2 - v^2}$) is defined geometrically, making this framework particularly suitable for high-uncertainty scenarios, such as medical diagnosis, pattern recognition, and decision-making under ambiguous conditions.

Recent research on Pythagorean fuzzy sets has explored their applications across various fields, emphasizing their superiority in handling complex decision-making problems. For example, Dutta et al. [6] utilized Pythagorean fuzzy sets for medical diagnosis and COVID-19 medicine selection, demonstrating their effectiveness in addressing intricate health-related decisions. Similarly, Ganie et al. [8] and Ganie [9] investigated distance measures and clustering techniques to enhance decision-making accuracy.

In this context, the study of FSs, IFSs, and PyFSs forms a progressive pathway for advancing decision-making models. This paper aims to provide a comprehensive overview of these theoretical foundations, complemented by a review of literature and illustrative examples. Furthermore, the comparative analysis presented in Table 1.1 highlights the distinctive features of each framework, underscoring the importance of selecting the appropriate model for specific applications.

Comparison of Fuzzy, Intuitionistic Fuzzy, and Pythagorean Fuzzy Sets

Aspect	**FS** [29]	**IFS** [5]	**PyFS** [27]
Membership degree μ	$0 \leq \mu \leq 1$	$0 \leq \mu \leq 1$	$0 \leq \mu \leq 1$
Non-membership degree v	Not defined	$0 \leq v \leq 1$	$0 \leq v \leq 1$
Hesitation degree π	Not applicable	$\pi = 1 - \mu - v$	$\pi = \sqrt{1 - \mu^2 - v^2}$
Key condition	μ only	$\mu + v \leq 1$	$\mu^2 + v^2 \leq 1$
Application suitability	Simple uncertainties	Moderate uncertainties	High uncertainties
Key condition	μ only	$\mu + v \leq 1$	$\mu^2 + v^2 \leq 1$
Application suitability	Simple uncertainties	Moderate uncertainties	High uncertainties

Table 1.Comparison of Fuzzy, Intuitionistic Fuzzy, and Pythagorean Fuzzy Sets

2. REVIEW 0F LITERATURE AND PRELIMINARIES

The use of PyFS as grown significantly in decision-making and various engineering applications due to theirability to handle uncertainty and imprecision. PyFS, which is an extension of FSs, provides a powerful frameworkfor dealing with uncertainty in a variety of complex problems. Several studies have contributed to advancing thetheoretical and practical applications of PyFS. Akram et al. (2022) [1] proposed a new optimization techniquefor group decision analysis using complex PyFSs. Their work introduced novel aggregation methods to enhancedecision-making under uncertainty, demonstrating the versatility of PyFS in real-world applications. Ali andYang (2024)[2] focused on circular Pythagorean fuzzy Hamacher aggregation operators, applying these operatorsto assess goldmines, which highlighted their importance in complex decision-making systems. Dutta et al. (2023)[6] explored nonlinear distance measures within the PyFSs framework, with applications in pattern recognition,medical diagnosis, and COVID-19 medicine selection, further illustrating the broad utility of PyFS in diversefields. Du et al. (2023) [7] proposed a weighting method for PyFS based on improved hesitation, which providesa more refined way to evaluate PyFS in decision-making.

Ganie et al. (2022) [8] discussed similarity and entropy measures for PyFSs, exploring their applications invarious decision-making problems. Their work established key principles that can be applied to a wide rangeof fuzzy-based systems. Similarly, Ganie (2022) [9] introduced t-conorm-based distance measures, which areessential for making knowledge-based decisions using PyFS. Ghosh et al. (2022) [10] focused on the multi-objective solid transportation problem utilizing PyFS, demonstrating how PyFS can handle uncertainties in logistics and optimization problems. Haq et al. (2023) [11] proposed a novel framework combining Pythagorean fuzzydominance-based rough sets, enhancing the ability to reduce knowledge in decision-making. Hezam et al. (2023)[12] introduced geometric aggregation operators for multicriteria group decision-making based on complex PyFSs,providing a more robust method for handling multiple conflicting criteria. Hua and Jing (2023) [13] presented aninterval-valued Pythagorean fuzzy PROMETHEE method based on the Shapley index for group decision-making.Their method provided an innovative approach for dealing with complex decision problems involving multiplecriteria. Jia and Herrera-Viedma (2023) [14] extended the use of PyFSs to solve Z-numbers in decision-makingmodels, providing a new perspective on integrating PyFS with other types of uncertainty.

Jin et al. (2022) [15] explored novel complex PyFSs under Aczel-Alsina operators, applying them to multi-attribute decision-making problems, and thereby contributing to the development of aggregation techniques forcomplex fuzzy systems. Kashyap et al. (2024) [16] proposed a novel trigonometric entropy measure for PyFSs,improving the efficiency of decision-making under uncertainty. Khan et al. (2023) [17] introduced the concept ofdisc PyFSs with distinctive radii, which are particularly useful in handling fuzzy sets with complex boundaries andapplications in decision analysis. Kumar and Chen (2022) [18] used entropy measures and aggregation operatorsfor group decision-making based on PyFSs, which provided a more reliable approach to combining individualjudgments in a decision-making process. Kumar et al. (2024) [19] introduced the concept of disc-based PyFSswith distinct radii, which provide additional flexibility for handling fuzzy data with varying radii. Liu (2024) [20]investigated Hellinger distance measures in a PyFSs environment, highlighting the importance of these measuresin decision-making applications. Liu et al. (2022) [21] introduced Archimedean aggregation operators based oncomplex PyFSs, offering new methods for decision-making in uncertain environments.

Meng et al. (2022) [22] applied PyFSs in graph theory to develop new methods for multi-criteria decision-making (MCDM), demonstrating how PyFS can improve decision-making in networked systems. Rani et al.(2023) [23] used a standard deviation-based method combined with PyFSs for multiple attribute decision-making(MADM), which offered a new approach for evaluating complex decision problems. Saikia et al. (2023) [24] introduced an advanced similarity measure for PyFSs, applying it to transportation problems. Their work provideddeeper insights into how PyFS can enhance decision-making in logistics and transportation. Shumrani and Gulistan (2022) [25] studied the similarity measures of N-cubic PyFSs using overlapping ratios, offering new waysto compare fuzzy sets in decision problems. Wang et al. (2022) [26] applied interval-valued PyFSs to conflictanalysis in decision-making, showing how prospect theory can be integrated with PyFS to better handle conflictsin multi-agent systems. Yang et al. (2024) [28] focused on clustering analysis for PyFSs and applied their methodto multiple attribute decision-making, offering a new tool for evaluating decision problems with diverse criteria.

