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A Comprehensive Study on Hexa Graph Topology and Its Applications

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Abstract: The role of this text is to introduce a new graph titled Hexa graph topological space. We define a Hexa graph topological space which has any six subgraphs and six graph decomposition of an undirected graph. Here we provide a new method form to generate topological induced by the edges of the graph and discuss some properties on it. Defining topolo- gies on discrete structures has been a challenging area of research that is relatively obscure. The concept of open sets defined on topology can be extended to the field of graph theory by defining the notion of new open subgraphs on Hexa graph topological spaces.

Keywords: Hexa graph topology, Hexa open subgraph, Hexa closed subgraph, Hexa graph Interior, Hexa graph closure.

1. Introduction and preliminaries

The relation between graph and topology led to a subfield called topo- logical graph theory. The graph is named Hexa graph because of its size since it has at most six subgraphs and six disjoint graph decomposition of G. we introduced a new form of topological spaces for graphs which is called ”Hexa Graph Topology”.Inspired by the work of Binoy Balan[2]

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in A New Graph Topology on Decomposition of Graph, we have intro- duced a novel concept called the hexa graph topological space. This idea is based on the recent advancements made by Mohanapriya and Santhiya [5] who introduced a new form of nanohexa topological space. Drawing from their foundational work, our contribution defines and develops the structure of hexagraph topological spaces, establishing new pathways for further exploration in graphbased topological frameworks. In this paper, we investigate the idea of the Hexa graph topology of a graph, where we consider the collection of six disjoint subgraphs of an undirected graph. We begin with the basic concepts of Hexa graph topological space and introduce Hexa closed subgraphs and Hexa open subgraphs in Hexa graph topological space. A graph topology on a graph is a group of subgraphs satisfying three axioms related to the axioms of topology.

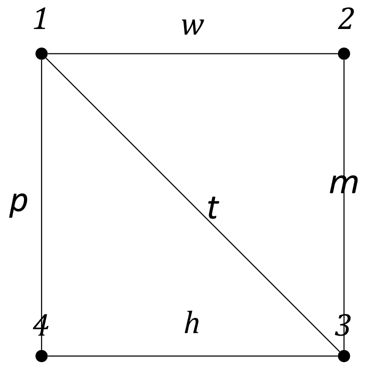
2. Lower Hexa and Upper Hexa subgraph

Throughout this part, we provide a new method to construct a new graph based on the concept of lower, upper and boundary of the graph called Lower Hexa subgraph and upper Hexa subgraph and boundary Hexa subgraph.

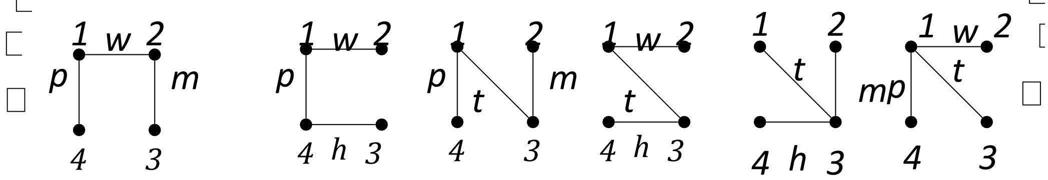
Definition 2.1. *Let G be an undirected graph with atleast 3 edges and it has a set of edges E(G).Consider six graph decomposition* $J_h(s)$ *for* $h = 1, 2, 3, 4, 5, 6$ *and the subgraph* $S_1, S_2, S_3, S_4, S_5, S_6$ *are subsets of the edge set* $E(G)$ *where each* S_i *is a subgraphs of G, then its lower hexa sub- graph and upper hexa subgraph is defined as* $H_-(S_i) = \bigcup_{s \in E(G)} \{s : J_1(s) \subseteq$

$S_1 \bigvee J_2(s) \subseteq S_2 \bigvee J_3(s) \subseteq S_3 \bigvee J_4(s) \subseteq S_4 \bigvee J_5(s) \subseteq S_5 \bigvee J_6(s) \subseteq S_6\}$, $H^{-}(S_i) = \bigcup_{s \in E(G)} \{s : J_1(s) \cap S_1 \neq \phi \bigvee J_2(s) \cap S_2 \neq \phi \bigvee J_3(s) \cap S_3 \neq \phi \bigvee J_4(s) \cap S_4 \neq \phi \bigvee J_5(s) \cap S_5 \neq \phi \bigvee J_6(s) \cap S_6 \neq \phi\}$. *The boundary hexa subgraph is defined as* $B_h(S) = H^{-}(S_i) - H_{-}(S_i)$.

