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PROPERTIES OF SOME NEW CLOSED SETS IN TOPOLOGICAL SPACES

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Abstract: In this article, we introduce a new class of generalized closed sets called g^-closed sets in topological spaces. We investigate the relationships among the related to generalized closed sets.

Keywords: Q-ker(M) $g^{\wedge}$-cld, $g_{\alpha}^{\wedge}$-cld, g-cld.

1. INTRODUCTION

In 1970 Levine [6] introduced the generalized closed sets and Crossley, S. G. and Hildebrand [3] introduced the semi-closure. Bhattacharrya, P and et al. [2] presented the sg-closed. Noiri, T and et al. [7] presented continuity. Ravi, O. and Ganesan, S. [9], introduced the $\ddot{g}$ -closed sets in topology.

In this article, we introduce a new class of generalized closed sets called g^-closed sets in topological spaces. We investigate the relationships among the related to generalized closed.

2. PRELIMINARIES

The definitions listed below are helpful in the properties and examples.

Definition 2.1

Consider M $\subseteq$ X is said to

(i) semi-open [1] if M $\subseteq$ cl(int(M));

(ii) preopen [5] if M $\subseteq$ int(cl(M));

(iii) α -open [4] if M $\subseteq$ int(cl(int(M)));

(iv) regular open [8] if M = int(cl(M)).

Definition 2.2 Let M $\subseteq$ X is called

(i) g-cld [8] if cl(M) $\subseteq$ H $\Rightarrow$ M $\subseteq$ H & H is open.

(ii) sg-cld [5] if scl(M) $\subseteq$ H $\Rightarrow$ M $\subseteq$ H & H is semi-open.

(iii) gs-cld [5] if scl(M) $\subseteq$ H $\Rightarrow$ M $\subseteq$ H & H is open.

(iv) α g-cld [4] if α cl(M) $\subseteq$ H $\Rightarrow$ M $\subseteq$ H & H is open.

(v) gsp-cld [5] if spcl(M) $\subseteq$ H $\Rightarrow$ M $\subseteq$ H & H is open.

(vi) $\ddot{g}$ -cld [9] if cl(M) $\subseteq$ H $\Rightarrow$ M $\subseteq$ H & H is sg-open.

(vii) $\hat{g}$ -cld [10] if cl(M) $\subseteq$ H $\Rightarrow$ M $\subseteq$ H & H is semi-open.

Definition 2.3 [9]

Let M $\subseteq$ X is

(i) P-cld if cl(M) $\subseteq$ H $\Rightarrow$ M $\subseteq$ H & H is $\ddot{g}$ -open.

(ii) Q-cld if scl(M) $\subseteq$ H $\Rightarrow$ M $\subseteq$ H & H is P-open.

Definition 2.4 [9]

Let M $\subseteq$ X is a $g^{\wedge}$-cld if scl(M) $\subseteq$ H $\Rightarrow$ M $\subseteq$ H & H is Q-open.

Definition 2.5 [9]

Let M $\subseteq$ X is a $g_{\alpha}^{\wedge}$-cld if α cl(M) $\subseteq$ H $\Rightarrow$ M $\subseteq$ H & H is Q-open .

3. PROPERTIES OF $g^{\wedge}$-CLOSED SETS

In this section, we discuss some basic properties of some new $\mathcal{g}^{\wedge}$-closed sets.

Definition 3.1

The intersection of all Q-open subsets in X containing M is called the Q-kernel of M and denoted by Q-ker(M).

Lemma 3.2

A subset M of X is $\mathcal{g}^{\wedge}$-cld if and only if scl(M) $\subseteq$ Q-ker(M).

Proof

Suppose that M is $\mathcal{g}^{\wedge}$-cld. Then scl(M) $\subseteq$ H whenever M $\subseteq$ H and H is Q-open. Let x$\in$scl(M). If x$\notin$ Q-ker(M), then there is an Q-open set H containing M such that x $\notin$ H. Since H is an Q-open set containing M, we have x $\notin$scl(M) and this is a contradiction.

Conversely, let scl(M) $\subseteq$ Q-ker(M). If H is any Q-open set containing M, then scl(M) $\subseteq$ Q-ker(M) $\subseteq$ H. Therefore, M is $\mathcal{g}^{\wedge}$-cld.

Proposition 3.3

If M and N are $\mathcal{g}^{\wedge}$-cld in X then M $\cup$ N is $\mathcal{g}^{\wedge}$-cld in X.

Proof

If M $\cup$ N $\subseteq$ K and G is Q-open, then M $\subseteq$ K and N $\subseteq$ K. Since M and N are $\mathcal{g}^{\wedge}$-cld, K $\supseteq$ scl(M) and K $\supseteq$ scl(N) and hence K $\supseteq$ scl(M) $\cup$ scl(N) = scl(M $\cup$ N). Thus M $\cup$ N is $\mathcal{g}^{\wedge}$-cld in X.

Proposition 3.4

If a set M is $\mathcal{g}^{\wedge}$-cld in X, then scl(M) – M contains no nonempty Q-cld in X.

