Fuzzy Transportation Optimization using Minimum Spanning Tree and Intuitionistic Fuzzy Ranking

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Abstract

This paper presents a novel approach for solving transportation problems involving trapezoidal intuitionistic fuzzy numbers (TIFNs) using the Minimum Spanning Tree (MST) procedure, along with value and ambiguity index ranking. The proposed method defuzzifies TIFNs to obtain crisp numbers, facilitating the determination of the optimal transportation plan via the MST procedure and value and ambiguity ranking. By accounting for the fuzzy nature of transportation costs, demands, and supplies, this approach provides a more realistic and robust solution. A numerical example demonstrates the effectiveness of the proposed method, showcasing its ability to efficiently handle fuzzy transportation problems and minimize total transportation cost. This research contributes to the development of fuzzy optimization techniques in transportation planning, offering a practical solution for decision-makers in logistics and supply chain management.

Keywords : Intuitionistic fuzzy numbers; transportation problem; Minimum Spanning Tree; Decision making; Supply Chain Management.

Introduction

The intuitionistic fuzzy sets were introduced by Atanassov [2] as a generalization of the fuzzy sets defined by Zadeh [28]. Unlike fuzzy sets, which are characterized solely by a membership function, intuitionistic fuzzy sets are characterized by both a membership function and a non-membership function. For this reason, intuitionistic fuzzy sets are more useful in expressing uncertainty and vagueness. Thus, defining the ranking of intuitionistic fuzzy numbers has represented a significant scientific pursuit for researchers, as they play an essential role, especially in 3 key areas: (i) Multi-Criteria Decision Making (MCDM), (ii) Economics, and (iii) Social Sciences and Engineering.

Since the inception of intuitionistic fuzzy numbers, researchers have approached them from various angles, proposing numerous classification methods. In 2003, Grzegrorzewski [7] proposed a ranking method using the expected interval, followed by Mitchell's [12] approach in 2004, which regarded intuitionistic fuzzy numbers as an ensemble of fuzzy numbers. In 2006, Nayagam, Venkateshwari, and Sivaraman [15] generalized Chen and Hwang's method, expanding its scope from fuzzy to intuitionistic fuzzy numbers. Subsequent years saw significant contributions, including Li's [10] ranking function in 2010, De and Das's [4] method in 2012, and several proposals in 2013, such as Rezvani's [22] value-index and ambiguity-index based approach, Roseline and Amirtharaj's [23] magnitude-based method, and Jafarian and Rezvani's [9] crisp value-based approach. Many papers addressing this research topic were published ([19], [16], [3], [24], [26], [1], [13]).

The purpose of classifying intuitionistic fuzzy numbers is to facilitate their utilization in real-life problems, especially in decision-making problems where information is uncertain. To address this, several ranking methods have been proposed, including Whang and Zhang's [25] method in 2009, Li et al.'s [11] method in 2010, and Ye's [8] method in 2011. These methods have been applied to solve MCDM problems in an intuitionistic fuzzy environment. In subsequent years, known methods for solving MCDM problems were adapted to an intuitionistic fuzzy context [17] and applied in various fields [21, 27, 29, 6, 5]. However, applying different classification methods for intuitionistic fuzzy numbers can lead to different ranking relationships. In transportation problems, 1. North West Corner method, 2. Least Cost method, and 3. Vogel’s Approximation method (VAM) [41] can be used to obtain basic feasible solutions. The optimality of the transportation problem can be checked using the MODI method [33]. Transportation problems can be classified into two types: 1. balanced transportation problems and 2. unbalanced transportation problems. The basic transportation problem was originally developed by Hitchcock [37]. Various methods have been proposed to solve transportation problems, including the simplex method [34], steppingstone method [32], and fuzzy optimization methods [30, 31, 35, 36, 38, 39, 40, 42, 44]. In real-life situations, uncertainty and imprecision in data are common, and fuzzy decision-making methods can be used to address these issues [43].

This paper is organized as follows: Section 2 provides preliminaries on intuitionistic fuzzy concepts, including intuitionistic trapezoidal fuzzy numbers and arithmetic operations. Section 3 explains optimizing fuzzy transportation with minimum spanning tree and intuitionistic fuzzy approach. Section 4 illustrates a numerical example applying this methodology to transportation problem with intuitionistic trapezoidal fuzzy numbers using minimum spanning tree and value and ambiguity index ranking. Comparative study presented in Section 5. The paper concludes with results and discussion in Section 6.

2. Intuitionistic Fuzzy Concepts

This section covers basic concepts of fuzzy sets including intuitionistic trapezoidal fuzzy numbers and arithmetic operations [2].

2.1 Intuitionistic fuzzy set

Intuitionistic fuzzy set $\widetilde{A}$ in universal set X is define as the following form $\widetilde{A} = \{(x, f_{\tilde{A}}(x), g_{\tilde{A}}(x)) : x \epsilon X\}$ where the function $f_{\tilde{A}}:$ X → [0, 1] define the degree of membership and the function $g_{\tilde{A}}$: X → [0, 1] define the degree of non-membership and $0 \leq f_{\tilde{A}}(x) + g_{\tilde{A}}(x) \leq 1$, for every x ∈ X.

Example : Let X be the set of all countries with elective governments. Assume that we know for every country x∈ X the percentage of the electorate who have voted for the corresponding government. Let it be denoted by M (x) and let $f(x) = \frac{M(x)}{100}$. Let $g(x) = 1 - f(x)$, This number corresponds to that part of electorate who have not voted for the government. By means of the fuzzy set theory we cannot consider this value in more detail. However, if we define g(x) as the number of votes given to parties or persons ouatside the government, then we can show the part of electorate who have not voted at all and the corresponding number will be $1 - f(x) - g(x)$. Thus we can construct the set $\{\langle x, f(x), g(x)\rangle | x \in X\}$ and obviously, $0 \leq f(x) + g(x) \leq 1$, for every x ∈ X.