Example 2.2. *Consider a undirected graph G with edges* $E(G) = \{m, w, h, p, t\}$ *and vertices* $V(G) = \{1, 2, 3, 4\}$



SIX DISJOINT SUBGRAPHS AS FOLLOWS:



$S_i, i = 1, 2, 3, 4, 5, 6$	$J_h(s), h = 1, 2, 3, 4, 5, 6$	$H_{-}(S_i)$	$H^{-}(S_i)$
$\{w, m, p\}$	$\{\{w\}, \{m, t\}, \{p, h\}\}$	$\{w\}$	G
$\{w, h, p\}$	$\{\{w\}, \{m, h\}, \{p, t\}\}$	$\{w\}$	G
$\{m, p, t\}$	$\{\{w\}, \{m, p\}, \{t, h\}\}$	$\{m, p\}$	$\{m, p, t, h\}$
$\{w, h, t\}$	$\{\{w, m, p\}, \{h, t\}\}$	$\{h, t\}$	G
$\{m, h, t\}$	$\{\{w, m, t\}, \{h, p\}\}$	φ	G
$\{w, p, t\}$	$\{\{w, h, t\}, \{m, p\}\}$	φ	G

Hence the Lower Hexa subgraph and the Upper Hexa subgraph and Bound- ary hexa subgraph is $H_{-}(S_i) = \{w, m, p, h, t\}$, $H^{-}(S_i) = \{m, p, t, h\}$, $B_N(S) = \{w\}$

Property 2.3. *Let G be an undirected graph with atleast 3 edges and with set of subgraphs* $S1, S2, S3, S4, S5, S6$ *are subsets of the edge set* $E(G)$*,*
Where each Si is a subgraphs of G.Then we have

(1) $H_{-}(S_i) \subseteq S_i \subseteq H^{-}(S_i)$.
(2) $H_{-}(G) = G = H^{-}(G)$.
(3) *If* $H = \phi$ *then* $H_{-}(\emptyset) = \emptyset = H^{-}(\emptyset)$
(4) $H^{-}(S_i)^c = [H_{-}(S_i)]^c$
(5) $H_{-}(S_i)^c = [H^{-}(S_i)]^c$
(6) $H^{-}(H^{-}(S_i)) = H^{-}(S_i)$
(7) $H_{-}(H_{-}(S_i)) = H_{-}(S_i)$

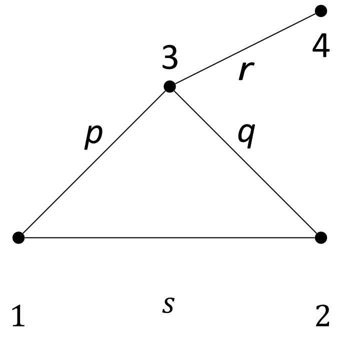
3. Hexa Graph Topological space

Definition 3.1. *Let G be an undirected graph with atleast 3 edges and let* $J_h(s)$*,* $h = 1, 2, 3, 4, 5, 6$ *denotes six graph decomposition of G. Consider six subgraph* $S_1, S_2, S_3, S_4, S_5, S_6$ *are subsets of the edge set* $E(G)$ *where each* $S_i \subseteq E(G)$ *for* $i = 1, 2, 3, 4, 5, 6$ *and* $\zeta_H(S_i) = \{G, \phi, H_{-}(S_i), H^{-}(S_i), B_h(S_i)\}$.
Then a pair $\zeta_H(S_i)$ *is known as Hexa graph topology on G with respect to the subgraph* S_i *of G provided that* $\zeta_H(S_i)$ *satisfies the following axioms*

