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Impact of Rotation on Heated Couple-Stress Ferromagnetic Fluid in a Variable Gravity Field

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Abstract:

The thermosolutal instability of a couple-stress ferromagnetic fluid subjected to rotation and varying gravity is explored through a linear stability analysis. The fluid layer, heated and soluted from below, is confined between two free surfaces. Using the normal mode technique, an exact solution for the stability of the couple-stress ferromagnetic fluid is derived. The critical Rayleigh number for the onset of instability is established using both numerical and graphical methods. In the case of stationary convection, a stable solute gradient acts to stabilize the system, while couple-stress effects can either stabilize or destabilize the fluid under certain conditions. Magnetization stabilizes the system regardless of rotation when there is no steady solute gradient. Additionally, the principle of exchange of stabilities applies under specific conditions where both rotation and a stable solute gradient are absent.

Keywords: Fluid flow, couple-stress fluid, ferromagnetic fluid, solute gradient, Rayleigh number.

1. INTRODUCTION

In the context of geophysical systems, thermo convective instability in rotating fluids is highly relevant for geothermal analysis. The rotation of the Earth, which occurs at a constant angular velocity around the vertical axis, introduces rotational effects that must be accounted for in such studies. This phenomenon also holds significant value for hydrologists and soil scientists investigating fluid movement and heat transfer in the Earth's crust. Traditionally, many researchers have aligned the axes of gravity and rotation in the vertical direction during their analysis. However, in real-world geophysical systems, the Earth's rotational axis is inclined to the direction of gravity by approximately 23.5 degrees. This inclination, known as the axial tilt, necessitates the consideration of obliqueness when performing stability analysis, as it impacts fluid dynamics and thermal convection within the system. Understanding the temperature gradient that drives thermal convection in mineral-rich fluids is crucial for optimizing energy extraction from geothermal systems. By considering the effect of the Earth's obliqueness and temperature variation, the thermo convective instability analysis can provide valuable insights into the behavior of geothermal systems and enhance energy efficiency in extraction processes.

Extrusion processes are heavily influenced by the fluid dynamics generated during the stretching of sheets. Recent research has made significant advancements in understanding the behavior of non-Newtonian fluids. The study of these fluids is essential in various technological and industrial applications, such as polymer surface manufacturing, continuous casting, paper production, and the extrusion of plastic sheets in aerospace engineering. Sharma et al. [1] have contributed both theoretical and experimental work to this field. Stokes' theory [2] addresses couple-stress fluids, which exhibit notable couple-stresses due to large molecular structures. Chandrasekhar [3] determined the critical Rayleigh number for the onset of convection currents in a non-porous rotating system using normal mode techniques. Kumar et al. [4]

explored the thermosolutal instability of rotating couple-stress fluids under a magnetic field, while Singh et al. [5] studied thermosolutal convection in compressible, rotating couple-stress fluids within a similar magnetic context. Kumar and Singh [6] investigated the dynamics of rotatory thermosolutal convection in these fluids.

Rosensweig [7] investigated a comprehensive introduction to magnetic liquids in his monograph. Finlayson examines convective instability in ferromagnetic fluids heated from below within a uniform magnetic field. Suresh and Vasanthakumari compare theoretical and computational approaches to ferroconvection induced by magnetic field-dependent viscosity in an anisotropic porous medium. Sunil et al. [10] investigated the effects of rotation on ferromagnetic fluids that are heated and soluted from below within a porous medium, while Sunil and Shandil [11] described the influence of rotation on ferromagnetic fluids in the presence of dust particles.

This study explores thermosolutal instability in couple-stress ferromagnetic fluids subjected to variable gravitational fields and rotational influences. It is assumed that gravitational acceleration, represented by its standard value at the Earth's surface, varies with altitude. This variation can be either positive or negative, depending on whether the gravitational field strengthens or weakens as altitude increases. Devi and Devi [12] investigated the effects of the Lorentz force on a 3D stretched sheet under Newtonian heating, focusing on thermo-physical properties. They compared the heat transfer characteristics of conventional nano-fluids with those of emerging hybrid nano-fluids. Arifin et al. [13] numerically studied the boundary layer flow and heat transfer of Casson fluid containing dust particles over a stretched surface. Kumar et al. [14] explored the onset of convection in a dusty couple-stress fluid with variable gravity through a porous medium in the context of hydro-magnetics. Vasantha Kumari et al. [15] examined the thermal instability of non-Newtonian fluid in a uniform magnetic field within a non-rotating medium. Kumar et al, [16] analyzed the implications of horizontal rotation and a horizontal magnetic field on the thermosolutal stability of a dusty couple-stress fluid moving through a material with pores. Afridi et al. [18] investigated the stability analysis and dual solutions of a hybrid nanofluid over a stretching/shrinking sheet carrying out MHD flow. Yashkun et al. [19] profounded the heat transfer and flow of a nano-fluid through a porous substance as a result of a stretching/shrinking sheet with thermal radiation, a magnetic field, and suction.