Definition 2.1.A FS F in a universe of discourse U is defined as:

$$F = \{(x, \mu_F(x)) | x \in U, 0 \leq \mu_F(x) \leq 1\}$$

Here, $\mu_F(x)$ is the membership function that quantifies the degree of membership of x in F.

Example 2.2. Let $U =$ {low, medium, high} be a set of risk levels. A fuzzy set F representing "medium risk"could be defined as:

$F =\{$ $(1ow, 0.2)$, (medium, 0.8), (high, 0.5)}.

This representation captures partial membership, reflecting uncertainty in risk evaluation.

Definition 2.3. An IFSI on U is given by:

$$I = \{(x, \mu_I(x), v_I(x)) | x \in U\},$$

where$0 \leq \mu_I(x), v_I(x) \leq 1$ and $\mu_I(x) + v_I(x) \leq 1$. The hesitation degree, denoted by $\pi_I(x)$, accounts for theuncertainty and is defined as:$\pi_I(x) = 1 - \mu_I(x) - v_I(x)$.

Example 2.4. Let $U =$ {low, medium, high} be a set of risk levels. An intuitionistic fuzzy set I representing"medium risk" could be:

$I =\{$ $(1ow, 0.2{,}0.6)$, (medium, 0.7, 0.1), (high, 0.4, 0.5)}.

Here, the hesitation degrees are $\pi_I(1ow) = 0.2, \pi_I$(medium)= 0.2, and π_I(high)= 0.1.

Definition 2.5. A PyFSP on U is defined as:

$$A = \{(x, \mu_P(x), v_P(x)) | x \in U, \mu_P(x)^2 + v_P(x)^2 \leq 1\}.$$

The hesitation degree is computed as:

$$\pi_P(x) = \sqrt{1 - \mu_P(x)^2 - v_P(x)^2}.$$

Example 2.6. Let $U=$ {low, medium, high}. A Pythagorean fuzzy set Prepresenting "medium risk" could be:$P=\{$ (low, 0.6,0.6) , (medium, 0.8, 0.3), (high, 0.4, 0.5)$\}$.
For medium, the hesitation degree is:π_P(medium)= $\sqrt{1-08^2-03^2}=\sqrt{1-064-009}=\sqrt{027}$.

3. DESCRIPTION OF THE PROBLEM

Consider a set of choices $U=\{U_1,U_2,\dots,U_m\}$ and a set of criteria $C=\{c_1,c_2,\dots,c_n\}$ that serve as thefoundation for assessing these alternatives in this decision-making situation. A Pythagorean fuzzy set (PyFS) isused to express each alternative U_i. For each criterion, the membership and non-membership degrees in such aset must meet the following requirement: $Py_{\mu_i^2}+Py_{v_i^2}\leq 1$. Compared to an intuitionistic fuzzy set, this system ismore dependable at handling uncertainty, making it perfect for decision-making situations where information maybe ambiguous or lacking. After ranking the options, the best one is found by modeling the dataset with PyFSs.
This method illustrates a problem involving decision-making in a fuzzy setting.

$\langle[Py_{\mu_{11}},Py_{v_{11}}]\rangle$	$\langle[Py_{\mu_{12}},Py_{v_{12}}]\rangle$	$\cdots$	$\langle[Py_{\mu_{1n}},Py_{v_{1n}}]\rangle$
$\langle[Py_{\mu_{21}},Py_{v_{21}}]\rangle$	$\langle[Py_{\mu_{22}},Py_{v_{22}}]\rangle$	$\cdots$	$\langle[Py_{\mu_{2n}},Py_{v_{2n}}]\rangle$
$\vdots$	$\vdots$		$\vdots$
$\langle[Py_{\mu_{m1}},Py_{v_{m1}}]\}$	$\langle[Py_{\mu_{m2}},Py_{v_{m2}}]\}$	$\cdots$	$\langle[Py_{\mu_{mn}},Py_{v_{mn}}]\}$

Table 2. Table of Pythagorean Fuzzy Elements $\langle[Py_{\mu_{ij}},Py_{v_{ij}}]\}$

3.1. **New method for decision making problem**. Pairs of elements from the closed interval $[0,1]$ can be usedas entries in a tabular representation of the complete data set.
Each column represents a PyFS corresponding to a specific criterion. The comparison between elements of two alternatives is carried out as follows: Let $\langle[Py_{\mu_1},Py_{v_1}]\}$ and $\langle[Py_{\mu_2},Py_{v_2}]\}$ represent the membership andnon-membership grades of the PyFSs for a given criterion.

Definition 3.1. The degree of membership $\langle Py_{\mu_1}\rangle$ is said to dominate $\langle Py_{\mu_2}\rangle$if $\langle Py_{\mu_1}\rangle\geq\langle Py_{\mu_2}\rangle$. Similarly, thedegree of non-membership $\langle Py_{v_1}\rangle$ is said to dominate $\langle Py_{v_2}\rangle$ if $\langle Py_{v_1}\rangle\leq\langle Py_{v_2}\rangle$.
To compare two rows, the membership and non-membership degrees for each criterion are evaluated independently to determine dominance. Each row is compared with all other rows, resulting in an$m\times m$ matrix. For an entry Py_{ij}, it is defined as a pair (Py_{ij}^M,Py_{ij}^N) , where:
$-Py_{ij}^M$denotes the count of criteria for which the membership degree of the i^{th} row dominates that of the j^{th} row.
$-Py_{ij}^N$represents the count of criteria for which the non-membership degree of the i^{th} row dominates that of the j^{th} row.
The cumulative domination value for an entry is defined as:

$$\Gamma_{ij}=Py_{ij}^M+Py_{ij}^N,\forall i,j=1,2,\dots,m.$$

The total domination entry is given by:

$$\eta=\Gamma_{ij}.$$

For each alternative, the row domination sum ζ and the column domination sum ℓ are calculated as:

$$\zeta=\sum_{j=1}^{m}\Gamma_{ij}\,,\ell=\sum_{j=1}^{m}\Gamma_{ji}.$$

The algebraic domination sum for each alternative is then computed as:

$$\aleph=\zeta-\ell,\forall i=1,2,\dots,m.$$

Finally, the algebraic domination sums are compared, and the alternative with the highest positive value of $\aleph$ isselected as the best among the set of alternatives.

