

Modeling and Analysis of Blood Movement in Stenosis vessels under the Stimulus of Permeability and Magnetization Parameters

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Abstract: The present paper are consider the two-phase typical, in which the blood is represented by a essential core of deferred erythrocytes bounded by a peripheral cell-free layer; the governing equations are developed and solved analytically to derive expressions for velocity, flow proportion, and viscosity in the existence of the outer layer; the effects of magnetization and permeability parameters on blood flow dynamics are systematically analyzed, and the results are presented and interpreted through graphical representations.

Key Words: Permeability parameter, Hartmann Number, viscosity, stenosed vessel, erythrocytes.

1. Introduction

One of the main causes of death in the globe is heart disease. These diseases usually result from a brief imbalance in the heart's oxygen or blood supply. A narrowing or blockage in the blood vessels, frequently brought on by a buildup of fatty compounds like cholesterol, cellular waste, calcium, and other elements, can induce this lack of something. According to Kumar and Saket [27, 20], this condition is known as stenosis.

Blood exhibits unusual gluey possessions, primarily due to the interruption of units in plasma. The existing literature includes studies exploring how the shape of stenosis influences overall blood flow dynamics, with a particular focus on the stress exerted on plaques. Several important analytical investigations have been conducted on blood flow within arterial stenosis. While the plasma component of blood follows the linear Newtonian model for viscosity (Kumar et al. [1]) blood as a whole is often treated as a non-Newtonian fluid, especially when the flow's characteristic dimensions approach that of the blood cells. To diagnose cardiovascular disorders and explain pathological changes in animal and human physiology, experimental research and theoretical assessments of blood flow are essential (Srivastava and Srivastava [2]).

The physiological and therapeutic significance of blood flow through small-diameter arteries is substantial. The complexities and non-Newtonian nature of blood flow makes investigation extremely challenging. Various shear effects contribute to these problems, as described by Chien et al. [3]. Blood behaves like Newtonian fluid at high shear rates in bigger arteries. The apparent viscosity of blood, however, reduces with decreasing channel diameter, particularly inside capillaries smaller than 300 micrometers, a phenomenon established by Fahraeus and Lindqvist [4].

Kumar [5] developed a computational model to investigate blood flow in the presence of atherosclerosis, revealing that blood viscosity remains constant for tube diameters greater than 1 mm. However, for diameters

smaller than 1 mm, viscosity exhibits a strong dependence on tube diameter. Additionally, their study demonstrated a nonlinear increase in viscosity with rising hematocrit levels (Udupa et al. [23]). Previous studies by Bugliarello and Sevilla [6], Cokelet [7], and others have shown that in narrow blood vessels, a plasma-rich peripheral layer surrounds a central core of suspended erythrocytes.

Red blood cells (RBCs), being inherently bio-magnetic, can influence blood flow under the application of a magnetic field (Kumar [11]). Theoretical analyses of the effects of magnetic fields on blood flow have been conducted by treating blood as an electrically conductive fluid (Chen [12]; Kumar [24]; Kumar et al. [23]). By considering blood as a magnetization liquid, it may be probable to regulate blood pressure and flow characteristics through the application of an appropriate magnetic field, presenting potential therapeutic applications for cardiovascular and vascular diseases.

A single-phase methodology is the foundation of a lot of blood flow models now in use [13]–[17]. But since blood is a interruption, a two-phase typical provides a more realistic picture. For the purpose to study blood flow in vessels with porous effects, Kumar et al. [18], [26] explored computational techniques. The most recent work uses a two-phase model which involves an applied magnetic field in addition to a central core of suspended erythrocytes encircled by a cell-free layer. Blood viscosity is primarily impacted by a static magnetic field with the interaction of the external magnetic field and the generated magnetic moment of red blood cells. Established on the anisotropic direction of RBCs, which prevents these from rolling, this interaction causes blood flow more frictionally, enhancing blood viscosity overall [22].

2. Key Components of the Model

(a) Magnetohydrodynamic (MHD) Flow:

Magnetohydrodynamics (MHD) deals with the behavior of electrically conductive fluids in the presence of magnetic fields. Since blood is a conductive fluid (due to the presence of ions in plasma), the magnetic field can interact with the flowing blood, altering its velocity and pressure distribution. The interaction between the magnetic field and the moving blood induces additional forces, which must be accounted for in the governing equations.