In the present study, we investigate the thermosolutal instability of couple-stress ferromagnetic fluid in the presence of a varying gravitational field and rotation ($g = \lambda g_0$). It is assumed that gravity varies as a function of altitude, with g_0 representing the value of gravity at the Earth's surface, which is always positive. The variation in gravity, denoted by g, can be either positive or negative, depending on whether gravity increases or decreases as altitude increases from its surface value g_0 .

2. Description of Mathematical Model

In the present communication, we consider an infinite horizontal layer of thickness (d) consisting of an electrically non-conducting, compressible couple-stress ferromagnetic fluid, heated and soluted from below. The layer is bounded by two planes at (z=0) and (z=d).

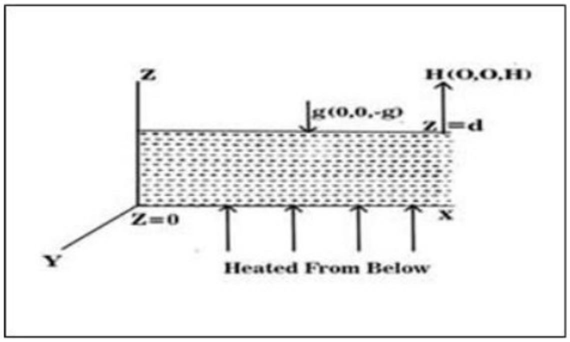


Figure 1. Geometrical configuration

Assuming the fluid is electrically non-conducting and the displacement current is negligible, Maxwell's equations simplify to:

$$\boldsymbol{M} \times \boldsymbol{H} = 0 \tag{1}$$

$$\nabla . B = 0 \quad \text{and} \quad \nabla \times H = 0 \tag{2}$$

In electrodynamics, the magnetic field H, magnetization M, and magnetic induction B are related in the following manner:

$$B = (H \times M)\mu_0 \tag{3}$$

We assume that the magnetization is aligned with the magnetic field, with its magnitude depending on the strength of the magnetic field, temperature, and salinity, such that

$$M = \frac{H}{M}(H, T, C) \tag{4}$$

The governing equations for the conservation of momentum, mass (continuity), energy (temperature), and solute concentration are as follows:

$$\frac{\partial q}{\partial t} + (q.\nabla)q = \frac{1}{\rho_0}\nabla p \left(1 + \frac{\delta\rho}{\rho_0}\right) + \frac{1}{\rho_0} M \nabla \mathrm{H} + \left(\mathrm{v} - \frac{\mu}{\rho_0}\nabla^2\right)\nabla^2 q + 2(qX \cap) - K \tag{5}$$

$$(\nabla . q) = 0, \tag{6}$$

$$\frac{\partial T}{\partial t} + q.\nabla T = K_T \nabla^2 T - K \tag{7}$$

$$\frac{\partial C}{\partial t} + q.\nabla \mathrm{C} = K_S \nabla^2 C - K \tag{8}$$

The equation of state is, ∇

$$\rho = \rho_0\left[1 - \alpha\left(T - T_0\right) + \alpha^{'}\left(C - C_0\right)\right], \tag{9}$$

In the present study, we assume that magnetization depends solely on temperature, i.e., (M = M (T)). Therefore, as the first approximation, we consider that:

$$M = M_0(1 - \gamma(T - T_0)\,) \tag{10}$$

3. PERTURBATION EQUATIONS

The properties of a fluid at rest seem to be the primary focus in the discussion of fluid dynamics. When the fluid is at rest, it is sometimes referred to as a fundamental state or basic flow in fluid mechanics.