Generalized Trapezoidal Intuitionistic Fuzzy Number (GTIFN)

A generalized trapezoidal intuitionistic fuzzy number $\widetilde{A} = \{(a_1, a_2, a_3, a_4, a'_1, a'_2, a'_3, a'_4); m_a, n_a\}$ is a fuzzy set on a set of real number R, whose membership function and non-membership function are defined as follows respectively as given in Figure 1.

$$f_{\tilde{A}} = \begin{cases} 0, & x < a_1 \\ \frac{x-a_1}{a_2-a_1} m_a, & a_1 \le x \le a_2 \\ m_a, & a_2 \le x \le a_3 \\ \frac{a_4-x}{a_4-a_3} m_a, & a_3 \le x \le a_4 \\ 0, & a_4 < x \end{cases} \quad \text{and} \quad g_{\tilde{A}} = \begin{cases} 1, & x < a'_1 \\ \frac{(a'_2-x)+n_a(x-a'_1)}{(a'_2-a'_1)}, & a'_1 \le x \le a'_2 \\ n_a, & a'_2 \le x \le a'_3 \\ \frac{(x-a'_3)+n_a(a'_4-x)}{a'_4-a'_3}, & a'_3 \le x \le a'_4 \\ 1, & a'_4 < x \end{cases}$$

The values m_a and n_a represents the maximum degree of membership function and minimum degree of non-membership, respectively, such that the conditions $0 \le m_a \le 1$, $0 \le n_a \le 1$, and $0 \le m_a + n_a \le 1$ are satisfied. The parameters m_a and n_a reflects the confidence level and non-confidence level of the elements x in $\tilde{A}$

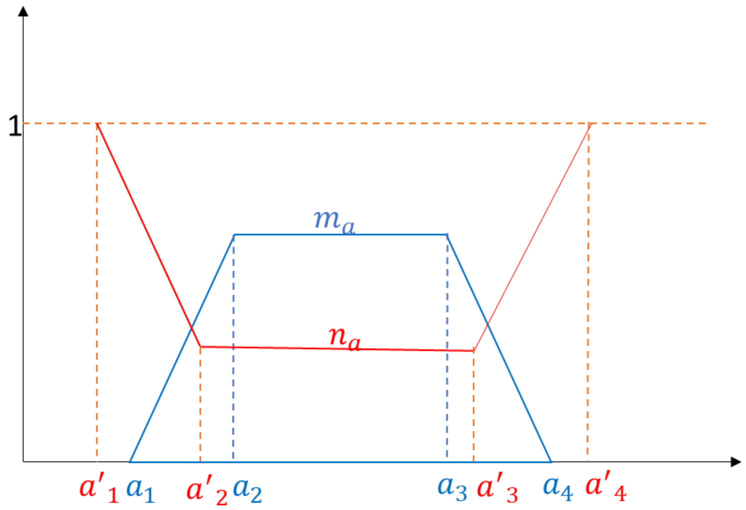

Figure 1: Generalized Trapezoidal Intuitionistic Fuzzy Number
$\tilde{A} = \{(a_1, a_2, a_3, a_4, a'_1, a'_2, a'_3, a'_4); m_a, n_a\}$

Positive GTIFN

If each number in $\tilde{A}$ is greater than or equal to zero and at least one of them is not equal to zero, then $\tilde{A}$ is called a positive GTIFN and it is denoted by $\tilde{A} > 0$.

Negative GTIFN

If each number in $\tilde{A}$ is less than or equal to zero and at least one of them is not equal to zero, then $\tilde{A}$ is called a negative GTIFN and it is denoted by $\tilde{A} < 0$.

Arithmetic operations:

Let $\tilde{A} = \{(a_1, a_2, a_3, a_4, a'_1, a'_2, a'_3, a'_4); m_a, n_a\}$

and $\widetilde{\mathrm{B}} = \{(b_1, b_2, b_3, b_4, b'_1, b_2, b_3, b'_4); m_b, n_b\}$ be two GTIFN's.

Then the arithmetic operations are

(i) Addition:

$$\tilde{A} \oplus \tilde{B} = \begin{Bmatrix} (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a'_1 + b'_1, a'_2 + b'_2 \\ a'_3 + b'_3, a'_4 + b'_4); \min(m_a, m_b), \max(n_a, n_b) \end{Bmatrix}$$

(ii) Subtraction:

$$\tilde{A} \,\Theta\, \tilde{B} = \begin{Bmatrix} (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1, a'_1 - b'_4, a_2 - b'_3, \\ a'_3 - b'_2, a'_4 - b'_1; \min(m_a, m_b), \max(n_a, n_b) \end{Bmatrix}$$

(iii) Scalar Multiplication:

$$k \otimes \tilde{A} = \begin{Bmatrix} \{(ka_1, ka_2, ka_3, ka_4, ka'_1, ka'_2, ka'_3, ka'_4); m_a, n_a\}\; if\; k \geq 0 \\ \{(ka_4, ka_3, ka_2, ka_1, ka'_4, ka'_3, ka'_2, ka'_1); m_a, n_a\}\; if\; k < 0 \end{Bmatrix}$$

(iv) Multiplication :

$$\tilde{A} \otimes \tilde{B} = \begin{cases} \begin{Bmatrix} (a_1b_1, a_2b_2, a_3b_3, a_4b_4, a'_1b'_1, a_2b_2, \\ a_3b_3, a'_4b'_4); \min(m_a, m_b), \max(n_a, n_b) \end{Bmatrix}; \tilde{A} > 0; \tilde{B} > 0 \\ \begin{Bmatrix} (a_1b_4, a_2b_3, a_3b_2, a_4b_1, a'_1b'_4, a_2b_3, \\ a_3b_2, a'_4b'_1); \min(m_a, m_b), \max(n_a, n_b) \end{Bmatrix}; \tilde{A} < 0, \tilde{B} > 0 \\ \begin{Bmatrix} (a_4b_4, a_3b_3, a_2b_2, a_1b_1, a'_4b'_4, a'_3b'_3, \\ a'_2b'_2, a'_1b'_1); \min(m_a, m_b), \max(n_a, n_b) \end{Bmatrix}; \tilde{A} < 0; \tilde{B} < 0 \end{cases}$$

α-cut

A $\alpha - cut$ set of a GTIFN $\widetilde{\mathrm{A}} = \{(a_1, a_2, a_3, a_4, a'_1, a'_2, a'_3, a'_4); m_a, n_a\}$ is a crisp subset of R defined as $\tilde{A}_\alpha = \{x / f_{\tilde{A}}(x) \geq \alpha\}$ where $0 \leq \alpha \leq m_a$

β **-cut**

A $\beta - cut$ set of a GTIFN $\widetilde{\mathrm{A}} = \{(a_1, a_2, a_3, a_4, a'_1, a'_2, a'_3, a'_4); m_a, n_a\}$ is a crisp subset of R defined as $\tilde{A}_\beta = \{x / g_{\tilde{A}}(x) \geq \beta\}$ where $n_a \leq \beta \leq 1$

By the definition of GTIFN , $\alpha - cut$ and $\beta - cut$ is easily followed that $\tilde{A}_\alpha$ and $\tilde{A}_\beta$ are closed sets and denoted by $\tilde{A}_\alpha = [L_{\tilde{A}}(\alpha), R_{\tilde{A}}(\alpha)]$ and $\tilde{A}_\beta = [L_{\tilde{A}}(\beta), R_{\tilde{A}}(\beta)]$

The value of $\tilde{A}_\alpha$ and $\tilde{A}_\beta$ are calculated as follows:

$$[L_{\tilde{A}}(\alpha), R_{\tilde{A}}(\alpha)] = \left[a_1 + \frac{\alpha(a_2 - a_1)}{m_a}, a_4 - \frac{\alpha(a_4 - a_3)}{m_a}\right]$$

$$[L_{\tilde{A}}(\beta), R_{\tilde{A}}(\beta)] = \left[\frac{(1-\beta)a'_2 + (\beta - n_a)a'_1}{m_a}, \frac{(1-\beta)a'_3 + (\beta - n_a)a'_4}{m_a}\right]$$

Support of GTIFN for the membership function

Support of GTIFN $\widetilde{A} = \{(a_1, a_2, a_3, a_4, a'_1, a'_2, a'_3, a'_4); m_a, n_a\}$ for the membership function is defined as $Support_f(\tilde{A}) = \{x / f_{\tilde{A}}(x) \geq 0\}$

Support of GTIFN for the non-membership function

Support of GTIFN $A = \{(a_1, a_2, a_3, a_4, a'_1, a'_2, a'_3, a'_4); m_a, n_a\}$ for the non-membership function is defined as $Support_g(\tilde{A}) = \{g_{\tilde{A}}(x) \leq 1\}$.