(b) Stenosed Artery:

The artery is modeled with a stenosis, or narrowing, which is a common condition that restricts blood flow. The stenosis is modeled with a cosine-shaped geometry to more realistically represent how the artery narrows (as opposed to a simple linear or abrupt narrowing). This type of shape helps in modeling gradual constrictions that more closely mimic pathological conditions such as atherosclerosis.

(c) Porous

Medium:

The porous medium assumption models the surrounding tissue of the artery as a permeable structure. This could account for the properties of the arterial wall, which may not be perfectly rigid but have some degree of permeability. The effect of this porous medium on blood flow is considered by modifying the usual governing equations for fluid flow, incorporating additional terms to account for the resistance due to the porous structure.

3. Mathematical Model

The mathematical model is developed by considering blood as a viscous, incompressible, electrically conducting fluid that undergoes steady, laminar flow under a constant magnetization parameter. The exemplary consists of an outer layer filled with plasma and a vital core of radius R_1 , which encompasses an erythrocyte interruption with a uniform hematocrit (Fig. 1).

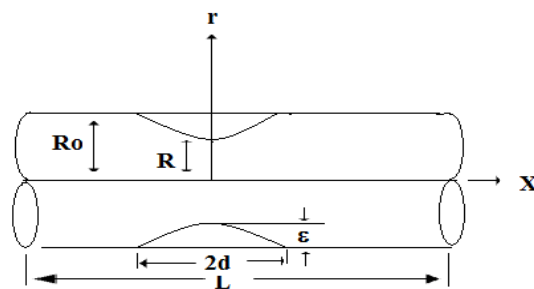


Fig.1: Geometrical Structure of blood flow in Stenosed artery.

4. Governing Equations

Incorporating the above assumptions, the governing equations for fluid flow are expressed as follows:

$$-\frac{\partial P}{\partial z} - \frac{\mu_0}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_0}{\partial r} \right), R_1 \leq r \leq R_0, \tag{1}$$

$$0 = (1 - \phi) \left[-\frac{\partial P}{\partial z} + \mu_s(\phi) \left(\frac{\partial^2 u_f}{\partial r^2} + \frac{1}{r} \frac{\partial u'_f}{\partial r} \right) \right] + \phi F_d (u'_p - u'_f) - K u_f \tag{2}$$

$$0 = \phi \left[-\frac{\partial P}{\partial z} - F_d (u'_p - u'_f) + k_0 M_p \frac{dH}{dz} \right] - K u_f \tag{3}$$

Here, the radial and axial coordinates are represented by r and z , respectively, while the fluid (plasma) and particle (red blood cells) velocity are displayed by u_f and u_p . The ϕ is the volume percentage of red blood cells and indicates the viscosity of solution form. The drag coefficient, indicated as F_d , describes the force of interaction between the two phases. M_p stands for red blood cell magnetization, k_0 for magnetic permeability. The permeability parameter is K . The formula for the drag coefficient is as described below:

$$F_d = \frac{9}{2} \frac{\mu_f}{R^2} \lambda(\phi), \text{ where } \lambda(\phi) = \frac{4 + 3(8\phi - 3\phi^2)^{\frac{1}{2}} + 3\phi}{(2 - 3\phi)^2}.$$

The boundary conditions are given by

$$\begin{aligned} u_0 &= 0, & \text{at } r &= R \\ \frac{\partial u_p}{\partial r} &= \frac{\partial u_f}{\partial r} = 0, & \text{at } r &= 0 \\ u_f &= u_0 \text{ and } \tau_f = \tau_0 & \text{at } r &= R_1 \end{aligned} \tag{4}$$

5. Computational Scheme

Introducing the following dimensionless scheme.

$$\begin{aligned} r' &= \frac{r}{R}, z' = \frac{z}{z_0}, R_1' = \frac{R_1}{R}, P' = \frac{P'}{\rho U^2} \\ u_f &= \frac{u'_f}{U_0}, u_p = \frac{u'_p}{U_0}, \mu = \frac{\mu_0}{\mu_s}, \text{Re} = \frac{U_0 R \rho}{\mu_0} \end{aligned} \tag{5}$$