$$\boldsymbol{q} = (0,0,0),\, p = p(z),\, \rho = \rho(z) = \rho_0\left[1 - \alpha\beta z + \alpha'\beta' z\right],\, T = T(z) = T_0 - \beta z,$$
$$C = C_0 - \beta' z,\, \boldsymbol{\Omega} = (0,0,\Omega),\, M = M_0\left(1 + \gamma'\beta z\right),\, \boldsymbol{M} = \boldsymbol{M}(z). \tag{11}$$

The flow that has been obstructed might be seen as given below

$$\boldsymbol{q} = (0,0,0) + (u_1, u_2, u_3),\, T = T(z) + \theta,\, C = C(z) + \gamma,\, \rho = \rho(z) + \delta\rho,$$
$$p = p(z) + \delta p \text{ and } M = M(z) + \delta M\ , \tag{12}$$

where, $\boldsymbol{q}(u_1, u_2, u_3), \theta, \gamma, \delta\rho, \delta p, \delta M$ are respectively the perturbation in fluid velocity $\boldsymbol{q}(0,0,0)$, temperature $\boldsymbol{T}$, solute gradient C, density ρ, pressure p and magnetization $\boldsymbol{M}$.

Now, using eq. (11) into governing equations (5) to (10), we get

$$\frac{\partial q}{\partial t} = -\frac{\nabla \delta p}{\rho_0} + \frac{\delta\rho}{\rho_0} g + \frac{\delta M}{\rho_0}\nabla H + \left(\nu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2 q + 2(q \times \Omega) \tag{13}$$

$$\nabla . q = 0 \tag{14}$$

$$\frac{\partial \theta}{\partial t} = \beta u_3 + \kappa_T \nabla^2 \theta, \tag{15}$$

$$\frac{\partial \gamma}{\partial t} = \beta' u_3 + \kappa_S \nabla^2 \gamma, \tag{16}$$

$$\delta\rho = -\rho_0\left(\alpha\theta - \alpha'\gamma\right), \tag{17}$$

$$\delta M = -\gamma' M_0 \theta. \tag{18}$$

Using eqn (17) and (18), The eqn (13) to (16) can be reduced to (19) to (24) below as,

$$\frac{\partial u_1}{\partial t} = -\frac{1}{\rho_0}\frac{\partial}{\partial x}\delta p + \left(\nu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2 u_1 + 2\Omega u_2, \tag{19}$$

$$\frac{\partial u_2}{\partial t} = -\frac{1}{\rho_0}\frac{\partial}{\partial y}\delta p + \left(\nu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2 u_2 - 2\Omega u_1, \tag{20}$$

$$\frac{\partial u_3}{\partial t} = -\frac{1}{\rho_0}\frac{\partial}{\partial z}\delta p - \frac{\delta\rho}{\rho_0} g + \frac{\delta M}{\rho_0}\nabla H + \left(\nu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2 u_3, \tag{21}$$

$$\frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial y} + \frac{\partial u_3}{\partial z} = 0, \tag{22}$$

$$\frac{\partial\theta}{\partial t}=\beta u_3+\kappa_T\nabla^2\theta, \tag{23}$$

$$\frac{\partial\gamma}{\partial t}=\beta' u_3+\kappa_S\nabla^2\gamma, \tag{24}$$

Differentiate partially with respect to $x\ and\ y$ eqns (19) and (20), respectively and adding, we obtain

$$\left[\frac{\partial}{\partial t}-\left(\nu-\frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2\right]\frac{\partial u_3}{\partial z}=\frac{1}{\rho_0}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\delta p-2\Omega\zeta-\mathrm{M} \tag{25}$$

Operating on eqn(21) by $\left[\nabla^2-\left(\partial^2/\partial z^2\right)\right]$ and Differentiate partially with respect to z eqn (25) and adding, we obtain

$$\frac{\partial}{\partial t}\nabla^2 u_3=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(\lambda g_0\alpha-\frac{\gamma' M_0\nabla H}{\rho_0}\right)\theta-\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\lambda g_0\alpha'\gamma+\left(\nu-\frac{\mu'}{\rho_0}\nabla^2\right)\nabla^4 u_3-2\Omega\frac{\partial\zeta}{\partial z}-\mathrm{M}. \tag{26}$$