2.2 Ranking GTIFNs using Value and Ambiguity with Theorems

Value of membership and non-membership functions

Let $\tilde{A}_\alpha$ and $\tilde{A}_\beta$ be an a $\alpha - cut$ set and a $\beta - cut$ set of GTIFN $\widetilde{A} =$ $\{(a_1, a_2, a_3, a_4, a'_1, a'_2, a'_3, a'_4); m_a, n_a\}$.

Then the value of membership function $f_{\tilde{A}}(x)$ and non-membership function $g_{\tilde{A}}(x)$ for GTIFN $\tilde{A}$ are defined as follows:

$$V_f(\tilde{A}) = \int_0^{m_a} \frac{L_{\tilde{A}}(\alpha) + R_{\tilde{A}}(\alpha)}{2} f(\alpha)\, d\alpha \quad , \quad V_g(\tilde{A}) = \int_{n_a}^{1} \frac{L_{\tilde{A}}(\beta) + R_{\tilde{A}}(\beta)}{2} g(\beta)\, d\beta$$

where we choose , $f(\alpha) = \frac{\alpha}{m_a}$ and $g(\beta) = \frac{1-\beta}{1-n_a}$

The value of membership function of GTIFN $\tilde{A}$ is calculated as follows:

$$V_f(\tilde{A}) = \int_0^{m_a} \left[a_1 + \frac{\alpha(a_2 - a_1)}{m_a} + a_4 - \frac{\alpha(a_4 - a_3)}{m_a}\right]\left(\frac{\alpha}{m_a}\right) d\alpha$$

$$= \left[\frac{a_1 + a_4}{2m_a}\, \alpha^2\right]_0^{m_a} + \left[\frac{a_2 - a_1 - a_4 - a_3}{3(m_a)^2}\right]_0^{m_a}$$

$$V_f(\tilde{A}) = \frac{[(a_1 + a_4) + 2(a_2 - a_3)]m_a}{6}$$

The value of non-membership function of GTIFN $\tilde{A}$ is calculated as follows:

$$V_g(\tilde{A}) = \int_{n_a}^{1} \left[\frac{(1-\beta)a'_2 + (\beta - n_a)a'_1}{m_a} + \frac{(1-\beta)a'_3 + (\beta - n_a)a'_4}{m_a}\right] \frac{1-\beta}{1-n_a}\, d\beta$$

$$= \int_{n_a}^{1} \left[\frac{(a'_2 + a'_3 - a'_1 - a'_4)(1-\beta)^2 + (1-n_a)(a'_1 + a'_4)(1-\beta)}{1-n_a}\right] d\beta$$

$$= -\left[\frac{(a'_2 + a'_3 - a'_1 - a'_4)(1-\beta)^3}{3(1-n_a)^2}\right]_{n_a}^{1} - \left[\frac{(1-n_a)(a'_1 + a'_4)(1-\beta)^2}{2(1-n_a)^2}\right]_{n_a}^{1}$$

$$V_g(\tilde{A}) = \frac{[(a'_1 + a'_4 + 2(a'_2 - a'_3)](1-n_a)}{6}$$

$$V(\tilde{A}) = \frac{V_f(\tilde{A}) + V_g(\tilde{A})}{2}$$

$$V(\tilde{A}) = \frac{[(\mathrm{a}_1 + \mathrm{a}_4 + 2(\mathrm{a}_2 - \mathrm{a}_3)]m_a + [(a'_1 + a'_4 + 2(a'_2 - a'_3)](1 - n_a)}{12}$$

Theorem 1:

Let $\widetilde{\mathrm{A}} = \{(a_1, a_2, a_3, a_4, a'_1, a'_2, a'_3, a'_4); m_a, n_a\}$and $\widetilde{\mathrm{B}} =$ $\{(b_1, b_2, b_3, b_4, b'_1, b_2, b_3, b'_4); m_b, n_b\}$ be two GTIFN's with $m_a = m_b$ and $n_a = n_b$.Then,

$$V_f(\tilde{A} + \tilde{B}) = V_f(\tilde{A}) + V_f(\tilde{B})\ and$$

$$V_g(\tilde{A} + \tilde{B}) = V_g(\tilde{A}) + V_g(\tilde{B})$$

Proof :

$$\tilde{A} \oplus \tilde{B} = \begin{Bmatrix} (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a'_1 + b'_1, a'_2 + b'_2 \\ a'_3 + b'_3, a'_4 + b'_4); \min(m_a, m_b), \max(n_a, n_b) \end{Bmatrix}$$

$$V_f(\tilde{A} + \tilde{B}) = \frac{[(\mathrm{a}_1 + b_1) + 2(\mathrm{a}_2 + b_2) + 2(\mathrm{a}_3 - \mathrm{b}_3) + (a_4 + b_4)]m_a}{6}$$

$$= \frac{[(\mathrm{a}_1 + \mathrm{a}_4 + 2(\mathrm{a}_2 - \mathrm{a}_3)]m_a}{6} + \frac{[(\mathrm{b}_1 + \mathrm{b}_4 + 2(\mathrm{b}_2 - \mathrm{b}_3)]m_a}{6}$$

$$= V_f(\tilde{A}) + V_f(\tilde{B})$$

Similarly,

$$V_g(\tilde{A} + \tilde{B}) = \frac{[(a'_1 + b'_1) + 2(a'_2 + b'_2) + 2(a'_3 + b'_3) + (a'_4 + b'_4)](1 - n_a)}{6}$$

$$= \frac{[(a'_1 + a'_4 + 2(a'_2 - a'_3)](1 - n_a)}{6} + \frac{[(b'_1 + b'_4 + 2(b'_2 - b'_3)](1 - n_a)}{6}$$

$$= V_g(\tilde{A}) + V_g(\tilde{B})$$

Ambiguity of membership and non-membership functions

Let $\tilde{A}_\alpha$ and $\tilde{A}_\beta$ be an $\alpha - cut$ set and $\beta - cut$ set of GTIFN

$\widetilde{\mathrm{A}} = \{(a_1, a_2, a_3, a_4, a'_1, a'_2, a'_3, a'_4); m_a, n_a\}$.