The velocities u_0 , u_f and u_p derived as the solution to equations (1)–(3) under the given boundary conditions, are expressed as follows:

$$u_0 = \frac{R_e}{4} \varepsilon \frac{\partial P}{\partial z} (R_1^2 - 1) - K_f \tag{6}$$

$$u_f = \frac{R_e}{4} \frac{\partial P}{\partial z} \left[\frac{1}{(1-\phi)\mu_s} \{e\phi R^2 + (1-\phi)\varepsilon\} (r^2 - R_1^2) + \varepsilon(R_1^2 - 1) \right] - \frac{\phi R_e R^2 e M_p k_0}{4(1-\phi)\mu_s} \cdot \frac{dH}{dz} (r^2 - R_1^2) \tag{7}$$

$$u_p = \frac{R_e}{4} \frac{\partial P}{\partial z} \left[\frac{1}{(1-\phi)\mu_s} \{e\phi R^2 + (1-\phi)\varepsilon\} (r^2 - R_1^2) + \varepsilon(R_1^2 - 1) - \frac{4e}{F_d} \right] + R_e M_p k_0 \frac{dH}{dz} \left[\frac{e}{F_d} - \frac{\phi R^2 e}{4(1-\phi)\mu_s} (r^2 - R_1^2) \right] \tag{8}$$

Now, the volumetric flow rate, or flow flux, is computed as

$$Q = Q_0 + Q_p + Q_f, \tag{9}$$

By substituting eqs (6) to (8) into eq (9), then reduce the flow rate are :

$$Q = \frac{\pi R^4}{8\mu_s} \frac{\partial P}{\partial z} \left[U_0 R_e \mu_s \varepsilon (1 - R_1^2) (R^2 + R_1^2 R^2 - 2) + \frac{2 R_1^4 U_0 R_e}{(1-\phi)} \cdot \left\{ (e\phi R^2 + (1-\phi)\varepsilon) (R^2 - 2) + \frac{2\varepsilon}{R_1^2} (R_1^2 - 1) \right\} + \frac{4\phi U_0 R_e \mu_s e}{F_d} + \frac{8\phi R_1^4 U_0 R_e M_p k_0 e \mu_s}{\left(\frac{\partial P}{\partial z}\right)} \cdot \frac{dH}{dz} \left\{ \frac{1}{F_d R_1^2} - (R^4 - 2R^2) \left(1 + \frac{4\phi}{(1-\phi)\mu_s} \right) \right\} \right] \tag{10}$$

The relation is determined by the fact that the total flux is the sum of the fluxes through the two regions: peripheral and core are given below

$$R_1 = \alpha R \tag{11}$$

(11)

The effective (apparent) viscosity can be calculated using the relations (10) and (11).

$$\mu_e = \mu_s \left[\frac{U_0 R_e \mu_s \varepsilon (1 - \alpha^2 R^2) (R^2 + R^4 \alpha^2 - 2) + \frac{2\alpha^4 R^4 U_0 R_e}{(1-\phi)} \left\{ (eR^2\phi + (1-\phi)\varepsilon) (R^2 - 2) + \frac{2\varepsilon (\alpha^2 R^2 - 1)}{\alpha^2 R^2} \right\}}{\frac{4\phi U_0 R_e \mu_s e}{F_d} + \frac{8\phi \alpha^4 R^4 U_0 R_e M_p k_0 e \mu_s}{\left(\frac{\partial P}{\partial z}\right)} \cdot \frac{dH}{dz} \left\{ \frac{1}{F_d \alpha^2 R^2} - (R^4 - 2R^2) \left(1 + \frac{4\phi}{(1-\phi)\mu_s} \right) \right\}} \right]^{-1} \tag{12}$$

6. Numerical Results

The most current algorithm demonstrates rapid convergence and is both cheap and efficient. This section provides computational results to examine how different parameters affect flow rate, velocity profiles, and other important properties. The outcomes have been calculated for several mixtures of the solution's parameters. In this work, we have created a mathematical model that incorporates a cosine-shaped geometry for the stenosis and describes magneto hydrodynamic blood flow in a stenosis vessel characterized as a permeable pedestrian. The axial velocity, pressure gradient, volumetric flow rate, and flow resistance are all analytically expressed by our analysis.

Certain of the information is illustrated graphically in Fig. 2-6 to provide a quantitative assessment for the different variables elaborate, particularly the hematocrit and magnetization ascent.