Differentiate partially with respect to $x\ and\ y$ eqns (20) and (19), respectively and subtracting them, we obtain

$$\frac{\partial\zeta}{\partial t}=\left(\nu-\frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2\zeta+2\Omega\frac{\partial u_3}{\partial z}. \tag{27}$$

3. RELATIOSHIP OF DISPERSION

Using normal mode technique, the eqns (26), (27), (23) and (24) reduces in non-dimensional form,

$$\left(D^2-a^2\right)\left[\sigma+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right]W+\frac{\lambda\alpha a^2d^2}{\nu}\left(g_0-\frac{\gamma' M_0\nabla H}{\rho_0\alpha\lambda}\right)\Theta\ -\frac{\lambda g_0\alpha' a^2d^2}{\nu}\Gamma+\frac{2\Omega d^3}{\nu}DZ=0, \tag{28}$$

Clearly all the even order derivatives of W vanish at the boundaries.

$$W=W_0\sin\pi z, \tag{29}$$

4. Numerical Results

A, STATIONARY CONVECTION

When the instability sets, the marginal state at stationary convection will be characterized by $\sigma_1=0$. Thus, put $\sigma_1=0$ in Eq. (36), we get,

$$R_1=\frac{1}{\lambda x}\left\{(1+x)^3\left[F_1(1+x)+1\right]+\frac{T_{A_1}}{\left[F_1(1+x)+1\right]}\right\}+S_1. \tag{30}$$

This (30) expresses modified Rayleigh number R_1 as a function of the parameters S_1, F_1, T_{A_1} and dimensionless wave number x.

Using (30) we have,

$$\frac{dR_1}{dS_1} = 1 \text{ (which is +ve)} \tag{31}$$

From (30), we have,

$$\frac{dR_1}{dF_1} = \frac{(1+x)}{\lambda x}\left\{(1+x)^3 - \frac{T_{A_1}}{\left[F_1(1+x)+1\right]^2}\right\}, \tag{32}$$

From (30), we have,

$$\frac{dR_1}{dT_{A_1}} = \frac{1}{\lambda x\left[F_1(1+x)+1\right]}, \tag{33}$$

An appropriate equation becomes

$$\frac{dR}{dM_0} = \left\{S_1\pi^4 + \frac{\pi^4}{\lambda x}(1+x)^3\left[F_1(1+x)+1\right] + \frac{T_{A_1}\pi^4}{\lambda x\left[F_1(1+x)+1\right]}\right\}\left(1 - \frac{\gamma' M_0 \nabla H}{\rho_0 \alpha \lambda g_0}\right)^{-2}\left(\frac{\gamma' \nabla H}{\rho_0 \alpha \lambda g_0}\right), \tag{34}$$

which shows that in the absence of solute gradient; magnetization has stabilizing effect on the system for both $\lambda > 0$ and $\lambda < 0$.

B. STABILITY OF THE SYSTEM AND OSCILLATORY MODES

The stability of system and oscillation mode is given by using appropriate boundary condition

$$\sigma I_1 + I_2 + FI_3 + d^2\left[\sigma^* I_4 + I_5 + FI_6\right] - \frac{\alpha a^2 \kappa_T \lambda}{\beta \nu}\left(g_0 - \frac{\gamma' M_0 \nabla H}{\rho_0 \alpha \lambda}\right)\left[I_7 + p_1 \sigma^* I_8\right] + \frac{\lambda g_0 \alpha' a^2 \kappa_S}{\beta' \nu}\left[I_9 + \sigma^* q I_{10}\right] = 0 \tag{35}$$

5. NUMERICAL COMPUTATIONS

The findings are consistent with prior studies. The latest findings appear to be in line with the previous studies. The numerical value of thermal Rayleigh number R_1 is determined for various values of solute gradient S_1, couple-stress F_1, rotation T_{A_1}, and magnetization M_0. Also, graphs have been plotted between R_1 and S_1, R_1 and F_1, R_1 and T_{A_1}, R_1 and M_0 as shown in figures (2)-(11).

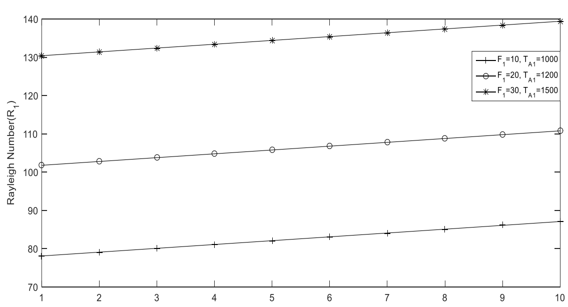


Figure 2. Variation of R_1 with S_1 for $\lambda > 0$.