Then the ambiguities of membership function $f_A(x)$ and non-membership function $g_A(x)$ for GTIFN of $\widetilde{\mathrm{A}}$ are defined as follows:

$I_f(\tilde{A}) = \int_0^{m_a} (R_{\tilde{A}}(\alpha) + L_{\tilde{A}}(\alpha))\ f(\alpha)\ d\alpha$, $I_g(\tilde{A}) = \int_{n_a}^{1} (R_{\tilde{A}}(\beta) + L_{\tilde{A}}(\beta))\ g(\beta)\ d\beta$

where $\mathrm{f}(\alpha) = \frac{\alpha}{\mathrm{m_a}}$ and $\mathrm{g}(\beta) = \frac{1-\beta}{1-n_a}$

The ambiguity of membership function for GTIFN of $\widetilde{\mathrm{A}}$ is calculated as follows:

$$I_f(\tilde{A}) = \int_0^{m_a} \left[a_4 + \frac{\alpha(a_4 - a_3)}{m_a} + a_1 - \frac{\alpha(a_2 - a_1)}{m_a}\right]\left(\frac{2\alpha}{m_a}\right) d\alpha$$

$$= \left[\frac{a_4 - a_1}{m_a}\ \alpha^2\right]_0^{m_a} - 2\left[\frac{a_2 - a_1 + a_4 - a_3}{3(m_a)^2}\alpha^3\right]_0^{m_a}$$

$$I_f(\tilde{A}) = \frac{[(\mathrm{a}_4 - \mathrm{a}_1) - 2(\mathrm{a}_2 - \mathrm{a}_3)]m_a}{3}$$

The ambiguity of non-membership function of for GTIFN of $\widetilde{\mathrm{A}}$ is calculated as follows:

$$I_g(\tilde{A}) = \int_{n_a}^{1}\left[\frac{(1-\beta)a_3' + (\beta - n_a)a_4'}{1-n_a} - \frac{(1-\beta)a_2' + (\beta - n_a)a_1'}{1-n_a}\right]\frac{2(1-\beta)}{1-n_a}\ d\beta$$

$$=\int_{n_a}^{1}\left[\frac{2(a_2'-a_3'-a_1'+a_4')(1-\beta)^2+(1-n_a)(a_4'-a_1')(1-\beta)}{(1-n_a)^2}\right]d\beta$$

$$=\left[\frac{2(a_4'-a_1'+a_2'-a_3')(1-\beta)^3}{3(1-n_a)^2}\right]_{n_a}^{1} - \left[\frac{(1-n_a)(a_4'-a_1')(1-\beta)^2}{(1-n_a)^2}\right]_{n_a}^{1}$$

$$I_g(\tilde{A}) = \frac{[(a_4'-a_1')-2(a_2'-a_3')](1-n_a)}{3}$$

$$I(\tilde{A}) = \frac{I_f(\tilde{A}) + I_g(\tilde{A})}{2}$$

$$I(\tilde{A}) = \frac{[(\mathrm{a}_4-\mathrm{a}_1-2(\mathrm{a}_2-\mathrm{a}_3)]m_a+[(a_4'-a_1'-2(a_2'-a_3')](1-n_a)}{6}$$

Theorem -2

Let $\widetilde{\mathrm{A}} = \{(a_1, a_2, a_3, a_4, a'_1, a'_2, a'_3, a'_4); m_a, n_a\}$

and $\widetilde{\mathrm{B}} = (b_1, b_2, b_3, b_4, b'_1, b_2, b_3, b'_4); m_b, n_b\}$ be two TRIFN's with $m_a = m_b$ and $n_a = n_b$

Then, $I_f(\tilde{A} + \widetilde{\mathrm{B}}) = I_f(\tilde{A}) + I_f(\widetilde{\mathrm{B}})$ *and*

$$I_g(\tilde{A} + \widetilde{\mathrm{B}}) = I_g(\tilde{A}) + I_g(\widetilde{\mathrm{B}})$$

Proof:

$$\tilde{A} \oplus \widetilde{\mathrm{B}} = \begin{Bmatrix}(a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a'_1 + b'_1, a'_2 + b'_2 \\ a'_3 + b'_3, a'_4 + b'_4); \min(m_a, m_b), \max(n_a, n_b)\end{Bmatrix}$$

From the definition of ambiguity of membership function

$$I_f(\tilde{A} + \widetilde{\mathrm{B}}) = \frac{[(\mathrm{a}_4 + b_4) - (\mathrm{a}_1 + b_1) - 2(\mathrm{a}_2 - \mathrm{b}_2) - (a_3 + b_3)]m_A}{3}$$

$$= \frac{[(\mathrm{a}_4-\mathrm{a}_1-2(\mathrm{a}_2-\mathrm{a}_3)]m_A}{3} + \frac{[(\mathrm{b}_4+\mathrm{b}_1-2(\mathrm{b}_2-\mathrm{b}_3)]m_A}{3}$$

$$=I_f(\tilde{A}) + I_f(\widetilde{\mathrm{B}})$$

Similarly,

$$I_g(\tilde{A} + \widetilde{\mathrm{B}}) = \frac{[(a_4'+b_4')-(a_1'+b_1')-2((a_2'+b_2')-(a_3'+b_3')](1-n_A)}{3}$$

$$= \frac{[(a_4'-a_1'-2(a_2'-a_3')](1-n_A)}{3} + \frac{[(b_4'+b_1'-2(b_2'-b_3')](1-n_A)}{3}$$

$$= I_g(\tilde{A}) + I_g(\tilde{\mathrm{B}})$$

Value index and Ambiguity index :

$$V(\tilde{A}, \lambda) = V_f(\tilde{A}) + \lambda\left(V_g(\tilde{A}) - V_f(\tilde{A})\right)$$

$$I(\tilde{A}, \lambda) = I_f(\tilde{A}) + \lambda\left(I_g(\tilde{A}) - I_f(\tilde{A})\right)$$

where $\lambda \in [0,1]$ is a weight represents the decision maker's preference information. Choose $\lambda = \frac{1}{2}$,

$$V\left(\tilde{A}, \frac{1}{2}\right) = V_f(\tilde{A}) + \frac{1}{2}\left(V_g(\tilde{A}) - V_f(\tilde{A})\right)$$

$$I\left(\tilde{A}, \frac{1}{2}\right) = I_f(\tilde{A}) + \frac{1}{2}\left(I_g(\tilde{A}) - I_f(\tilde{A})\right)$$

$$R(\tilde{A}) = V(\tilde{A}) + I(\tilde{A})$$

Rank of $\tilde{A} = R(\tilde{A}) = V(\tilde{A}) + I(\tilde{A}) = \frac{[(-\mathrm{a}_1 - 2\mathrm{a}_2 + 6\mathrm{a}_3 + 3\mathrm{a}_4]m_a + [(-a_1' - 2a_2' + 6a_3' + 3a_4'](1 - n_a)}{12}$