3.2. **Algorithm for the proposed method**. The algorithm to solve the decision-making problem is as follows:

(1) Construct a table for the PyFSs based on the available data.

(2) Populate the table by calculating the values of Py_{ij}^{M} and Py_{ij}^{N} for the given PyFSs.

(3) Compute the values of η and construct the cumulative domination matrix.

(4) Calculate the algebraic domination sum $\aleph_i$ by determining ζ_i and ℓ_j for $,j=1,2,\dots\ ,\ m.$

(5) Compare the values of $\aleph_i$, and conclude that the alternative corresponding to the most positive $\aleph_i$ is theoptimal choice.

3.3. **Applications based on the proposed method**.

Example 3.2. Assume that an outbreak of a highly contagious disease has occurred. A team of experts from theGlobal Health Monitoring Center (GHMC) must identify the most vulnerable zones among nine regions $U=\{U_1,U_2,\dots\ ,U_9\}$ that could potentially face the greatest impact of the disease. In the context of a GHMC, thegoal is to assess and monitor various health indicators across different regions or countries. These indicators helptrack disease outbreaks, healthcare system effectiveness, and other health-related metrics. The use of PyFSs in thiscontext allows for the representation of uncertain, imprecise, or partial information, which is common in healthdata analysis. The team evaluates the regions based on a set of criteria $C=\{c_1,c_2,c_3,\dots\ ,c_9\}$, where the criteriaare defined as follows:

(1) **Population Density** $(\boldsymbol{c_1})$ **:** It measures the number of people living per unit area. The higher the population density, the more likely it is that the disease will spread because of close contact among individuals.

(2) **Access to Clean Water** $(\boldsymbol{c_2})$ **:** It evaluates the availability of safe and potable water. Poor access to cleanwater increases the likelihood of waterborne diseases such as cholera and diarrhea.

(3) **Availability of Healthcare Facilities** $(\boldsymbol{c_3})$ **:** Assesses the presence and accessibility of hospitals, clinics,and medical personnel. Limited healthcare facilities can delay treatment and exacerbate outbreaks.

(4) **Level of Sanitation** $(\boldsymbol{c_4})$ **:** Examines the adequacy of sanitation systems such as waste disposal

andsewage treatment. Poor sanitation leads to unhygienic conditions, fostering the spread of diseases.

(5) **Climate Conditions** (c_5) **:** Considers weather factors like temperature, humidity, and rainfall. Someclimatic conditions favor vector-borne diseases like malaria and dengue.

(6) **Vulnerability of Affected Population** (c_6) **:** Identifies the percentage of populations who are more vulnerable, like children, older adults, or immunocompromised people. More vulnerable populations areeasily prone to infections.

(7) **Distance from Disaster Origin** (c_7) **:** Measures the distance of the area from the natural disasterepicenter. Areas closer to the disaster origin are usually more affected due to infrastructure damage and poorliving conditions.

(8) **Hygiene Awareness** (c_8) **:** Evaluates public awareness and practices concerning hygiene both at theindividual and community levels. Unhygienic habits accelerate the spread of infections.

(9) **Availability of Emergency Resources** (c_9) **:** Evaluates the availability of resources including vaccines,medicines, clean shelters, and emergency teams. Such resources can help contain the effects of an out-break.

Step 1: The table is constructed using the PyFSs as below:

Regions	c_1	c_2	c_3	c_4	c_5
U_1	$\langle[0.2385, 0.8563]\rangle$	$\langle[0.0994, 0.6307]\rangle$	$\langle[0.8117, 0.0791]\rangle$	$\langle[0.7178, 0.3402]\rangle$	$\langle[0.8842, 0.4242]\rangle$
U_2	$\langle[0.4586, 0.3478]\rangle$	$\langle[0.0720, 0.2190]\rangle$	$\langle[0.6745, 0.6324]\rangle$	$\langle[0.6137, 0.0251]\rangle$	$\langle[0.4750, 0.4004]\rangle$
U_3	$\langle[0.2599, 0.2589]\rangle$	$\langle[0.0023, 0.4123]\rangle$	$\langle[0.1281, 0.7653]\rangle$	$\langle[0.4118, 0.8287]\rangle$	$\langle[0.6441, 0.2761]\rangle$
U_4	$\langle[0.1028, 0.0980]\rangle$	$\langle[0.7632, 0.4775]\rangle$	$\langle[0.0201, 0.7460]\rangle$	$\langle[0.1327, 0.9118]\rangle$	$\langle[0.9519, 0.2870]\rangle$
U_5	$\langle[0.0205, 0.8765]\rangle$	$\langle[0.8530, 0.1018]\rangle$	$\langle[0.8274, 0.1079]\rangle$	$\langle[0.1569, 0.1929]\rangle$	$\langle[0.2152, 0.9385]\rangle$
U_6	$\langle[0.1110, 0.0651]\rangle$	$\langle[0.7607, 0.2904]\rangle$	$\langle[0.4698, 0.1277]\rangle$	$\langle[0.1464, 0.6309]\rangle$	$\langle[0.0065, 0.3623]\rangle$
U_7	$\langle[0.1806, 0.4337]\rangle$	$\langle[0.8774, 0.4333]\rangle$	$\langle[0.2357, 0.1661]\rangle$	$\langle[0.6719, 0.2460]\rangle$	$\langle[0.1325, 0.8193]\rangle$
U_8	$\langle[0.2034, 0.2564]\rangle$	$\langle[0.3163, 0.2806]\rangle$	$\langle[0.5842, 0.6338]\rangle$	$\langle[0.1081, 0.1758]\rangle$	$\langle[0.1700, 0.8564]\rangle$
U_9	$\langle[0.0376, 0.2770]\rangle$	$\langle[0.0167, 0.0871]\rangle$	$\langle[0.6587, 0.6049]\rangle$	$\langle[0.3203, 0.5809]\rangle$	$\langle[0.6817, 0.0057]\rangle$
U_{10}	$\langle[0.2988, 0.7731]\rangle$	$\langle[0.3564, 0.9154]\rangle$	$\langle[0.8043, 0.0723]\rangle$	$\langle[0.4035, 0.5086]\rangle$	$\langle[0.9091, 0.1691]\rangle$