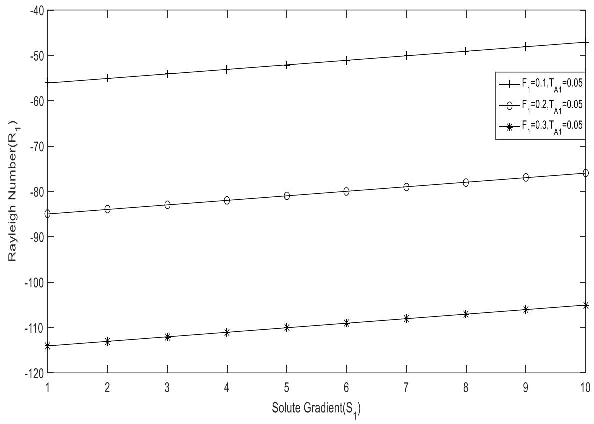


Figure 3. Variation of R_1 with S_1 for $\lambda < 0$.

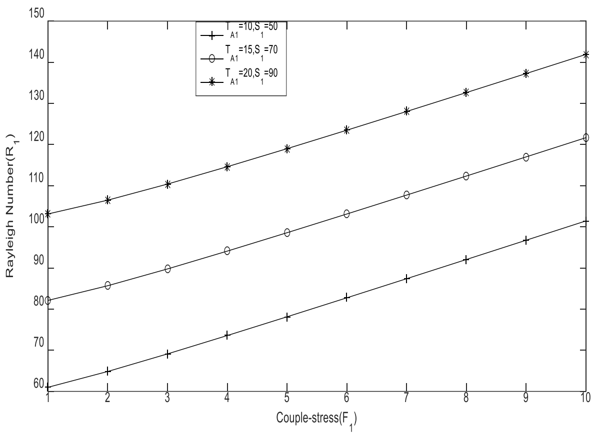


Figure 4. Variation of R_1 with F_1 for $\lambda > 0$.

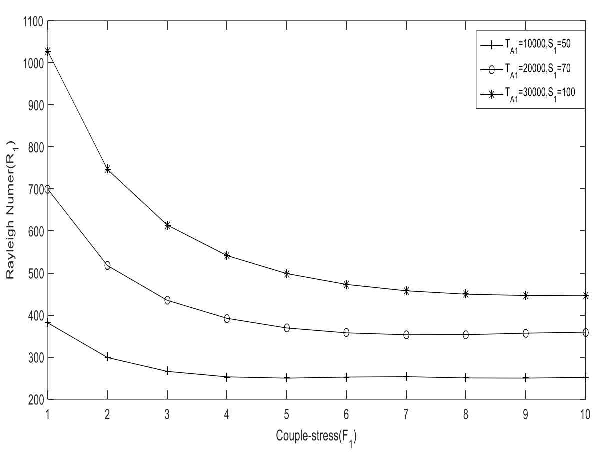
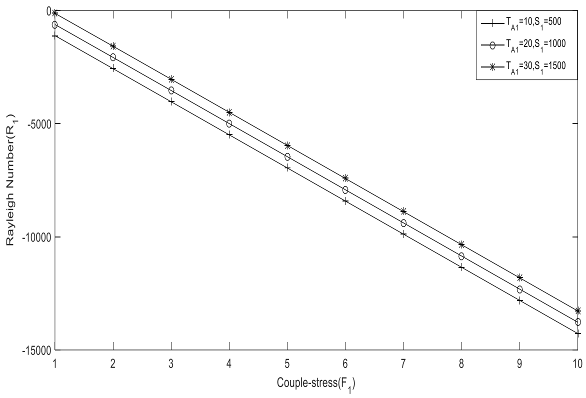


Figure 5. Variation of R_1 with F_1 for $\lambda > 0$.