Theorem 3 :

Let $\widetilde{\mathrm{A}} = \{(a_1, a_2, a_3, a_4, a'_1, a'_2, a'_3, a'_4); m_a, n_a\}$

and $\widetilde{\mathrm{B}} = \{(b_1, b_2, b_3, b_4, b'_1, b_2, b_3, b'_4); m_b, n_b\}$ be two GTRIFN's with $m_a = n_a$ and $m_b = n_b$. Then, $V(\tilde{A} + \tilde{B}) = V(\tilde{A}) + V(\tilde{B})$, $I(\tilde{A} + \tilde{B}) = I(\tilde{A}) + I(\tilde{B})$ *and* $R(\tilde{A} + \tilde{B}) = R(\tilde{A}) + R(\tilde{B})$

Proof:

$$V(\tilde{A} + \tilde{B}) = + \frac{V_f(\tilde{A}+\tilde{B}) + V_g(\tilde{A}+\tilde{B})}{2}$$

$$= \frac{V_f(\tilde{A}) + V_f(\tilde{B}) + V_g(\tilde{A}) + V_g(\tilde{A})}{2} \quad \text{(using Theorem -1)}$$

$$= \frac{V_f(\tilde{A}) + V_g(\tilde{A})}{2} + \frac{V_f(\tilde{B}) + V_g(\tilde{B})}{2} = V(\tilde{A}) + V(\tilde{B})$$

$$I(\tilde{A} + \tilde{B}) = + \frac{I_f(\tilde{A} + \tilde{B}) + I_g(\tilde{A} + \tilde{B})}{2}$$

$$= \frac{I_f(\tilde{A}) + I_f(\tilde{B}) + I_g(\tilde{A}) + I_g(\tilde{B})}{2} \quad \text{(using Theorem -2)}$$

$$= \frac{I_f(\tilde{A}) + I_g(\tilde{A})}{2} + \frac{I_f(\tilde{B}) + I_g(\tilde{B})}{2} = I(\tilde{A}) + I(\tilde{B})$$

$$R(\tilde{A} + \tilde{B}) = V(\tilde{A} + \tilde{B}) + I(\tilde{A} + \tilde{B})$$

$$= V(\tilde{A}) + V(\tilde{B}) + I(\tilde{A}) + I(\tilde{B})$$

$$= R(\tilde{A}) + R(\tilde{B})$$

Comparing Generalized intuitionistic trapezoidal fuzzy numbers using the Ranking function :

Let $\tilde{A}$ and $\tilde{B}$ be two GITFNs. Then :

(i) If $R(\tilde{A}) > R(\tilde{B})$ then $\tilde{A} > \tilde{B}$
(ii) If $R(\tilde{A}) < R(\tilde{B})$ then $\tilde{A} < \tilde{B}$
(iii) If $R(\tilde{A}) = R(\tilde{B})$ then $\tilde{A} = \tilde{B}$

Numerical Examples :

1. Let $\tilde{A}$ = ((2,7,11,15),(1,6,12,16);0.5,0.3) and $\tilde{B}$ = ((2,5,8,10),(1,3,9,11);0.6,0.1)
   Then $R(\tilde{A})$ = 10.2 , $R(\tilde{B})$ = 9.3, since $R(\tilde{A}) > R(\tilde{B})$ we have $\tilde{A}>\tilde{B}$.
2. Let $\tilde{A}$ = ((2,2.5,3,3.5),(1.5,2.2,3.2,4);0.7,0.2) and
   $\tilde{B}$ = ((2.5,3.2,3.8,4.2),(2,3,4,4.5);0.7,0.2)
   Then $R(\tilde{A})$ = 2.940833, $R(\tilde{B})$ = 3.5125, since $R(\tilde{A}) < R(\tilde{B})$ we have $\tilde{A}<\tilde{B}$.
3. Let $\tilde{A}$ = ((2,3.5,4.5,6), (2,3.5,4.5,6);0.5,0.2) and
   $\tilde{B}$ = ((2,3,5,6), (2,3,5,6);0.5,0.2)
   Then $R(\tilde{A})$ = 3.9, $R(\tilde{B})$ = 4.3333, since $R(\tilde{A}) < R(\tilde{B})$ we have $\tilde{A}< \tilde{B}$.

3. Optimizing Fuzzy Transportation with Minimum Spanning Tree and Intuitionistic Fuzzy Approach

3.1 Transportation Problem

The transportation problem for trapezoidal intuitionistic fuzzy numbers (TIFNs) involves finding the minimum cost of transporting goods from sources to destinations while considering the fuzzy nature of costs, demands, and supplies. A transportation problem is a type of linear programming problem that focuses on finding an optimal solution for transporting and allocating resources to various destinations from different sites while minimizing expenditure. The transportation problem is an application of graph theory. The initial basic feasible solution of the transportation problem can be calculated using methods such as the Northwest Corner Method, Least Cost Method, Vogel's Approximation Method, Row Minima Method, and Column Minima Method. For the optimal solution of the transportation problem, the Modified Distribution (MODI) method is used.

3.2 A Review of Minimum Spanning Tree Applications in Transportation Problems

A spanning tree of a graph G is a subgraph of G that is a tree and contains all the vertices of G. A minimum spanning tree of a weighted graph G is a spanning tree with the smallest total weight. Kruskal's and Prim's algorithms, classified as greedy algorithms, are used to find the minimum spanning tree. To study transportation problems using a graphical approach, minimum spanning tree algorithms are employed to minimize transportation costs. Kruskal's and Prim's algorithms are used to obtain a minimum spanning tree, leading to the minimum cost. Additionally, the Northwest Corner Method, Least Cost Method, Vogel’s Approximation Method, Row Minima Method, and Column Minima Method are utilized to find the initial basic feasible solution. The Modified Distribution (MODI) method is applied for the optimal solution, using TORA software to solve the transportation problem. Akpan and Iwok [45] worked on solving a transportation problem using a minimum panning tree, mainly employing Kruskal's algorithm [47]. They converted the given data into a bipartite network representation diagram and applied Kruskal's algorithm. Then, they used TORA software to confirm the solution and compared the solution of Kruskal's algorithm with the heuristic solution. Finally, they concluded that the solution obtained using Kruskal's algorithm in graph theory was less than the heuristic method.