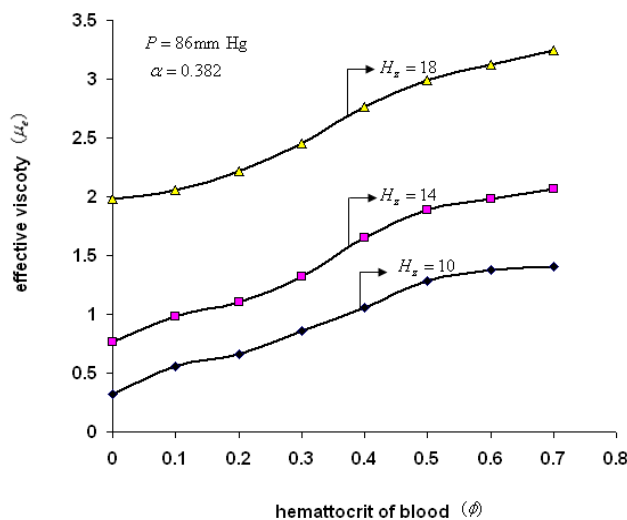


Fig. 2 Plotted diagram on Viscosity versus hematocrit of blood for various values of Magnetization Parameter

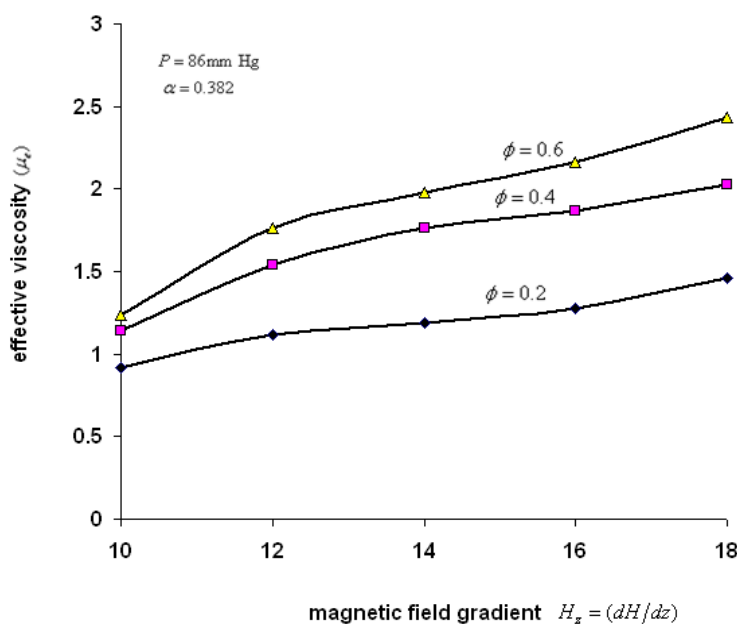


Fig. 3 Plotted diagram on Viscosity with Magnetization parameter of blood for various values of the Hematocrit

The deviation of effective viscosity (μ_e) for altered values of magnetization parameter and $K=1$ is depicted in fig.2 and for altered values of hematocrit (ϕ) is depict in fig.3.

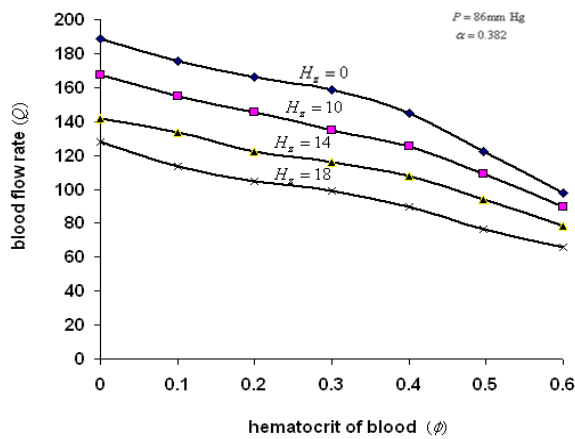


Fig. 4 Plotted diagram of blood flow to connect hematocrit of blood for various values of Magnetization Parameter

