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SOME PROPERTIES OF (1,2)*-ρ-CLOSED SETS IN BITOPOLOGICAL SPACES

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Abstract: In this paper, we introduce a new class of sets namely (1,2)*-ρ-closed sets in bitopological spaces. This class lies between the class of $\tau_{1,2}$-closed sets and the class of (1,2)*-g-closed sets

Keywords: (1,2)*-ρ-closed set, (1,2)*-ρ-open set.

1. INTRODUCTION

Levine [5] also introduced the notion of g-closed sets and investigated its fundamental properties. This notion was shown to be productive and very useful. Kelly [4] introduced the concepts of bitopological spaces.Recently, Bhattacharya and Lahiri [2], Andrijevic, D [1], introduced the Semi-preopen sets and Duszynski, Z et. all [3], presented the new generalization of closed sets in bitopological spaces.

In this paper, we introduce a new class of sets namely (1,2)*-ρ-closed sets in bitopological spaces. This class lies between the class of $\tau_{1,2}$-closed sets and the class of (1,2)*-g-closed sets.

2. PRELIMINARIES

Throughout this paper, (X, τ_1, τ_2) (briefly, X) will denote bitopological space (briefly, BTPS).

Definition 2.1

Let H be a subset of X. Then H is said to be $\tau_{1,2}$-open [6] if $H = P \cup Q$ where $P \in \tau_1$ and $Q \in \tau_2$.

The complement of $\tau_{1,2}$-open set is called $\tau_{1,2}$-closed.

Notice that $\tau_{1,2}$-open sets need not necessarily form a topology.

Definition 2.2 [6]

Let H be a subset of a bitopological space X. Then

(i) the $\tau_{1,2}$-closure of H, denoted by $\tau_{1,2}$-cl(H), is defined as $\cap$ {F : H $\subseteq$ F and F is $\tau_{1,2}$-closed}.

(ii) the $\tau_{1,2}$-interior of H, denoted by $\tau_{1,2}$-int(H), is defined as $\cup$ {F : F $\subseteq$ H and F is $\tau_{1,2}$-open}.

Definition 2.3

A subset H of a BTPS X is called:

(i) (1,2)*-semi-open [8] if H $\subseteq$ $\tau_{1,2}$-cl($\tau_{1,2}$-int(H));

(ii) (1,2)*-preopen [12] if H $\subseteq$ $\tau_{1,2}$-int($\tau_{1,2}$-cl(H));

(iii) (1,2)*-α-open [7] if H $\subseteq$ $\tau_{1,2}$-int($\tau_{1,2}$-cl($\tau_{1,2}$-int(H)));

(iv) regular (1,2)*-open [9] if H = $\tau_{1,2}$-int($\tau_{1,2}$-cl(H)).

The complements of the abovementioned open sets are called their respective closed sets.

Definition 2.4

A subset H of a BTPS X is called

(i) (1,2)*-generalized closed (briefly,(1,2)*-g-cld) [12] if $\tau_{1,2}$-cl(H) $\subseteq$ U whenever H $\subseteq$ U and U is $\tau_{1,2}$-open in X.

(ii) (1,2)*-semi-generalized closed (briefly,(1,2)*-sg-cld) [8] if (1,2)*-scl(H) $\subseteq$ U whenever H $\subseteq$ U and U is (1,2)*-semi-open in X.

(iii) (1,2)*-generalized semi-closed (briefly,(1,2)*-gs-cld) [10] if (1,2)*-scl(H) $\subseteq$ U whenever H $\subseteq$ U and U is $\tau_{1,2}$-open in X.

(iv) (1,2)*-α-generalized closed (briefly,(1,2)*-αg-cld) set [7] if (1,2)*-αcl(H) $\subseteq$ U whenever H $\subseteq$ U and U is $\tau_{1,2}$-open in X.

(v) (1,2)*-generalized semi-preclosed (briefly,(1,2)*-gsp-cld) set [11] if (1,2)*-spcl(H) $\subseteq$ U whenever H $\subseteq$ U and U is $\tau_{1,2}$-open in X.

(vi) (1,2)*-ĝ-closed set ((1,2)*-ω-cld)) [11] if $\tau_{1,2}$-cl(H) $\subseteq$ U whenever H $\subseteq$ U and U is (1,2)*-semi-open in X.