Angulakshmi et al. [48] studied various graph algorithms, including Dijkstra's algorithm, Breadth-First Traverse, Depth-First Traverse, and Prim's algorithm. They considered an example of a directed graph and an undirected graph and applied graph theory algorithms. The paper concluded that the shortest path graph algorithm is effective in networking and has better time complexity than other algorithms. Aljanabi and Jasim [49] developed a new Kruskal's algorithm to solve transportation problems. They proposed a new approach for finding a minimum feasible solution for transportation problems with different types. They applied Kruskal's algorithm to find the minimum edges, sorted them, and transferred the amount available in the left-side source node to the demand node on the right side.

In paper [50], a study on finding the minimum spanning tree and shortest distance between two places using Kruskal's and Prim's algorithms was presented. They developed a network model to find the minimum distance and cost for transporting a product from place A to place B. By applying Kruskal's and Prim's algorithms, they obtained a minimum cost path. Ramya and Indira [51] focused on using the Penalty Difference Algorithm to transport goods from source to destination. This algorithm was compared with other algorithms, and the author concluded that the Penalty Difference Algorithm generated a solution is precisely the same as using Vogel's Approximation method.

3.3 Methodology

To solve the transportation problem using a minimum spanning tree (MST), follow these steps:

1. Construct a graph: Represent the sources, destinations, and transportation links as nodes and edges in a graph.
2. Assign fuzzy costs: Label each edge with a TIFN representing the fuzzy transportation cost.
3. Convert TIFNs to crisp numbers: Defuzzify the TIFNs using a method of value and ambiguity index ranking.
4. Find the MST: Apply Kruskal's [47] and Prim's [46] algorithms to find the MST of the graph, considering the crisp costs.

We can use Kruskal's and Prim's algorithms to find the minimum spanning tree.

Kruskal's Algorithm

Kruskal's algorithm is a greedy algorithm used to find the minimum spanning tree of a given graph.

Implementation:

1. Sort all the edges of the given graph in increasing order of their weights.
2. Select the edge with the minimum weight and use it to connect the vertices of the graph, ensuring that no loop is formed. If a loop occurs, cancel that edge and move to the next minimum weight edge.

3. Add all the edges until all the vertices are connected, and a minimum spanning tree is obtained.

Prim's Algorithm

Prim's algorithm is a greedy algorithm used to find the minimum spanning tree of a given graph.

Implementation:

1. Choose any vertex randomly, usually the vertex connected to the edge with the minimum weight.
2. Select all the edges that connect the tree to new vertices, and find the minimum weight edge among those edges. Include it in the current tree, ensuring that no loop is formed. If a loop occurs, cancel that edge and move to the next minimum weight edge.
3. Repeat step 2 until all the vertices are included, and a minimum spanning tree is obtained.

Kruskal's and Prim's in transportation problem:-

Steps to Find the Minimum Spanning Tree:

1. Represent the transportation problem using a network flow diagram.
2. Apply Kruskal's and Prim's algorithms to find the least edge node and transfer the required cost from the source node to the demand node.
3. Delete the vertices that have been fulfilled and the edges with no amount to transfer.
4. Continue this process until all demand and source nodes are satisfied, and find the minimum spanning tree to determine the minimum cost.
5. Calculate the total cost: Sum the crisp costs of the edges in the MST to obtain the minimum total transportation cost.
6. Interpret the result: The MST represents the optimal transportation plan, and the total cost is the minimum fuzzy cost.

4 Numerical Example

A Generalized ITFN transportation problem (Table 1) consists of three origins $S_1, S_2, and\ S_3$ and three destinations $D_1, D_2, and\ D_3$. Ranks of each cell in Table 2 is obtained using ranking method (value and ambiguity) and solutions are obtained using TORA and minimum spanning tree approach.

Table 1 : Generalized Intuitionistic Trapezoidal Fuzzy Transportation Problem

Destinations → Origins ↓	D_1	D_2	D_3	Supply s_i
S_1	(2,4,8,15;0.6) (1,4,8,18;0.3)	(3,5,7,12; 0.5) (1,5,7,15;0.3)	(2,5,9,16;0.7) (1,5,9,18;0,3)	25
S_2	(2,5,8,10;0.6) (1,5,8,12;0.2)	(4,8,10,13;0.4) (3,8,10,15,0.3)	(4,8,10.13;0.4) (3,8,10,15;0.3)	30
S_3	(2,7,11,15;0.5) (1,7,11,18;0.3)	(5,9,12,16;0.7) (3,9,12,19;0.2)	(4,6,8,10;0.6) (3,6,8,12;0.3)	40
Demand d_j	35	45	15	**95**

Table 2 : Ranks of each cell of table 1 using ranking method

Destinations → Origins	D_1	D_2	D_3	Supply s_i
S_1	9.575	7.141667	10.90833	25
S_2	8.166667	7.65	7.65	30
S_3	10.08333	12.85833	7.125	40
Demand d_j	35	45	15	**95**

4.1 Existing Method :

North-West Corner Rule and MODI

First Iteration (Table 3)

Table 3 : North-West Corner Rule and MODI (First Iteration)

Destinations → Origins ↓	D_1 **v₁ = 9.58**	D_2 **v₂ = 9.06**	D_3 **v₃ = 3.32**	Supply s_i
S_1 **u₁ = 0.00**	(2,4,8,15;0.6) (1,4,8,18;0.3) **(25)**	(3,5,7,12; 0.5) (1,5,7,15;0.3)	(2,5,9,16;0.7) (1,5,9,18;0,3)	25
S_2 **u₂ = 1.41**	(2,5,8,10;0.6) (1,5,8,12;0.2) **(10)**	(4,8,10,13;0.4) (3,8,10,15,0.3) **(20)**	(4,8,10.13;0.4) (3,8,10,15;0.3)	30
S_3 **u₃ = 3.80**	(2,7,11,15;0.5) (1,7,11,18;0.3)	(5,9,12,16;0.7) (3,9,12,19;0.2) **(25)**	(4,6,8,10;0.6) (3,6,8,12;0.3) **(15)**	40

Demand d_j	35	45	15	**95**

Second Iteration (Table 4)

Table 4 : North-West Corner Rule and MODI (Second Iteration)

Destinations → Origins ↓	D_1 **v₁ = 9.58**	D_2 **v₂ = 12.36**	D_3 **v₃ = 6.62**	Supply s_i
S_1 **u₁ = 0.00**	(2,4,8,15;0.6) (1,4,8,18;0.3) **(25)**	(3,5,7,12; 0.5) (1,5,7,15;0.3)	(2,5,9,16;0.7) (1,5,9,18;0,3)	25
S_2 **u₂ = 4.71**	(2,5,8,10;0.6) (1,5,8,12;0.2)	(4,8,10,13;0.4) (3,8,10,15,0.3) **(30)**	(4,8,10.13;0.4) (3,8,10,15;0.3)	30
S_3 **u₃ = 0.50**	(2,7,11,15;0.5) (1,7,11,18;0.3) **(10)**	(5,9,12,16;0.7) (3,9,12,19;0.2) **(15)**	(4,6,8,10;0.6) (3,6,8,12;0.3) **(15)**	40
Demand d_j	35	45	15	**95**