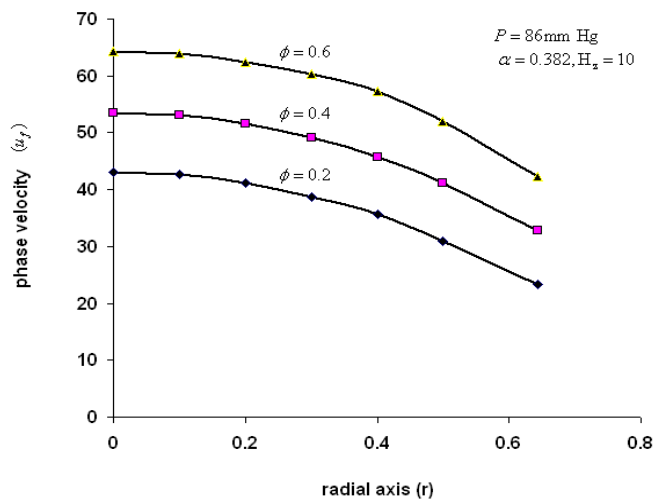


Fig. 5 Plotted diagram of phase velocity with radial axis for various values of hematocrit of blood.

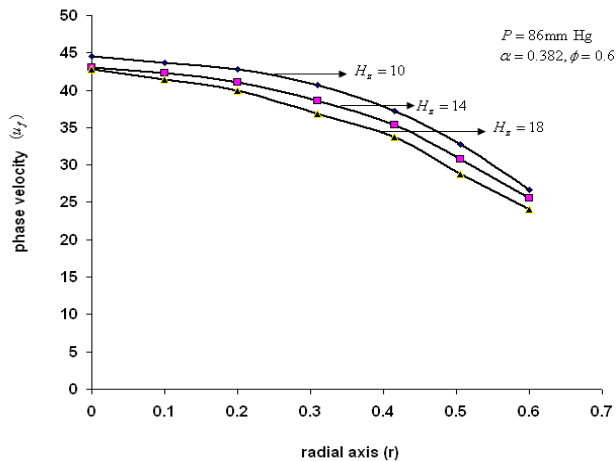


Fig. 6 Plotted diagram of phase velocity with radial axis for many values of Magnetization parameter

The variant of axial velocities outlines u_f and u_p for together plasma and erythrocyte stage with radial axis for altered values of hematocrit (ϕ) are plotted in fig. 5.

Fig. 6 depicts that the variation of axial velocity componets u_f and u_p for both plasma and erythrocyte stage with radial axis (r) for dissimilar values of magnetization parameter and permeability parameter.

7. Conclusions

This study investigate fluid mechanical phenomena influenced by the magnetization and permeability parameters, considering the presence of a peripheral layer. Blood is modeled as a two-fluid system, with the core region comprising an erythrocyte suspension, while the plasma in the peripheral region is treated as a Newtonian fluid. The findings reveal that as magnetic field strength and hematocrit levels increase, velocity and flow rate decrease, whereas viscosity rises. [Kumar et al. 25].

From this analysis, it is evident that the magnetic field significantly influences axial flow velocities and effective viscosity. By appropriately adjusting the magnetic field, it is possible to regulate both these parameters. This study examines the impact of magnetization and permeability on blood flow in small vessels, a topic of considerable interest in medical sciences. The application of a magnetic field alters blood flow in small vessels, offering insights into conditions such as blood pressure regulation and hypertension.

Additionally, the study of magnetization effects on blood flow reveals temperature variations, which may be beneficial for thermal therapies in treating tumors, glands, and other medical conditions. Understanding these effects could also contribute to advancements in treatments for small vessel disease, where medications are often used to prevent vessel narrowing, reducing the risk of heart attacks.

References:

1. Anil Kumar, GC Sharma and M. Jain: MHD flows in stenosed artery, Proceedings in International Conference on Mathematical Modeling, organized by Indian Institute of Technology Roorkee, pp 683-687, 2001.
2. V.P.Srivastava and R.Srivastava: Particulate interruption blood flow through a narrow catheterized artery, *Compt. and Math. With appl.*,58(2009),227-238.
3. S.Chien,S.Usami and R.Skalak: Blood flow in small tubes”,In Renkins E.M.Michel,C.C.(Eds.), *American Physiological Society Handbook of Physiology,Section 2,The Cardiovascular System*,4,Bethesda MD;American Physiological Society,(1984), 217-249.