(vii) (1,2)*-αgs-closed (briefly, (1,2)*- αgs-cld) set [7] if (1,2)*-αcl(H) $\subseteq$ U whenever H $\subseteq$ U and U is (1,2)*-semi-open in X.

(viii) (1,2)*-g*s-closed (briefly, (1,2)*-g*s-cld) set [6] if (1,2)*-scl(H) $\subseteq$ U whenever H $\subseteq$ U and U is (1,2)*-gs-open in X.

The complements of the abovementioned closed sets are called their respective open sets.

Definition 2.5 [11]

Subset H of a BTPS X is said to be (1,2)*-locally closed if H = U $\cap$ F, where U is $\tau_{1,2}$-open and F is $\tau_{1,2}$-closed in X.

3. (1,2)*-ρ-CLOSED SETS IN BITOPOLOGICAL SPACES

We introduce the following definition.

Definition 3.1

A subset H of a BTPS X is called a (1,2)*-ρ-closed (briefly, (1,2)*-ρ-cld) set if $\tau_{1,2}$-scl(H) $\subseteq$ U whenever H$\subseteq$ U and U is(1,2)*-gs-open in X.

Proposition 3.2

Every $\tau_{1,2}$-closed set is (1,2)*-ρ-cld.

Proof:

If His any $\tau_{1,2}$-closed set in X and G is any (1,2)*-gs-open set containing H, then G $\supseteq$ H = $\tau_{1,2}$-scl(H). Hence H is (1,2)*-ρ-cld.

The converse of Proposition 3.2 need not be true as seen from the following example.

Example 3.3

Let X = {x, y, z}, τ_1 = {ϕ, X, {x, y}} and τ_2 = {ϕ, X}. Then the sets in {ϕ, {x, y}, X} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {z}} are called $\tau_{1,2}$-closed.Here, H = {x, z} is(1,2)*-ρ-cld set but not $\tau_{1,2}$-closed.

Proposition 3.4

Every (1,2)*-ρ-cld set is (1,2)*-g*s-cld.

Proof:

If H is a (1,2)*-ρ-cld subset of X and G is any (1,2)*-gs-open set containing H, then G $\supseteq$ $\tau_{1,2}$-scl(H). Hence H is (1,2)*-g*s-cld in X.

The converse of Proposition 3.4 need not be true as seen from the following example.

Example 3.5

In Example 3.3,Here, H = {z} is(1,2)*-g*s-cld but not (1,2)*-ρ-cld set in X.

Proposition 3.6

Every (1,2)*-ρ-cld set is (1,2)*-ω-cld.

Proof:

Suppose that H $\subseteq$ G and G is (1,2)*-semi-open in X. Since every (1,2)*-semi-open set is (1,2)*-gs-open and H is (1,2)*-ρ-cld, therefore $\tau_{1,2}$-scl(H) $\subseteq$ G. Hence H is (1,2)*-ω-cld in X.

The converse of Proposition 3.6 need not be true as seen from the following example.

Example 3.7

Let X = {x, y, z}, τ_1 = {ϕ, X, {x}} and τ_2 = {ϕ, X, {y, z}}. Then the sets in {ϕ, {x}, {y, z}, X} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {x}, {y, z}} are called $\tau_{1,2}$-closed.Here, H = {x, z} is (1,2)*-ω-cld but not (1,2)*-ρ-cld set in X.

Proposition 3.8

Every (1,2)*-ρ-cld set is(1,2)*-g-cld.

Proof:

If H is a (1,2)*-ρ-cld subset of X and G is any $\tau_{1,2}$-open set containing H, since every $\tau_{1,2}$-open set is (1,2)*-gs-open, we have G $\supseteq$ $\tau_{1,2}$-scl(H). Hence H is (1,2)*-g-cld in X.

The converse of Proposition 3.8 need not be true as seen from the following example.

Example 3.9

Let X = {x, y, z}, τ_1 = {ϕ, X, {x}} and τ_2 = {ϕ, X, {y, z}}. Then the sets in {ϕ, {x}, {y, z}, X} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {x}, {y, z}} are called $\tau_{1,2}$-closed.Here, H = {x, y} is (1,2)*-g-cld but not (1,2)*-ρ-cld set in X.