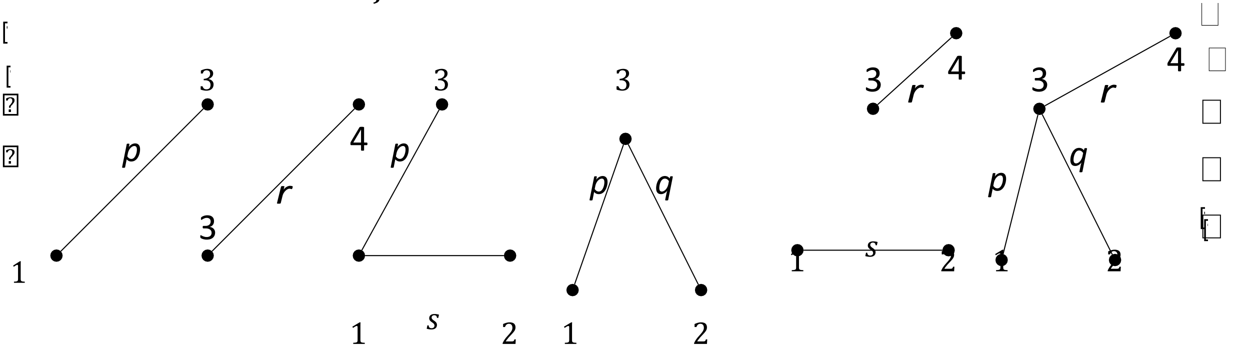
(1) $G, V_o \in \zeta_H(S_i)$, *where G = full graph ,* V_o *= Null graph*
(2) *Any union of elements of* $\zeta_H(S_i)$ *is in* $\zeta_H(S_i)$
(3) *The finite intersection of the elements of* $\zeta_H(S_i)$ *is in* $\zeta_H(S_i)$.

We call the pair $(G, \zeta_H(S_i))$ *as the Hexa graph topological space on G.The elements of Hexa graph topological spaces are regard as Hexa open subgraph of G and the complement is the Hexa open subgraph of G induced by the* *edge set is called Hexa closed subgraph of G.A subgraph which is Hexa open and Hexa closed is called Hexa clopen subgraph.*

Example 3.2. *Consider a undirected graph G with edges* $E(G) = \{p, q, r, s\}$ *and vertices* $V(G) = \{1, 2, 3\}$



SIX DISJOINT SUBGRAPHS AS FOLLOWS:



$S_i, i = 1, 2, 3, 4, 5, 6$	$J_h(s), h = 1, 2, 3, 4, 5, 6$	$H_-(S_i)$	$H^-(S_i)$
$\{p\}$	$\{\{p, q\}, \{r\}, \{s\}\}$	φ	$\{p, q\}$
$\{r\}$	$\{\{p, q, r\}, \{s\}\}$	φ	$\{p, q, r\}$
$\{p, s\}$	$\{\{p\}, \{q\}, \{r, s\}\}$	$\{p\}$	$\{p, r, s\}$
$\{p, q\}$	$\{\{p\}, \{q, r, s\}\}$	$\{p\}$	G
$\{r, s\}$	$\{\{p, r\}, \{q\}, \{s\}\}$	$\{s\}$	$\{p, s, r\}$
$\{p, q, r\}$	$\{\{p, q, s\}, \{r\}\}$	$\{r\}$	G

Here $S_i, i = 1, 2, 3, 4, 5, 6$ *be any six disjoint subgraph of* G *with edges* $\{p\}, \{r\}, \{p, s\}, \{p, q\}, \{r, s\}, \{p, q, r\}$ *and* $J_h(G), h = 1, 2, 3, 4, 5, 6$ *be any six disjoint graph decomposition of G. Hence the Lower Hexa subgraph and Upper Hexa subgraph and Boundary hexa subgraph is* $H_-(S_i) = \{p, s, r\}$, $H^-(S_i) = \{p\}$, $B_h(S_i) = \{s, r\}$ *then* $\zeta_H(S_i) = \{G, \emptyset, \{p\}, \{s, r\}, \{p, s, r\}\}$ *is a Hexa graph topology on* G *and the elements are Hexa open subgraph and the complement* $\zeta_H(S_i)^c = \{G, \emptyset, \{q\}, \{p, q\}, \{q, r, s\}\}$ *is Hexa closed subgraph of G.*