'**Figure 6. Variation of R_1 with F_1 for $\lambda < 0$.**

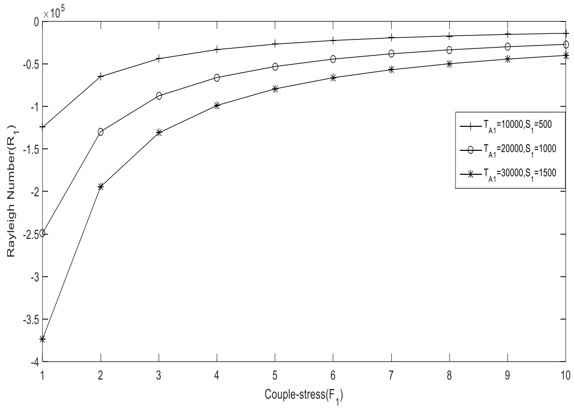


Figure 7. Variation of R_1 with F_1 for $\lambda < 0$.

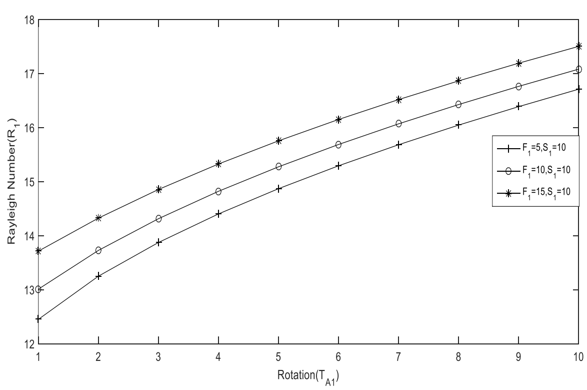


Figure 8. Variation of R_1 with T_{A_1} for $\lambda > 0$.

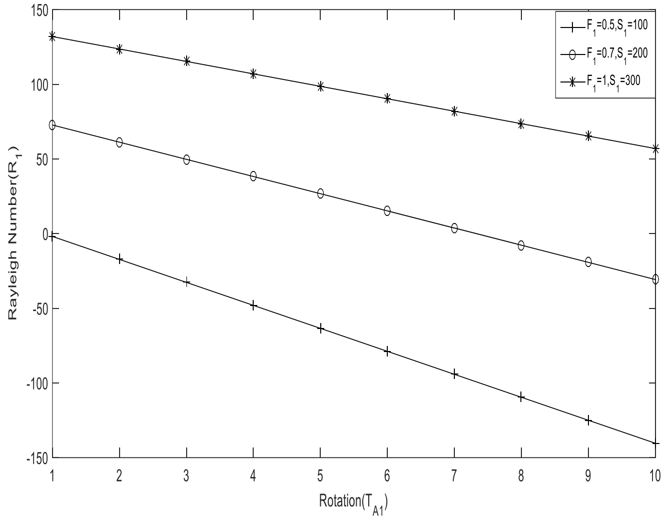


Figure 9. Variation of R_1 with T_{A_1} for $\lambda < 0$.

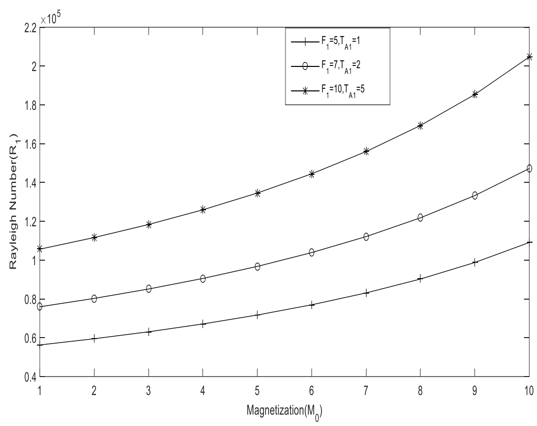


Figure 10. Variation of R_1 with M_0 for $\lambda > 0$.

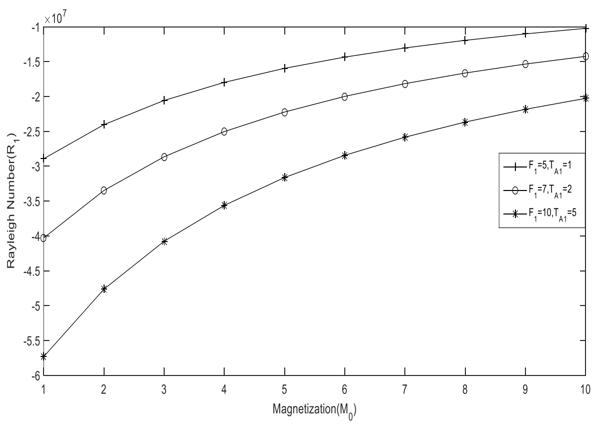


Figure 11. Variation of R_1 with M_0 for $\lambda < 0$.