Proof

Suppose that M is $\mathcal{g}^{\wedge}$-cld. Let K be a Q-cld subset of scl(M) – M. Then M $\subseteq K^c$. Since M is $\mathcal{g}^{\wedge}$-cld, scl(M) $\subseteq K^c$. Consequently, K $\subseteq$ $(scl(M))^c$. We already have K $\subseteq$ scl(M). Thus K $\subseteq$ scl(M) $\cap$ $(scl(M))^c$ and K is empty.

The converse of Proposition 3.4 need not be true as seen from the following example.

Example 3.5

Let $X = \{a_1, b_1, c_1, d_1\}$ with $\tau = \{\phi, \{a_1, d_1\}, X\}$. $g^\wedge C(X) = \{\phi, \{b_1, c_1\}, \{a_1, b_1, c_1\}, \{b_1, c_1, d_1\}, X\}$ and $BC(X) = \{\phi, \{a_1\}, \{d_1\}, \{a_1, b_1\}, \{a_1, c_1\}, \{a_1, d_1\}, \{b_1, c_1\}, \{a_1, b_1, c_1\}, \{a_1, b_1, d_1\}, \{a_1, c_1, d_1\}, X\}$. If $M = \{b_1\}$ then $scl(M) - M = \{c_1\} \not\subset$ Q-cld. But M is not $g^\wedge$-cld.

Theorem 3.6

If a set M is $g^\wedge$-cld, then $scl(M) - M$ contains no nonempty closed set.

Proof

Suppose that M is $g^\wedge$-cld. Let K be a closed subset of $scl(M) - M$. Then $M \subseteq K^c$. Since M is $g^\wedge$-cld, we have $scl(M) \subseteq K^c$. Consequently, $K \subseteq (scl(M))^c$. Hence, $K \subseteq scl(M) \cap (scl(M))^c = \phi$. Therefore K is empty.

Proposition 3.7

If M is $g^\wedge$-cld in X and $M \subseteq N \subseteq scl(M)$, then N is $g^\wedge$-cld in X.

Proof

Let $N \subseteq H$ where H is Q-open in X. Since $M \subseteq N$, $M \subseteq H$. Since M is $g^\wedge$-cld, $scl(M) \subseteq H$. Since $N \subseteq scl(M)$, $scl(N) \subseteq scl(M) \subseteq H$. Therefore N is $g^\wedge$-cld in X.

Proposition 3.8

Let $M \subseteq Y \subseteq X$ and suppose that M is $g^\wedge$-cld in X. Then M is $g^\wedge$-cld relative to Y.

Proof

Let $M \subseteq Y \cap K$, where K is Q-open in X. Then $M \subseteq K$ and hence $scl(M) \subseteq K$. This implies that $Y \cap scl(M) \subseteq Y \cap K$. Thus M is $g^\wedge$-cld relative to Y.

Proposition 3.9

If M is an Q-open and $g^\wedge$-cld in X, then M is semi-closed in X.

Proof

Since M is Q-open and $g^\wedge$-cld, $scl(M) \subseteq M$ and hence M is semi-closed in X.

Theorem 3.10

Let M be a locally closed. Then M is semi-closed if and only if M is $g^\wedge$-cld.

Proof

It is fact that every semi-closed is $g^{\wedge}$-cld.

Conversely, we have M ∪ (X – scl(M)) is open in X, since M is locally closed. Now M ∪ (X – scl(M)) is Q-open set of X such that M ⊆ M ∪ (X – scl(M)). Since M is $g^{\wedge}$-cld, then scl(M) ⊆ M ∪ (X – scl(M)). Thus, we have scl(M) ⊆ M and hence M is a semi-closed.

Proposition 3.11

For each x ∈ X, either {x} is Q-cld or $\{x\}^c$ is $g^{\wedge}$-cld in X.

Proof

Suppose that {x} is not Q-closed in X. Then $\{x\}^c$ is not Q-open and the only Q-open set containing $\{x\}^c$ is the space X itself. Therefore scl($\{x\}^c$) ⊆ X and so $\{x\}^c$ is $g^{\wedge}$-cld in X.

Lemma 3.12

Let K be a closed set of X. Then the following properties hold:

If M is Q-cld in X, then M ∩ K is Q-cld in X.

Corollary 3.13

If M is $g^{\wedge}$-cld and K is a closed, then M ∩ K is $g^{\wedge}$-cld.

Proof

Let H be a Q-open of X such that M ∩ K ⊆ H. It shows that M ⊆ H ∪ (X/K) and H ∪ (X/K) is Q-open in X. Since M is $g^{\wedge}$-cld in X, we have scl(M) ⊆ H ∪ (X\K) and so scl(M ∩ K) ⊆ scl(M) ∩ scl(K) = scl(M) ∩ K ⊆ (H ∪ (X\K)) ∩ K = H ∩ K ⊆ H. Therefore M ∩ K is $g^{\wedge}$-cld in X.

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