Regions	c_6	c_7	c_8	c_9
U_1	$\langle[0.6137,0.7837]\rangle$	$\langle[0.9769,0.0296]\rangle$	$\langle[0.8374,0.0169]\rangle$	$\langle[0.2630,0.9585]\rangle$
U_2	$\langle[0.5757,0.7629]\rangle$	$\langle[0.7704,0.0295]\rangle$	$\langle[0.7391,0.0225]\rangle$	$\langle[0.4681,0.4904]\rangle$
U_3	$\langle[0.0104,0.2855]\rangle$	$\langle[0.3712,0.8888]\rangle$	$\langle[0.3083,0.4910]\rangle$	$\langle[0.0271,0.3243]\rangle$
U_4	$\langle[0.2393,0.1903]\rangle$	$\langle[0.7848,0.3969]\rangle$	$\langle[0.8353,0.3761]\rangle$	$\langle[0.5373,0.5821]\rangle$
U_5	$\langle[0.5305,0.3576]\rangle$	$\langle[0.0761,0.5647]\rangle$	$\langle[0.2129,0.5029]\rangle$	$\langle[0.2007,0.2202]\rangle$
U_6	$\langle[0.6119,0.4786]\rangle$	$\langle[0.3079,0.3516]\rangle$	$\langle[0.7085,0.6283]\rangle$	$\langle[0.7461,0.6542]\rangle$
U_7	$\langle[0.4692,0.0878]\rangle$	$\langle[0.0592,0.0475]\rangle$	$\langle[0.4959,0.8665]\rangle$	$\langle[0.2353,0.3828]\rangle$
U_8	$\langle[0.1584,0.7752]\rangle$	$\langle[0.3478,0.6546]\rangle$	$\langle[0.7402,0.3148]\rangle$	$\langle[0.3158,0.7444]\rangle$

U_9	⟨[0.3238,0.4375]⟩	⟨[0.9345,0.0466]⟩	⟨[0.0216,0.1586]⟩	⟨[0.4396,0.7392]⟩
U_{10}	⟨[0.1425,0.3474]⟩	⟨[0.6567,0.3858]	⟨[0.1258,0.1132]⟩	⟨[0.0734,0.1188]⟩

Step 2: By using Definition 1.1.1, the domination matrix is obtained by each row with other rows as follows:

Regions	U_1	U_2	U_3	U_4	U_5	U_6	U_7	U_8	U_9	U_{10}
U_1	⟨[2,2]⟩	⟨[1,0]⟩	⟨[4,1]⟩	⟨[3,1]⟩	⟨[6,2]⟩	⟨[3,0]⟩	⟨[5,1]⟩	⟨[4,1]⟩	⟨[3,0]⟩	⟨[3,1]⟩
U_2	⟨[3,5]⟩	⟨[2,5]⟩	⟨[8,5]⟩	⟨[4,6]⟩	⟨[6,5]⟩	⟨[4,6]⟩	⟨[5,5]⟩	⟨[7,5]⟩	⟨[6,4]⟩	⟨[6,4]⟩
U_3	⟨[2,6]⟩	⟨[1,5]⟩	⟨[5,9]⟩	⟨[4,7]⟩	⟨[6,5]⟩	⟨[3,7]⟩	⟨[5,5]⟩	⟨[4,7]⟩	⟨[3,5]⟩	⟨[3,5]⟩
U_4	⟨[1,6]⟩	⟨[1,6]⟩	⟨[3,9]⟩	⟨[2,9]⟩	⟨[2,9]⟩	⟨[1,8]⟩	⟨[1,7]⟩	⟨[0,9]⟩	⟨[3,6]⟩	⟨[1,8]⟩
U_5	⟨[0,1]⟩	⟨[0,0]⟩	⟨[2,1]⟩	⟨[1,1]⟩	⟨[1,2]⟩	⟨[1,0]⟩	⟨[0,0]⟩	⟨[0,0]⟩	⟨[1,0]⟩	⟨[0,1]⟩
U_6	⟨[1,7]⟩	⟨[1,6]⟩	⟨[3,9]⟩	⟨[2,9]⟩	⟨[2,9]⟩	⟨[2,9]⟩	⟨[1,8]⟩	⟨[1,9]⟩	⟨[3,7]⟩	⟨[1,9]⟩
U_7	⟨[1,4]⟩	⟨[1,3]⟩	⟨[4,4]⟩	⟨[3,4]⟩	⟨[3,4]⟩	⟨[3,4]⟩	⟨[3,3]⟩	⟨[3,5]⟩	⟨[3,4]⟩	⟨[3,3]⟩
U_8	⟨[1,6]⟩	⟨[1,5]⟩	⟨[4,9]⟩	⟨[3,7]⟩	⟨[4,5]⟩	⟨[3,7]⟩	⟨[3,5]⟩	⟨[4,8]⟩	⟨[3,5]⟩	⟨[3,5]⟩
U_9	⟨[0,6]⟩	⟨[0,5]⟩	⟨[3,7]⟩	⟨[1,7]⟩	⟨[1,5]⟩	⟨[1,7]⟩	⟨[0,5]⟩	⟨[0,7]⟩	⟨[3,5]⟩	⟨[0,5]⟩
U_{10}	⟨[3,3]⟩	⟨[1,0]⟩	⟨[5,2]⟩	⟨[4,1]⟩	⟨[6,2]⟩	⟨[3,0]⟩	⟨[5,2]⟩	⟨[4,2]⟩	⟨[3,0]⟩	⟨[4,2]⟩

Step 3: The cumulative domination value η is constructed by computing as follows:

Regions	U_1	U_2	U_3	U_4	U_5	U_6	U_7	U_8	U_9	U_{10}	Total
U_1	4	1	5	4	8	3	6	5	3	4	43
U_2	8	7	13	10	11	10	10	12	10	10	101
U_3	8	6	14	11	11	10	10	11	8	8	97
U_4	7	7	12	11	11	9	8	9	9	9	92
U_5	1	0	3	2	3	1	0	0	1	1	12
U_6	8	7	12	11	11	11	9	10	10	10	99
U_7	5	4	8	7	7	7	6	8	7	6	65
U_8	7	6	13	10	9	10	8	12	8	8	91
U_9	6	5	10	8	6	8	5	7	8	5	68
U_{10}	6	1	7	5	8	3	7	6	3	6	52
Total	60	44	97	79	85	72	69	80	67	67	720

