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Equitable Total Coloring of Spider Graph

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Abstract

An equitable total coloring (χ_{et})of a graph was introduced by Fu[4] in 1994. He gave the Conjecture that For any simple graph G satisfies condition $\chi_{et} \leq \Delta(G)+2$. The graph G (V,E) is called equitably Total k – Colorable if the vertex set and edge set of the graph can be partitioned into k non empty independent sets $T_1, T_2, T_3, \ldots, T_k$ such that $\|T_i - T_j\| \leq 1$ for every i and j. If the connected graph G is neither a complete graph nor an odd cycle then it satisfies the Equitable Coloring Conjecture. In this paper We examine and establish equitable total coloring of Spider Graph.

Keywords: Equitable Total Coloring, Spider Graph.

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1.Introduction:

A graph $G = (V(G), E(G))$ consists of a vertex set $V(G)$ and an edge set $E(G)$, where all graphs considered are finite simple Graph. A proper k-coloring of a graph G is a function $f: V(G) \to \{1,2, \ldots, k\}$, such that adjacent vertices and edges are assigned distinct colors. An equitable k-coloring is a proper coloring where the sizes of any two color classes differ by at most one. Girija and Veninstine Vivik [3] are determine the equitable total chromatic number χ_{etc} for the double star graph $K_{1,n,n}$ and the fan graph $F_{m,n}$. [1] Bor-Liang Chena and Chih-Hung Yenb gives necessary conditions for a graph Δ(G) ≥ χ(G) to be equitably Δ(G) – colorable. Geetha and Somasundaram [6] are determine the power graphs of the path and cycle, Complement of graphs are Total coloring. In 2013 Kaliraj,Vernold Vivin and Akbar Ali [7] are resolved the equitable color for the Mycielskian of wheel graph μ(Wn) and Mycielskian of complete bipartite graph μ(Km,n).

2.Preliminaries

Definition 2.1: [9] A spider graph SP (1^n2^m)is a graph formed from a star $K_{1,n+m}$ and each of its m vertices having degree 1 is joined to a new vertex.

Definition 2.2 : A spider graph is a tree with one vertex atmost n-degree and remaining all the vertices of degree two. A notation of the spider graph $S_{n,m}$ where n and m are any positive integers. The vertex set of the spider graph V($S_{m,n}$)= $\{v_0 \cup v_{i,j}; i = 1,2,\ldots,n , j = 1,2,\ldots,m\}$ and the Edge set E($S_{m,n}$)=$\{v_0\, v_{i,1} \cup v_{i,j}v_{i,j+1}; i = 1,2,\ldots,n , j = 1,2,\ldots,m\}$.

Here we given generalized spider graph figure:1

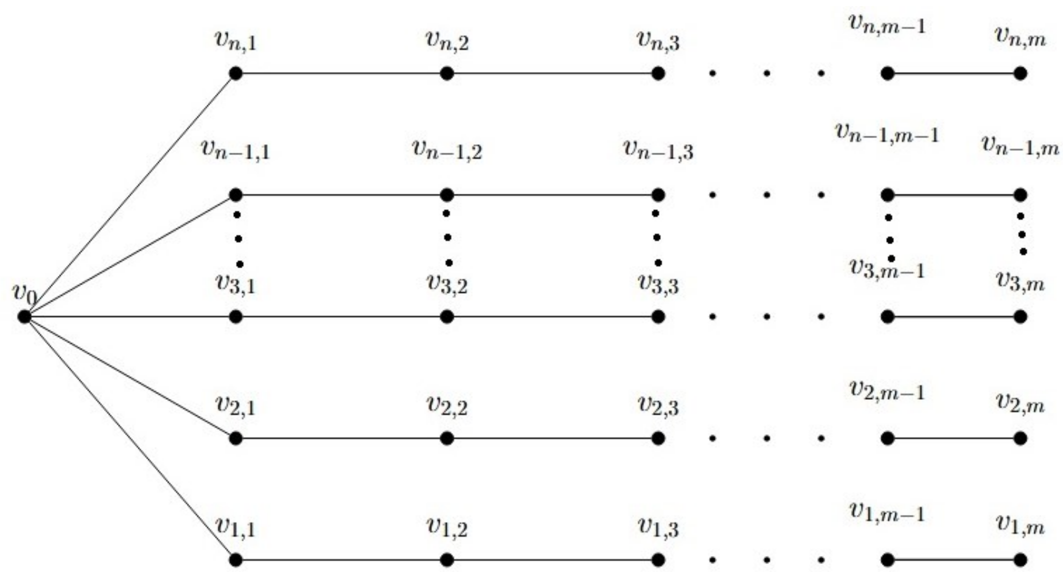

Figure:1 Generalized spider graph $S_{n,m}$.