Property 3.3. *Let G be an undirected graph with atleast 3 edges and with set of subgraphs S1, S2, S3, S4, S5, S6 are subsets of the edge set E(G), Where each Si is a subgraphs of G satisfies the following properties:*

(1) *If* $H_-(S_i) = \phi$ *and* $H^-(S_i) = G$ *then* $\zeta_H(S_i) = \{\phi, G\}$ *be the Indis- crete Hexa graph topology on G*

(2) *If* $H_-(S_i) \neq \phi$ *and* $H^-(S_i) = G$ *then* $\zeta_H(S_i) = \{G, \phi, H_-(S_i), B_h(S_i)\}$ *be the Hexa graph topology on G*

(3) *If* $H_-(S_i) = \phi$ *and* $H^-(S_i) \neq G$ *then* $\zeta_H(S_i) = \{G, \phi, H^-(S_i)\}$ *be the Hexa graph topology on G*

Definition 3.4. *Let G be an undirected graph with atleast 3 edges and with set of subgraphs S1, S2, S3, S4, S5, S6 are subsets of the edge set E(G), Where each Si is a subgraphs of G.Then we have ,*$\zeta_H(S_i) = \{\phi, G\}$ *be a collection of the trivial subgraphs of G, then* $\zeta_H(S_i)$ *is a Hexa graph topology on G and is said to be the Indiscrete Hexa graph topology on G.*

Remark 3.5. *The discrete Hexa graph topology is defined as* $\zeta_H(S_i) = \{G, \phi, H_-(S_i), H^-(S_i), B_h(S_i)\}$.

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Definition 3.6. *Let G be an undirected graph with atleast 3 edges and with set of subgraphs S1, S2, S3, S4, S5, S6 are subsets of the edge set $E(G)$, Where each Si is a subgraphs of G.If $\zeta_H[S_i]$ is the Hexa Graph topology on G concerning to S_i, then* $B = \{G, H_{-}(S_i), B_h(S_i)\}$ *is the basis for* $\zeta_H(S_i)$.

Definition 3.7. *Let $\zeta_H(S_i)$ be a Hexa graph topological space concerning to the set of subgraphs S1, S2, S3, S4, S5, S6 are subsets of the edge set $E(G)$. The Hexa graph interior of a subgraph P of G is described as the union of all Hexa open subgraph which is an edge induced subgraph of P and it is identified by* $N_hInt(P)$. *That is* $N_hInt(P)$ *is the greatest Hexa open subgraph of P. The Hexa graph closure of P is specific as the intersection of all Hexa closed subgraph which is a supergraph of P and it is identified by* $N_hCl(P)$. *Therefore,* $N_hCl(P)$ *is the lowest Hexa closed subgraph of P.*

Example 3.8. *Consider example 3.2 with* $E(G) = \{p, q, r, s\}$ *and* $V(G) = \{1, 2, 3\}$ *and S_i be any six subgraph and $J_h(s)$ be any six graph decomposi- tion of G then its Hexa graph topology is defined as* $\zeta_H(S_i) = \{G, \emptyset, \{p\}, \{s, r\}, \{p, s, r\}\}$ *and* $\zeta_H(S_i)^c = \{G, \emptyset, \{q\}, \{p, q\}, \{q, r, s\}\}$. *For a subgraph* $P = \{r\}$ *then its* $N_hInt(P)$ *and* $N_hCl(P)$ *is defined as follows:*

Property 3.9. *Let G be the undirected graph with Hexa graph topological space* $(G, \zeta_H(S_i))$ *and* $T, P \subseteq G$ *then the following properties holds:*