Step 4: The algebraic sums is computed as below:

Regions	Total (Row)	Total (Column)	Difference (Row -Column)	Rank
U_1	43	60	-17	9
U_2	101	44	57	1
U_3	9	7	9	7
U_4	9	2	7	9
U_5	1	2	8	5
U_6	9	9	7	2
U_7	6	5	6	9
U_8	9	1	8	0
U_9	6	8	6	7
U_{10}	5	2	6	7

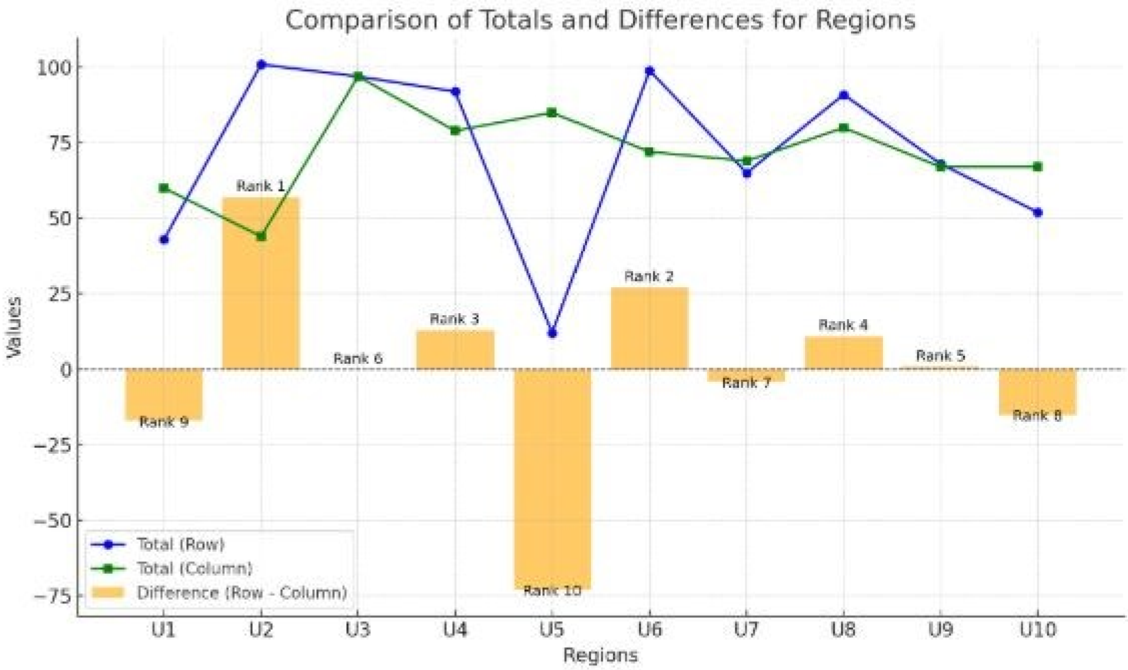

Figure 1. Graphical Representation of the Natural Disaster Management Center

Step 5: Based on the rank derived from the table. The region U_2 is identified as the most vulnerable, with thehighest positive difference between the total row and total column values (57), indicating it is the top priority forintervention. Regions $U_6(27)$ and $U_4(13)$ follow as the second and third most vulnerable regions, respectively,based on their rankings. These regions should also be given significant attention due to their relatively high positivedifferences. On the other hand, U_5 has the largest negative difference (-73) and is ranked the lowest, indicatingit is the least affected or least vulnerable region in this analysis. Overall, priority should be given to the regions with higher positive differences to mitigate the impact of the disease outbreak effectively.

Analysis: The table ranks regions $U_1, U_2, \ldots\ , U_{10}$ based on the difference between their row totals (indicatingvulnerability scores) and column totals (representing resilience or health support measures). A positive differencehighlights higher vulnerability, while a negative difference suggests relative stability or preparedness.

· **Most Vulnerable Region:**U_2 ranks first with the highest positive difference $(+57)$. This suggests that U_2 faces the greatest vulnerability due to its high vulnerability score and comparatively lower resiliencescore. Immediate interventions such as enhancing healthcare facilities, improving sanitation, and deploying emergency resources are critical for this region.

· **Moderately Vulnerable Regions:**U_6 and U_4 rank second and third, with positive differences of $+27$and $+13$, respectively. These regions also require prioritized efforts, focusing on improving access toclean water, healthcare facilities, and public hygiene awareness.

· **Stable or Resilient Regions:** Regions such as U_8(Rank $4, Difference$: $+11$) and U_9 (Rank 5, Difference:$+1$) show moderate stability. While they require ongoing monitoring and preventive measures, theircurrent health systems appear to manage vulnerabilities better.

· **Least Vulnerable Region:**U_5 ranks last with the most negative difference (-73) . This indicates that U_5benefits from a combination of strong health infrastructure and low vulnerability factors. Lessons learnedfrom U_5 should be analyzed and considered for replication in other vulnerable regions.

Recommendations

(1) **Urgent Focus on $\boldsymbol{U_2}$:** Deploy emergency resources, improve healthcare accessibility, and enhance sanitation systems.

(2) **Address Gaps in $\boldsymbol{U_6}$ and $\boldsymbol{U_4}$:** Strengthen public health awareness, ensure clean water availability, andimprove resilience against outbreaks.

(3) **Sustain Stability in $\boldsymbol{U_8}$and $\boldsymbol{U_9}$:** Monitor disease trends and maintain health infrastructure investments.

(4) **Replicate Best Practices from $\boldsymbol{U_5}$:** Study $U_5 s$ successful strategies and implement them across othervulnerable regions to minimize future risks.

This evaluation provides a clear, data-driven roadmap to guide resource allocation and decision-making inmanaging the outbreak across the regions.

Example 3.3. Assume that a Natural Disaster Management Center (NDMC) needs to identify the most vulnerable regions among nine areas: $U = \{U_1, U_2, \ldots, U_9\}$ These regions will be evaluatedbased on a set of criteria$C = \{c_1, c_2, c_3, \ldots, c_9\}$, where the criteria are as follows:

(1) **Seismic Risk($\boldsymbol{c_1}$) :** This criterion assesses the seismic activity of a region, including the frequency andintensity of earthquakes. Higher seismic risk increases the likelihood of severe damage in the event of anearthquake.