Third Iteration (Table 5)

Table 5 : North-West Corner Rule and MODI (Third Iteration)

Destinations → Origins ↓	D_1 **v₁ = 9.58**	D_2 **v₂ = 7.14**	D_3 **v₃ = 6.62**	Supply s_i
S_1 **u₁ = 0.00**	(2,4,8,15;0.6) (1,4,8,18;0.3) **(10)**	(3,5,7,12; 0.5) (1,5,7,15;0.3) **(15)**	(2,5,9,16;0.7) (1,5,9,18;0,3)	25
S_2 **u₂ = 0.51**	(2,5,8,10;0.6) (1,5,8,12;0.2)	(4,8,10,13;0.4) (3,8,10,15,0.3) **(30)**	(4,8,10.13;0.4) (3,8,10,15;0.3)	30
S_3 **u₃ = 0.50**	(2,7,11,15;0.5) (1,7,11,18;0.3) **(25)**	(5,9,12,16;0.7) (3,9,12,19;0.2)	(4,6,8,10;0.6) (3,6,8,12;0.3) **(15)**	40
Demand d_j	35	45	15	**95**

Fourth Iteration (Table 6)

Table 6 : North-West Corner Rule and MODI (Optimal Iteration)

Destinations → Origins ↓	D_1 **v₁ = 7.66**	D_2 **v₂ = 7.14**	D_3 **v₃ = 4.70**	Supply s_i
S_1 **u₁ = 0.00**	(2,4,8,15;0.6) (1,4,8,18;0.3)	(3,5,7,12; 0.5) (1,5,7,15;0.3) **(25)**	(2,5,9,16;0.7) (1,5,9,18;0,3)	25
S_2 **u₂ = 0.51**	(2,5,8,10;0.6) (1,5,8,12;0.2) **(10)**	(4,8,10,13;0.4) (3,8,10,15,0.3) **(20)**	(4,8,10.13;0.4) (3,8,10,15;0.3)	30

S_3 **u₃ = 2.42**	(2,7,11,15;0.5) (1,7,11,18;0.3) **(25)**	(5,9,12,16;0.7) (3,9,12,19;0.2)	(4,6,8,10;0.6) (3,6,8,12;0.3) **(15)**	40
Demand d_j	35	45	15	**95**

Least Cost Method

First Iteration (Table 7)

Table 7 : Least Cost Method and MODI (First and Optimal Iteration)

Destinations → Origins ↓	D_1 **v_1 = 7.66**	D_2 **v_2 = 7.14**	D_3 **v_3 = 4.70**	Supply s_i
S_1 **u_1 = 0.00**	(2,4,8,15;0.6) (1,4,8,18;0.3)	(3,5,7,12; 0.5) (1,5,7,15;0.3) **(25)**	(2,5,9,16;0.7) (1,5,9,18;0,3)	25
S_2 **u_2 = 0.51**	(2,5,8,10;0.6) (1,5,8,12;0.2) **(10)**	(4,8,10,13;0.4) (3,8,10,15,0.3) **(20)**	(4,8,10.13;0.4) (3,8,10,15;0.3)	30
S_3 **u_3 = 2.42**	(2,7,11,15;0.5) (1,7,11,18;0.3) **(25)**	(5,9,12,16;0.7) (3,9,12,19;0.2)	(4,6,8,10;0.6) (3,6,8,12;0.3) **(15)**	40
Demand d_j	35	45	15	**95**

Vogel's Approximation Method (VAM)

First Iteration (Table 8)

Table 8 : Vogel's Approximation Method and MODI (First and Optimal Iteration)

Destinations → Origins ↓	D_1 **v_1 = 7.66**	D_2 **v_2 = 7.14**	D_3 **v_3 = 4.70**	Supply s_i
S_1 **u_1 = 0.00**	(2,4,8,15;0.6) (1,4,8,18;0.3)	(3,5,7,12; 0.5) (1,5,7,15;0.3) **(25)**	(2,5,9,16;0.7) (1,5,9,18;0,3)	25
S_2 **u_2 = 0.51**	(2,5,8,10;0.6) (1,5,8,12;0.2) **(10)**	(4,8,10,13;0.4) (3,8,10,15,0.3) **(20)**	(4,8,10.13;0.4) (3,8,10,15;0.3)	30
S_3 **u_3 = 2.42**	(2,7,11,15;0.5) (1,7,11,18;0.3) **(25)**	(5,9,12,16;0.7) (3,9,12,19;0.2)	(4,6,8,10;0.6) (3,6,8,12;0.3) **(15)**	40
Demand d_j	35	45	15	**95**

Optimal assignment of three initial feasible solutions using MODI is presented in Table 9.

Table 9 : Optimal Assignment of all Methods

Destinations → Sources ↓	D_1	D_2	D_3	Supply s_i
S_1	(2,4,8,15;0.6) (1,4,8,18;0.3)	(3,5,7,12; 0.5) (1,5,7,15;0.3) **(25)**	(2,5,9,16;0.7) (1,5,9,18;0,3)	25
S_2	(2,5,8,10;0.6) (1,5,8,12;0.2) **(10)**	(4,8,10,13;0.4) (3,8,10,15,0.3) **(20)**	(4,8,10.13;0.4) (3,8,10,15;0.3)	30
S_3	(2,7,11,15;0.5) (1,7,11,18;0.3) **(25)**	(5,9,12,16;0.7) (3,9,12,19;0.2)	(4,6,8,10;0.6) (3,6,8,12;0.3) **(15)**	40
Demand d_j	35	45	15	**95**

The minimum fuzzy transportation cost achieved as follows :

$x_{12} = 25$, $x_{21} = 10$, $x_{22} = 20$, $x_{31} = 25$, $x_{33} = 15$

Optimal Cost =

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij} \otimes x_{ij} = \tilde{c}_{12} \otimes x_{12} \oplus \tilde{c}_{21} \otimes x_{21} \oplus \tilde{c}_{22} \otimes x_{22} \oplus \tilde{c}_{31} \otimes x_{31} \oplus \tilde{c}_{33} \otimes x_{33}$$

$= (3,5,7,12; 0.5)(1,5,7,15;0.3) \otimes 25 \oplus (2,5,8,10;0.6)(1,5,8,12;0.2) \otimes 10 \oplus (4,8,10,13;0.4)(3,8,10,15;0.3) \otimes 20 \oplus (2,7,11,15;0.5)(1,7,11,18;0.3) \otimes 25 \oplus (4,6,8,10;0.6)(3,6,8,12;0.3) \otimes 15$

$= (225,510,730,1035;0.6)(120,510,730,1245;0.3)$

4.2 Proposed Method using Kruskal's and Prim's algorithms in Transportation problem

Kruskal's and Prim's algorithm in transportation problem:

Step 1: Represent transportation problem using network flow diagram (Figure 3)