4. R.Fahraeus and R.Lindqvist: Viscosity blood in narrow capillary tube”, American Journal Physiology ,96 (1931),562-568.
5. Anil Kumar: Computational model of blood flow in the presence of atherosclerosis, International Federation for Medical and Biological Engineering (IFMBE), Conference 6th World Congress on Biomechanics, IFMBE Proceedings 31, Springer Publishing 1591–1594, 2010
6. G.Bugliarello,J.Sevilla: Velocity distribution and other characteristics of steady and pulsatile blood flow in fine glass, Biorheology,7 (1970),85-107.
7. G.R.Cokelet: The rheology of human blood, In Fung Y.C.(Eds.),Biomechanics, Prentice-Hall,Englewood Cliffs N.J.(1972).
8. M.Sharan and A.S.Popel:A two-phase model for flow of blood in narrow tubes with increased effective viscosity near the wall, Biorheology,38(2001),415-428.
9. V.P.Srivastava: A theoretical model for blood flow in small vessels, International Journal of Application and Applied Mathematics,2(1),(2007),51-65.
10. D.S.Sankar and U.Lee: Two-fluid Herchel-Bilkey model for blood flow in catheterized arteries, J. of Mechanical Sciences and Technology,22(2008),1008- 1018.
11. Anil Kumar (2021): Mathematical model for porous influence through mild stenosis thrombosis over delivery of oxygen and nutrients arteries , Advances in Mathematics: Scientific Journal 10 (1), 463-469 , ISSN: 1857-8365,
12. I.H.Chen,“Analysis of an intensive magnetic field on blood flow :part- 2” J.Bioelect.,4(1985),55-61.
13. Y.Haik,V.Pai and C.J.Chen: Apparent viscosity of human blood in a high static magnetic field”,Journal of magnetism and magnetic materials,225 (2001),180-186.
14. J.R.Womersley: Method for the calculation of velocity rate of flow and viscous drag in arteries when the pressure gradient is known, J.Physiol.,127(1955),533- 563.
15. Varshney, G., Katiyar, V., and Kumar, S. : Effect of magnetic field on the blood flow in artery having multiple stenosis: a numerical study. *International Journal of Engineering, Science and Technology*, 2, 67-82 , (2010).
16. V.K.Sud and G.S.Sekhon: Arterial flow under periodic body acceleration, Bullmath.Biol. 47(1985),35-52.
17. P.Chaturani and V.Palanisamy: Pulsatile flow of blood with periodic body acceleration, International Journal of .Engng Sci.,29(1991),113-119.
18. Anil Kumar, CL Varshney and GC Sharma (2005): Computational technique for flow in blood vessels with porous effects, Applied Mathematics and Mechanics 26(1) , 63-72, 2005
19. P.G.Saffman,“On the stability of laminar flow of a dusty gas, Journal of Fluid Mech.,13 (1962),120-128.
20. Anil Kumar and S P Agarwal (2015): Computer Modeling and analysis of the hydrodynamic parameters of blood flow through stenotic artery, Third International conference on recent trends in computing , India Procedia Computer Science, 57 (2015) 403 – 410
21. D.C.Sanyal, K.Das and S.Debnath: Effect of magnetic field on pulsatile blood flow through an inclined circular tube with periodic body acceleration, Journal of Physio.Sci.,11(2007),43-56.
22. T.Yamamoto,Y.Nagayama and M. Tamura: A blood –oxygenation dependent increase in blood viscosity due to a static magnetic field,Phys.Med.Biol.,49(2004),3267-3277.
23. Anil Kumar, CL Varshney and GC Sharma (2005): Computational technique for flow in blood vessels with porous effects, Applied Mathematics and Mechanics 26(1) pp 63-72, 2005.
24. Anil Kumar Gupta (2011): Performance and analysis of blood flow through carotid artery, International Journal of Engineering and Business Management Vol. 3, No. 4, pp 1-6, ISSN: 1847-9790, Croatia. Cited 12 Published, January 1, 2011
25. Sharma, G. C., Jain, M., and Kumar, A.: MHD flow in stenosed artery, Proceedings in International conference on Mathematical modelling Jan. 29-31 Roorkee, (2001), 683-687.
26. Sharma, G.C Jain, M. and Kumar, A.: Finite element Galerkin’s approach for a computational study of arterial flow, Applied Mathematics and Mechanics, 22 (9), (2001), 1012-1018.
27. Kumar, A., and Saket R.K., Reliability of Convective Diffusion Process in Stenosis Blood Vessels”, Chemical product and process modeling,. 3 (1), (2008), 1- 17