(2) **Flood Risk ($\boldsymbol{c_2}$) :** This evaluates the likelihood of flooding in the region, considering factors like theproximity to rivers, coastal lines, and historical flood data. Regions at higher risk for flooding are morevulnerable to disasters like tsunamis and flash floods.

(3) **Infrastructure Quality ($\boldsymbol{c_3}$) :** This criterion measures the strength and quality of the regions infrastructure, including buildings, roads, and bridges. Weak infrastructure is more prone to collapse

during naturaldisasters, exacerbating the overall damage.

(4) **Population Density** (c_4) **:** Evaluates how densely populated a region is. Higher population densities canresult in more casualties and greater challenges for evacuation and recovery.

(5) **Emergency Response Time** (c_5) **:** This criterion measures how quickly emergency services can respondto a disaster. A shorter response time can significantly reduce the impact of a disaster.

(6) **Access to Emergency Supplies** (c_6) **:** This evaluates the availability and accessibility of emergencysupplies like food, water, medical supplies, and temporary shelters. Regions with limited access to theseresources are more likely to suffer from prolonged consequences.

(7) **Vulnerability of Critical Infrastructure** (c_7) **:** This assesses how vulnerable critical infrastructures (e.g.,power plants, hospitals, communication systems) are to natural disasters. Disruptions to critical infrastructure can delay recovery efforts and worsen the effects of the disaster.

(8) **Topography** (c_8) **:** Evaluates the natural features of the land, such as mountains, plains, or valleys. Certain topographical features can either mitigate or worsen the impact of natural disasters (e.g., floodplainsare more vulnerable to floods).

(9) **Public Awareness of Disaster Preparedness** (c_9) **:** This criterion assesses the level of public awareness and preparedness for natural disasters. Regions with better public preparedness, including training,evacuation plans, and disaster drills, are more resilient.

Step 1: The table is constructed using the PyFSs as below:

Regions	c_1	c_2	c_3	c_4	c_5
Cl	$\langle[0.0507,0.5935]\rangle$	$\langle[0.6362,0.3511]\rangle$	$\langle[0.2939,0.0579]\rangle$	$\langle[0.4227,0.7931]\rangle$	$\langle[0.7805,0.11]\rangle$
C2	$\langle[0.7873,0.3068]\rangle$	$\langle[0.7852,0.529]\rangle$	$\langle[0.3761,0.3407]\rangle$	$\langle[0.2511,0.1405]\rangle$	$\langle[0.3454,0.1997]\rangle$
C3	$\langle[0.3065,0.9208]\rangle$	$\langle[0.2243,0.3963]\rangle$	$\langle[0.0364,0.3449]\rangle$	$\langle[0.448,0.7278]\rangle$	$\langle[0.8468,0.4085]\rangle$
C4	$\langle[0.6624,0.6779]\rangle$	$\langle[0.4711,0.6735]\rangle$	$\langle[0.5582,0.7604]\rangle$	$\langle[0.0752,0.6623]\rangle$	$\langle[0.1835,0.7013]\rangle$
C5	$\langle[0.2242,0.3338]\rangle$	$\langle[0.9297,0.0186]\rangle$	$\langle[0.5723,0.8125]\rangle$	$\langle[0.6602,0.0112]\rangle$	$\langle[0.3829,0.6161]\rangle$
C6	$\langle[0.9647,0.1245]\rangle$	$\langle[0.7555,0.5765]\rangle$	$\langle[0.3237,0.5256]\rangle$	$\langle[0.5139,0.3114]\rangle$	$\langle[0.2892,0.9396]\rangle$
C7	$\langle[0.5062,0.8192]\rangle$	$\langle[0.0714,0.622]\rangle$	$\langle[0.9286,0.3208]\rangle$	$\langle[0.2372,0.659]\rangle$	$\langle[0.523,0.3762]\rangle$
C8	$\langle[0.15,0.6166]\rangle$	$\langle[0.7377,0.3434]\rangle$	$\langle[0.4049,0.1419]\rangle$	$\langle[0.8508,0.0141]\rangle$	$\langle[0.8825,0.415]\rangle$
C9	$\langle[0.0617,0.2204]\rangle$	$\langle[0.5823,0.5048]\rangle$	$\langle[0.4562,0.8388]\rangle$	$\langle[0.7065,0.4044]\rangle$	$\langle[0.422,0.6449]\rangle$
C10	$\langle[0.3397,0.6501]\rangle$	$\langle[0.3672,0.868]\rangle$	$\langle[0.4447,0.5856]\rangle$	$\langle[0.7709,0.4366]\rangle$	$\langle[0.6698,0.0381]\rangle$

Regions	c_6	c_7	c_8	c_9
C1	$\langle[0.3205, 0.5626]\rangle$	$\langle[0.0229, 0.2742]\rangle$	$\langle[0.5415, 0.3742]\rangle$	$\langle[0.4722, 0.1617]\rangle$
C2	$\langle[0.1997, 0.8427]\rangle$	$\langle[0.4624, 0.7076]\rangle$	$\langle[0.1943, 0.2354]\rangle$	$\langle[0.4819, 0.5779]\rangle$
C3	$\langle[0.8593, 0.4316]\rangle$	$\langle[0.7468, 0.0670]\rangle$	$\langle[0.0094, 0.8684]\rangle$	$\langle[0.6321, 0.0055]\rangle$
C4	$\langle[0.4911, 0.4760]\rangle$	$\langle[0.1982, 0.1781]\rangle$	$\langle[0.0823, 0.7913]\rangle$	$\langle[0.4516, 0.7854]\rangle$
C5	$\langle[0.7950, 0.2705]\rangle$	$\langle[0.5972, 0.3230]\rangle$	$\langle[0.7997, 0.5636]\rangle$	$\langle[0.1367, 0.3297]\rangle$
C6	$\langle[0.5456, 0.3586]\rangle$	$\langle[0.8541, 0.4973]\rangle$	$\langle[0.2973, 0.0586]\rangle$	$\langle[0.6910, 0.3997]\rangle$
C7	$\langle[0.3999, 0.9075]\rangle$	$\langle[0.3512, 0.5510]\rangle$	$\langle[0.6452, 0.2217]\rangle$	$\langle[0.6529, 0.0899]\rangle$
C8	$\langle[0.0978, 0.4955]\rangle$	$\langle[0.6999, 0.4598]\rangle$	$\langle[0.3647, 0.4066]\rangle$	$\langle[0.1787, 0.5539]\rangle$
C9	$\langle[0.2383, 0.0762]\rangle$	$\langle[0.7868, 0.3762]\rangle$	$\langle[0.3131, 0.4038]\rangle$	$\langle[0.0449, 0.8615]\rangle$
C10	$\langle[0.2268, 0.0451]\rangle$	$\langle[0.6096, 0.5784]\rangle$	$\langle[0.8061, 0.2846]\rangle$	$\langle[0.0177, 0.3961]\rangle$