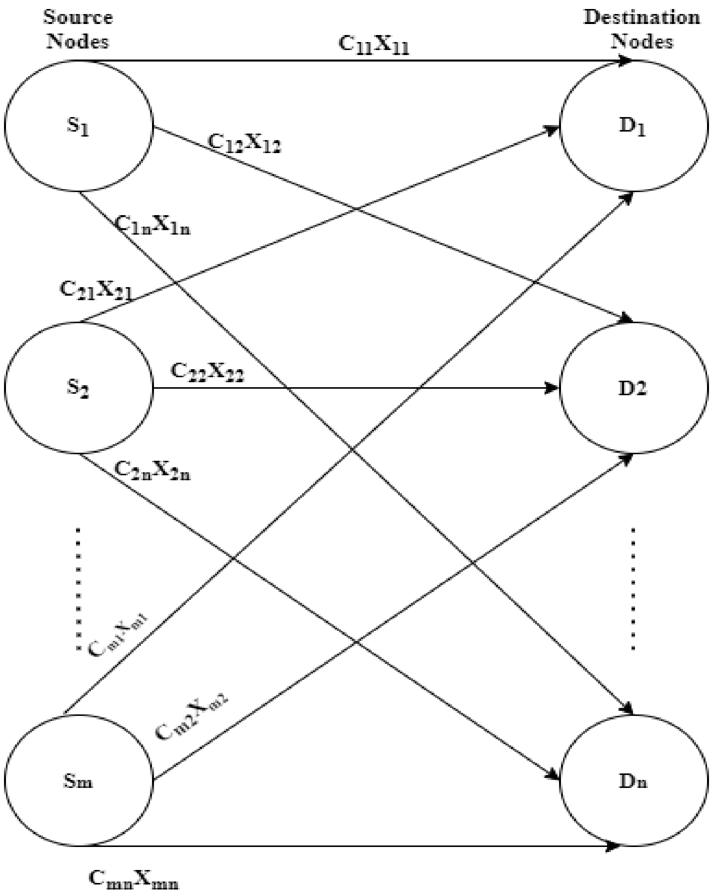

Figure 3 : Network flow diagram

Step 2: By applying the Kruskal's and Prim's algorithm find the least edge node and transfer the required cost from source node to demand node.

For example problem : Least edge = $S_3 - D_3$ (15) (according to Table 2)

Step 3: Delete the vertices which have been fulfilled and the edges which has no amount to transfer

For example problem : continuing step 2,

Second edge = $S_1 - D_2$ (25)

Third edge = $S_2 - D_2$ (20)

Fourth edge = $S_2 - D_1$ (10)

Fifth edge = $S_3 - D_1$ (25)

Step 4: Continue this process until all the demand and source nodes are satisfied and then find the minimum spanning tree (Figure 4) to find the minimum cost.

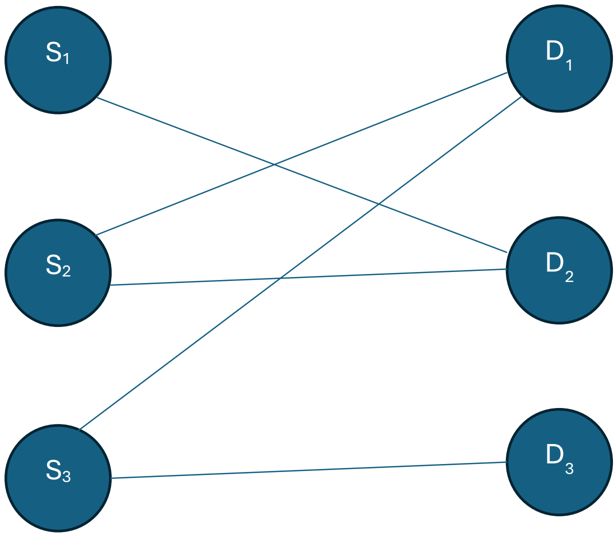

Figure 4 : Minimum Spanning Tree

Step5 : Calculate the total cost: Sum the fuzzy costs of the edges in the MST to obtain the minimum total transportation cost.

The minimum fuzzy transportation cost achieved as follows :

$x_{12} = 25$, $x_{21} = 10$, $x_{22} = 20$, $x_{31} = 25$, $x_{33} = 15$

Optimal Cost =

$$\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij} \otimes x_{ij}$$

$$= \tilde{c}_{12} \otimes x_{12} \oplus \tilde{c}_{21} \otimes x_{21} \oplus \tilde{c}_{22} \otimes x_{22} \oplus \tilde{c}_{31} \otimes x_{31} \oplus \tilde{c}_{33} \otimes x_{33}$$

= (3,5,7,12; 0.5)(1,5,7,15;0.3) ⊗ 25 ⊕ (2,5,8,10;0.6) (1,5,8,12;0.2)⊗ 10 ⊕ (4,8,10,13;0.4)(3,8,10,15;0.3)⊗ 20 ⊕ (2,7,11,15;0.5) (1,7,11,18;0.3)⊗ 25 ⊕ (4,6,8,10;0.6)(3,6,8,12;0.3)⊗ 15 = (225,510,730,1035;0.6) (120,510,730,1245;0.3)

4. Comparative Study

Ranking Procedure	Fuzzy Transportation Method	No. of Iterations for Optimal	Fuzzy Optimal Cost

Gani and Mohammed [25]	North west corner rule and MODI - Three iterations	Fourth	(225,510,730,1035;0.6) (120,510,730,1245;0.3)
	Least Cost Method and MODI – One iteration	One	
	Vogel's Approximation Method and MODI	One	
Value and Ambiguity index ranking	Kruskal's and Prim's algorithm	***	(225,510,730,1035;0.6) (120,510,730,1245;0.3)

6.Results and Discussion

The proposed approach, which combines the Minimum Spanning Tree (MST) procedure with value and ambiguity index ranking for trapezoidal intuitionistic fuzzy numbers (TIFNs), yields promising results in solving transportation problems. Our numerical example demonstrates that the method efficiently handles fuzzy transportation problems, minimizing total transportation cost while accounting for the uncertainty and vagueness inherent in real-world transportation systems.The results show that the proposed approach provides a more realistic and robust solution compared to traditional methods, which often rely on crisp numbers and neglect the fuzzy nature of transportation costs, demands, and supplies. By defuzzifying TIFNs and applying the MST procedure, we can determine the optimal transportation plan that minimizes costs and maximizes efficiency.The value and ambiguity index ranking plays a crucial role in handling the uncertainty associated with TIFNs, enabling decision-makers to make informed choices in logistics and supply chain management. The proposed approach offers a practical solution for real-world transportation problems, where uncertainty and vagueness are prevalent.Overall, the results demonstrate the effectiveness of the proposed approach in solving fuzzy transportation problems, and its potential to contribute to the development of fuzzy optimization techniques in transportation planning.

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