(1) $T \subseteq N_hCl[T]$

(2) T *is Hexa closed subgraph* $\Longleftrightarrow \mathrm{N}_h Cl[T] = T$

(3) $\mathrm{N}_h Cl[\phi] = \phi$ *and* $\mathrm{N}_h Cl[T] = T$

(4) $T \subseteq P \implies \mathrm{N}_h Cl[T] \subseteq \mathrm{N}_h Cl[P]$

(5) $\mathrm{N}_h Cl[T \cup P] = \mathrm{N}_h Cl[T] \cup \mathrm{N}_h Cl[P]$

(6) $\mathrm{N}_h Cl[T \cap P] \subseteq \mathrm{N}_h Cl[T] \cap \mathrm{N}_h Cl[P]$

Property 3.10. *Let G be the undirected graph with Hexa graph topological space* $(G, \zeta_H(S_i))$ *and* $T, P \subseteq G$ *then the following properties holds:*

(1) T *is Hexa open subgraph* $\Longleftrightarrow \mathrm{N}_h Int[T] = T$

(2) $\mathrm{N}_h Int[\phi] = \phi$ *and* $\mathrm{N}_h Int[T] = T$

(3) $T \subseteq P \implies \mathrm{N}_h Int[T] \subseteq \mathrm{N}_h Int[P]$

(4) $\mathrm{N}_h Int[T \cup P] \subseteq \mathrm{N}_h Int[T] \cup \mathrm{N}_h Int[P]$

(5) $\mathrm{N}_h Int[T \cap P] = \mathrm{N}_h Int[T] \cap \mathrm{N}_h Int[P]$

Theorem 3.11. *Let G be the undirected graph with Hexa graph topolog- ical space* $(G, \zeta_H(S_i))$ *and for a subgraph T,P of G and* $T \subset P$ *then the following conditions are hold:*

(1) $\mathrm{N}_h Int[T] = \mathrm{N}_h Int[\mathrm{N}_h Int[T]] = \mathrm{N}_h Cl[\mathrm{N}_h Int[T]] = \mathrm{N}_h Int[\mathrm{N}_h Cl[T]]$

(2) $\mathrm{N}_h Cl[T] = \mathrm{N}_h Cl[\mathrm{N}_h Cl[T]]$

(3) $\mathrm{N}_h Int[\mathrm{N}_h Int[T]] \subseteq \mathrm{N}_h Int[\mathrm{N}_h Int[P]]$

(4) $\mathrm{N}_h Cl[\mathrm{N}_h Cl[T]] \subset \mathrm{N}_h Cl[\mathrm{N}_h Cl[P]]$

proof: Obvious.

Applications*:*

Hexa Graph Topology extends Nano Graph Topology by shifting focus from *vertices to edges, emphasizing hexagonal structures where each edge con- nects to six surrounding subgraphs. This sixfold symmetry enables deeper analysis in various fields:*

Medical Diagnosis: *Models blood vessels or nerves as edge-based systems with six connections, aiding microcirculatory analysis.*

Pattern Recognition: *Hexagonal grids help detect structural patterns in image processing and cellular automata.*

Blood Circulation: *Represents vascular networks with enhanced flow modeling and optimization.*

Social Networks: *Captures multi-level group interactions and infor- mation flow via six-edge connectivity.*

Maps GIS: *Enhances spatial data modeling with hexagonal grids for urban planning and route optimization.*

Geospatial/Network Analysis: *Enables precise routing, connectiv- ity, and flow analysis in complex networks.*

Website Design: *Offers aesthetically engaging layouts, better naviga- tion, and user interaction through hex-based structures.*

Hexa Discrete Topology: *Applies hexagonal grids to analyze discrete topological properties, improving computational modeling and simulation in mathematics and geometry.*

CONCLUSION

Analogous to the similar concepts in point set topology, the notions of Hexa open subgraphs and Hexa closed subgraphs in Hexa graph topological space have been introduced and studied in this paper. The topic is assuring for further analysing as topological structures of graphs are relatively new and have enough within themselves.

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