Step 2: By using Definition 1.1.1, the domination matrix is obtained by each row with other rows as follows:

Regions	U_1	U_2	U_3	U_4	U_5	U_6	U_7	U_8	U_9	U_{10}
U_1	$\langle[2,2]\rangle$	$\langle[0,2]\rangle$	$\langle[2,3]\rangle$	$\langle[0,7]\rangle$	$\langle[0,2]\rangle$	$\langle[0,1]\rangle$	$\langle[0,4]\rangle$	$\langle[0,1]\rangle$	$\langle[1,3]\rangle$	$\langle[1,2]\rangle$
U_2	$\langle[9,5]\rangle$	$\langle[9,7]\rangle$	$\langle[7,7]\rangle$	$\langle[9,8]\rangle$	$\langle[6,6]\rangle$	$\langle[7,7]\rangle$	$\langle[8,7]\rangle$	$\langle[7,7]\rangle$	$\langle[9,7]\rangle$	$\langle[8,6]\rangle$
U_3	$\langle[3,0]\rangle$	$\langle[4,0]\rangle$	$\langle[4,1]\rangle$	$\langle[4,0]\rangle$	$\langle[2,0]\rangle$	$\langle[2,1]\rangle$	$\langle[2,0]\rangle$	$\langle[3,0]\rangle$	$\langle[3,0]\rangle$	$\langle[2,0]\rangle$
U_4	$\langle[8,1]\rangle$	$\langle[7,2]\rangle$	$\langle[6,3]\rangle$	$\langle[9,5]\rangle$	$\langle[6,1]\rangle$	$\langle[5,1]\rangle$	$\langle[8,2]\rangle$	$\langle[5,0]\rangle$	$\langle[7,2]\rangle$	$\langle[6,1]\rangle$
U_5	$\langle[2,5]\rangle$	$\langle[3,6]\rangle$	$\langle[2,7]\rangle$	$\langle[4,8]\rangle$	$\langle[2,4]\rangle$	$\langle[0,6]\rangle$	$\langle[1,6]\rangle$	$\langle[3,7]\rangle$	$\langle[2,7]\rangle$	$\langle[1,6]\rangle$
U_6	$\langle[9,7]\rangle$	$\langle[9,9]\rangle$	$\langle[9,7]\rangle$	$\langle[9,9]\rangle$	$\langle[9,7]\rangle$	$\langle[9,8]\rangle$	$\langle[9,8]\rangle$	$\langle[9,8]\rangle$	$\langle[9,8]\rangle$	$\langle[9,7]\rangle$
U_7	$\langle[6,0]\rangle$	$\langle[7,1]\rangle$	$\langle[5,2]\rangle$	$\langle[7,0]\rangle$	$\langle[3,0]\rangle$	$\langle[3,1]\rangle$	$\langle[5,2]\rangle$	$\langle[5,0]\rangle$	$\langle[6,2]\rangle$	$\langle[5,1]\rangle$
U_8	$\langle[2,1]\rangle$	$\langle[1,2]\rangle$	$\langle[2,3]\rangle$	$\langle[2,7]\rangle$	$\langle[1,1]\rangle$	$\langle[0,1]\rangle$	$\langle[1,4]\rangle$	$\langle[2,1]\rangle$	$\langle[2,3]\rangle$	$\langle[1,2]\rangle$
U_9	$\langle[2,6]\rangle$	$\langle[0,8]\rangle$	$\langle[2,7]\rangle$	$\langle[0,8]\rangle$	$\langle[0,7]\rangle$	$\langle[0,7]\rangle$	$\langle[0,8]\rangle$	$\langle[0,7]\rangle$	$\langle[2,8]\rangle$	$\langle[1,7]\rangle$
U_{10}	$\langle[4,1]\rangle$	$\langle[4,2]\rangle$	$\langle[4,3]\rangle$	$\langle[4,7]\rangle$	$\langle[2,1]\rangle$	$\langle[3,1]\rangle$	$\langle[2,3]\rangle$	$\langle[3,0]\rangle$	$\langle[4,2]\rangle$	$\langle[3,2]\rangle$

Step 3: The cumulative domination value η is constructed by computing as follows:

Regions	U_1	U_2	U_3	U_4	U_5	U_6	U_7	U_8	U_9	U_{10}	Total
U_1	4	2	5	7	2	1	4	1	4	3	33
U_2	14	16	14	17	12	14	15	14	16	14	146
U_3	3	4	5	4	2	3	2	3	3	2	31
U_4	9	9	9	14	7	6	10	5	9	7	85
U_5	7	9	9	12	6	6	7	10	9	7	82
U_6	16	18	16	18	16	17	17	17	17	16	168
U_7	6	8	7	7	3	4	7	5	8	6	61
U_8	3	3	5	9	2	1	5	3	5	3	39
U_9	8	8	9	8	7	7	8	7	10	8	80
U_{10}	5	6	7	11	3	4	5	3	6	5	55
Total	75	83	86	107	60	63	80	68	87	71	780

Step 4: The algebraic sums is computed as below:

Regions	Total (Row)	Total (Column)	Difference (Row -Column)	Rank
U_1	33	75	-42	9
U_2	146	83	63	2
U_3	31	86	-55	10
U_4	85	107	-22	7
U_5	82	60	22	3
U_6	168	63	105	1
U_7	61	80	-19	6
U_8	39	68	-29	8
U_9	80	87	-7	4
U_{10}	55	71	-16	5

Analysis: The table ranks regions $U_1, U_2, \ldots$, U_{10} based on the difference between their row totals (representing overall vulnerability scores) and column totals (indicating resilience or support levels). A positive differencesuggests higher vulnerability, while a negative difference indicates relative stability or preparedness.