Proposition 3.10

Every (1,2)*-ρ-cld set is (1,2)*-αgs-cld.

Proof:

If H is a (1,2)*-ρ-cld subset of X and G is any (1,2)*-semi-open set containing H, since every (1,2)*-semi-open set is (1,2)*-gs-open, we have G ⊇ $\tau_{1,2}$-scl(H) ⊇ (1,2)*-αcl(H). Hence H is (1,2)*-αgs-cld in X.

The converse of Proposition 3.10 need not be true as seen from the following example.

Example 3.11

Let X = {x, y, z}, τ_1 = {ϕ, X, {x}} and τ_2 = {ϕ, X, {y, z}}. Then the sets in {ϕ, {x}, {y, z}, X} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {x}, {y, z}} are called $\tau_{1,2}$-closed.Here,H = {x, z} is (1,2)*-αgs-cld but not (1,2)*-ρ-cld set in X.

Proposition 3.12

Every (1,2)*-ρ-cld set is (1,2)*-αg-cld.

Proof:

If H is a (1,2)*-ρ-cld subset of X and G is any $\tau_{1,2}$-open set containing H, since every $\tau_{1,2}$-open set is (1,2)*-gs-open, we have G ⊇ $\tau_{1,2}$-scl(H) ⊇ (1,2)*-αcl(H). Hence H is (1,2)*-αg-cld in X.

The converse of Proposition 3.12 need not be true as seen from the following example.

Example 3.13

Let X = {x, y, z}, τ_1 = {ϕ, X, {z}} and τ_2 = {ϕ, X, {x, y}}. Then the sets in {ϕ, {z}, {x, y}, X} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {z}, {x, y}} are called $\tau_{1,2}$-closed.Here, H = {x, z} is (1,2)*-αg-cld but not (1,2)*-ρ-cld set in X.

Proposition 3.14

Every (1,2)*-ρ-cld set is (1,2)*-gs-cld.

Proof:

If H is a (1,2)*-ρ-cld subset of X and G is any $\tau_{1,2}$-open set containing H, since every $\tau_{1,2}$-open set is (1,2)*-gs-open,we have G ⊇ $\tau_{1,2}$-scl(H). Hence H is (1,2)*-gs-cld in X.

The converse of Proposition 3.14 need not be true as seen from the following example.

Example 3.15

Let X = {x, y, z}, τ_1 = {ϕ, X, {x}} and τ_2 = {ϕ, X}. Then the sets in {ϕ, {x}, X} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {y, z}} are called $\tau_{1,2}$-closed.Here, H = {z} is (1,2)*-gs-cld but not (1,2)*-ρ-cld set in X.

Proposition 3.16

Every (1,2)*-ρ-cld set is (1,2)*-gsp-cld.

Proof:

If H is a (1,2)*-ρ-cld subset of X and G is any $\tau_{1,2}$-open set containing H, every $\tau_{1,2}$-open set is (1,2)*-gs-open, we have G ⊇ $\tau_{1,2}$-scl(H) ⊇ (1,2)*-spcl(H). Hence H is (1,2)*-gsp-cld in X.

The converse of Proposition 3.16 need not be true as seen from the following example.

Example 3.17

In Example 3.15, Here H = {z} is (1,2)*-gsp-cld but not (1,2)*-ρ-cld set in X.

Remark 3.18

The following example shows that (1,2)*-ρ-cld sets are independent of (1,2)*-α-cld sets and (1,2)*-semi-cld sets.

Example 3.19

Let X = {x, y, z}, τ_1 = {ϕ, X, {x, y}} and τ_2 = {ϕ, X}. Then the sets in {ϕ, {x, y},X} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {z}} are called $\tau_{1,2}$-closed.Here, H = {x, z} is (1,2)*-ρ-cld but it is neither (1,2)*-α-cld nor (1,2)*-semi-cld in X.

Example 3.20

Let X = {x, y, z}, τ_1 = {ϕ, X, {x}} and τ_2 = {ϕ, X}. Then the sets in {ϕ, {x}, X} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {y, z}} are called $\tau_{1,2}$-closed.Here, H = {y} is (1,2)*-α-cld as well as(1,2)*-semi-cld in X but it is not(1,2)*-ρ-cld in X.

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