In Fig.2, critical Rayleigh number R_1 is depicted against solute gradient parameter S_1 for $F_1 = 10, 20, 30; T_{A_1} = 1000, 1200, 1500$ and $\lambda > 0(\lambda = 3)$, which shows that critical Rayleigh number R_1 increases with increase in solute gradient parameter S_1.

In Fig.3, critical Rayleigh number R_1 is depicted against solute gradient parameter S_1 for $F_1 = 0.1, 0.2, 0.3; T_{A_1} = 0.05, 0.05, 0.05$ and $\lambda < 0(\lambda = -5)$, which shows that critical Rayleigh number R_1 increases with increase in solute gradient parameter S_1.

In Fig.5, critical Rayleigh number R_1 is depicted against couple-stress parameter F_1 for $T_{A_1} = 10000, 20000, 30000; S_1 = 50, 70, 100$ and $\lambda > 0(\lambda = 1000)$, which shows that critical Rayleigh number R_1 decreases with sin the system.

In Fig.6, critical Rayleigh number R_1 is plotted against couple-stress parameter F_1 for $T_{A_1} = 10, 20, 30; S_1 = 500, 1000, 1500$ and $\lambda < 0(\lambda = -1)$, which depicts that critical Rayleigh number R_1 decreases with increase in couple-stress parameter F_1.

In Fig.7, critical Rayleigh number R_1 is plotted against couple-stress parameter F_1 for $T_{A_1} = 10000, 20000, 30000; S_1 = 500, 1000, 1500$ and $\lambda < 0(\lambda = -15)$, which shows that critical Rayleigh number R_1 increases with increase in couple-stress parameter F_1.

In Fig.8, critical Rayleigh number R_1 is plotted against rotation parameter T_{A_1} for $F_1 = 5, 10, 15; S_1 = 10, 10, 10$ and $\lambda > 0(\lambda = 50)$, which shows that critical Rayleigh number R_1 increases with increase in rotation parameter T_{A_1}.

In Fig.9, critical Rayleigh number R_1 is plotted against rotation parameter T_{A_1} for $F_1 = 0.5, 0.7, 1; S_1 = 100, 200, 300$ and M= 1, $\lambda < 0(\lambda = -10)$, which shows that critical Rayleigh number R_1 decreases with increase in rotation parameter T_{A_1}.

In Fig.10, critical Rayleigh number R_1 is plotted against magnetization parameter M_0 for $F_1 = 5, 7, 10; T_{A_1} = 1, 2, 5$ and $\lambda > 0(\lambda = 0.1)$, which shows that critical Rayleigh number R_1 increases with increase in magnetization parameter M_0.

In Fig.11, critical Rayleigh number R_1 is plotted against magnetization parameter M_0 for $F_1 = 5, 7, 10; T_{A_1} = 1, 2, 5$ and $\lambda < 0(\lambda = -0.02)$, which shows that critical Rayleigh number R_1 increases with increase in magnetization parameter M_0.

6. **Conclusion**

Thermal convection was studied in a spinning Newtonian ferromagnetic fluid that is sensitive to temperature and a magnetic field. Problems with large-scale changes, such as material processing, earth crust, and crystal growth, may be treated with the assistance of such analysis. The system's normal mode techniques caused by external constraints are studied using the perturbation approximation. The main results from the analysis of this present problem are given below:

For stationary convection,

(i)Solute gradient has a stabilizing effect on the system.

(ii) Couple-stress has a stabilizing effect on the system for $\lambda > 0, T_{A_1} < \left[F_1(1+x)+1\right]^2 (1+x)^3$ and' Also, couple-stress has a destabilizing effect on the system for $\lambda > 0, T_{A_1} > \left[F_1(1+x)+1\right]^2 (1+x)^3$ and $\lambda < 0, T_{A_1} < \left[F_1(1+x)+1\right]^2 (1+x)^3$. The exchange principle ceases to exist when a viscoelastic parameter is present since the magnetic field produces oscillating modes. The stabilizing effect grows so strong for even bigger rotations that no inclination causes the destabilizing effect to be felt.

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