· **Most Vulnerable Region:** U_6 ranks first with the highest positive difference $(+105)$. This indicates thatRegion U_6 is the most vulnerable due to its high vulnerability score and comparatively low resiliencescore. Immediate focus should be directed to enhancing emergency preparedness, infrastructure, andresource accessibility in this region.

· **Moderately Vulnerable Regions:** U_2 and U_5 follow with ranks 2 and 3, having positive differences of $+63$ and $+22$, respectively. While less critical than U_6, these regions also exhibit significant vulnerabilities, requiring prioritized interventions.

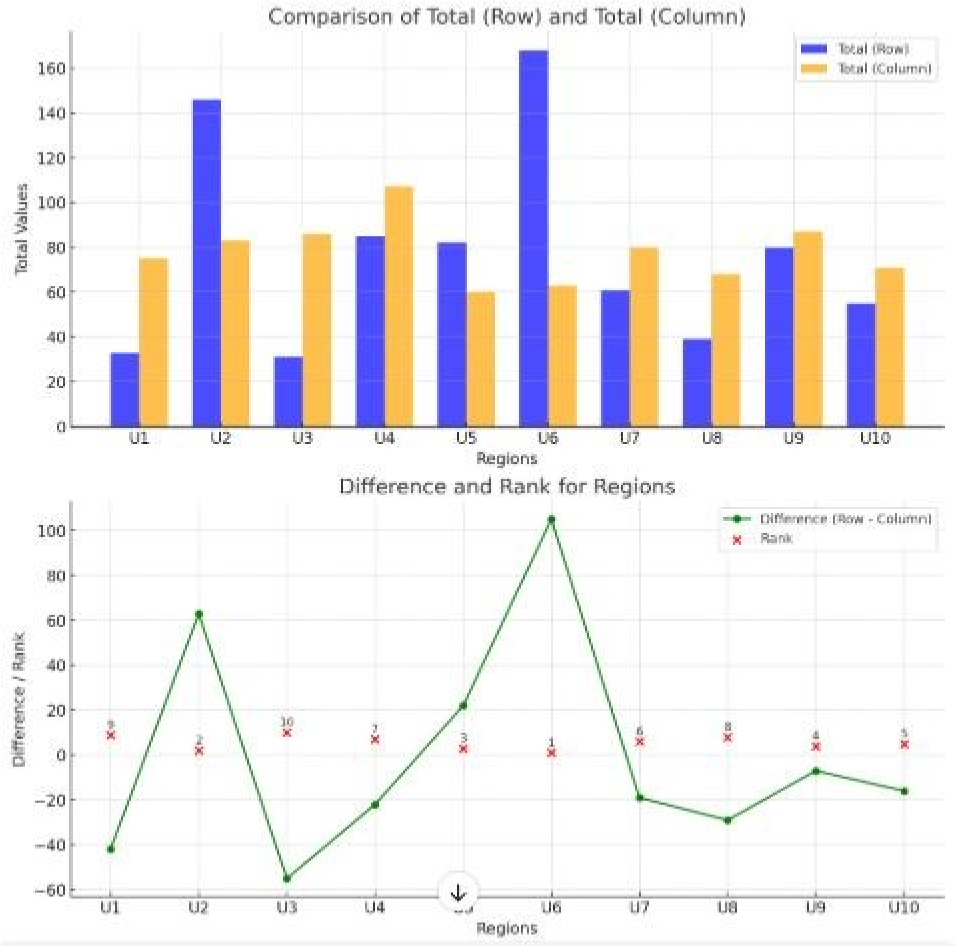

Figure 2. Graphical Representation of the Natural Disaster Management Center

·**Stable or Resilient Regions:** Regions such as U_9(Rank 4, $Difference: -7$) and U_{10} (Rank5, Difference: -16) show moderate stability. However, continued monitoring and incremental improvements in disasterpreparedness are recommended.

·**Least Vulnerable Region:**U_3 ranks last with the most negative difference (-55) . This suggests that U_3benefits from relatively high resilience and lower vulnerability scores. It serves as a model for disastermanagement strategies that could be replicated in more vulnerable regions.

Recommendations

(1) **Focus on U_6 :** Strengthen emergency response systems, infrastructure, and disaster preparedness training

(2) **Enhance $U_2$and U_5:** Address gaps in critical infrastructure and access to emergency supplies to reducevulnerability.

(3) **Maintain Stability in $U_9$and U_{10}:** Invest in long-term resilience measures while leveraging theircurrent strengths.

(4) **Learn from U_3:** Analyze U_3 success factors and apply these insights to other regions where applicable.This structured approach ensures a data-driven prioritization of resources and efforts to minimize vulnerabilitiesacross all regions.

4. CONCLUSION

In this work, we explored the progressive evolution of fuzzy set theory, transitioning from classical fuzzy setsto intuitionistic fuzzy sets and subsequently to Pythagorean fuzzy sets. Each of these frameworks enhances thecapacity to handle uncertainty and imprecision in data, with Pythagorean fuzzy sets offering a higher degree offlexibility due to their relaxed constraint $\mu^2 + v^2 \leq 1$. This property makes them particularly effective for modelingcomplex decision-making scenarios where ambiguity and incomplete information are prevalent.

The comparative analysis presented highlights the distinct characteristics of these frameworks, emphasizingtheir suitability for different levels of uncertainty. Fuzzy sets remain a reliable choice for simple uncertainties,while intuitionistic fuzzy sets and Pythagorean fuzzy sets extend the applicability to moderate and high-uncertaintysituations, respectively. Through examples and a review of recent literature, we illustrated how Pythagoreanfuzzy sets outperform their predecessors in diverse applications, ranging from health monitoring and disastermanagement to clustering and decision-making.

This study underscores the importance of selecting an appropriate mathematical framework tailored to thespecific nature of the problem. The increasing adoption of Pythagorean fuzzy sets in various fields signifiestheir potential to address real-world challenges effectively. Future research could focus on developing advancedoperations, distance measures, and aggregation operators within the PyFS framework, further expanding theirapplicability and enhancing decision-making accuracy.

In summary, the integration of fuzzy sets, intuitionistic fuzzy sets, and Pythagorean fuzzy sets into decision-making processes provides a robust toolkit for addressing uncertainty in complex systems, paving the way forinnovative solutions in a wide range of domains.

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