Conjecture 2.3:[8] For any simple graph $G(V,E)\chi''_{et}$(G)≤ χ''_{et}(G)≤ Δ(G) + 1.

Conjecture 2.4: [5] For every graph G, G has an equitable total k- coloring for each

k ≥ max{ χ''_{et}(G), Δ(G) + 2}.

3.Main Theorem

Here we are proving Equitable total coloring of χ_{et} ($S_{n,m}$) ≤ Δ+1.

Theorem:3.1 A spider graph $S_{n,m}$ are equitable total coloring n, m ≥ 5.

Proof: Let $S_{n,m}$ be a spider graph have a vertex set $V(S_{n,m}) = \{v_0 \cup v_{i,j}; i = 1,2,\ldots,n, j = 1,2,\ldots,m\}$ and the Edge set $E(S_{n,m}) = \{v_0\, v_{i,1} \cup v_{i,j}v_{i,j+1}; i = 1,2,\ldots,n, j = 1,2,\ldots,m\}$.

Each vertex and edges are a pairing of elements at the same index only used in the sequences of 'i' and 'j'. Which is followed all the sequences in this paper.

Case:1 n = m

Sub-Case: 1 n is Odd

$$T_1 = \left\{\begin{matrix} (v_{1,1}), (v_{0,}v_{n-1,1}) \\ v_{i,j}; i = 1,3,5,\ldots,2\,; j = \left\lceil\frac{n+3}{2}\right\rceil, \left\lceil\frac{n+1}{2}\right\rceil, \left\lceil\frac{n-1}{2}\right\rceil, \ldots, 2 \\ v_{i,j}v_{i,j+1}; i = 2,4,6,\ldots,2\,; j = \left\lceil\frac{n+1}{2}\right\rceil, \left\lceil\frac{n-1}{2}\right\rceil, \left\lceil\frac{n-3}{2}\right\rceil, \ldots, 2 \\ v_{i,j}; i = 5,7,9,\ldots,n; j = m, m-1, m-2, \ldots, \left\lceil\frac{n+5}{2}\right\rceil \\ v_{i,j}v_{i,j+1}; i = 6,8,10,\ldots,n-1; j = m-1, m-2, m-3, \ldots, \left\lceil\frac{n+5}{2}\right\rceil \end{matrix}\right\}$$

$$T_2 = \left\{\begin{matrix} (v_{2,1}), (v_{0,}v_{n,1,}), (v_{1,1})(v_{1,2}) \\ v_{i,j}; i = 1,3,5,\ldots,2\,; j = \left\lceil\frac{n+3}{2}\right\rceil, \left\lceil\frac{n+1}{2}\right\rceil, \left\lceil\frac{n-1}{2}\right\rceil, \ldots, 2 \\ v_{i,j}v_{i,j+1}; i = 2,4,6,\ldots,2\,; j = \left\lceil\frac{n+3}{2}\right\rceil, \left\lceil\frac{n+1}{2}\right\rceil, \left\lceil\frac{n-1}{2}\right\rceil, \ldots, 2 \\ v_{i,j}; i = 6,8,10,\ldots,n-1; j = m, m-1, m-2, \ldots, \left\lceil\frac{n+5}{2}\right\rceil \\ v_{i,j}v_{i,j+1}; i = 7,9,11,\ldots,n; j = m-1, m-2, m-3, \ldots, \left\lceil\frac{n+5}{2}\right\rceil \end{matrix}\right\}$$

$$T_3 = \left\{\begin{matrix} (v_{3,1}), (v_{0,}v_{1,1}), (v_{2,1})(v_{2,2}), (v_{1,2}) \\ v_{i,j}; i = 1,3,5,\ldots,n\,; j = \left\lceil\frac{n+5}{2}\right\rceil, \left\lceil\frac{n+3}{2}\right\rceil, \left\lceil\frac{n+1}{2}\right\rceil, , \ldots, 3 \\ v_{i,j}v_{i,j+1}; i = 2,4,6,\ldots,n-1\,; j = \left\lceil\frac{n+3}{2}\right\rceil, \left\lceil\frac{n+1}{2}\right\rceil, \left\lceil\frac{n-1}{2}\right\rceil, \ldots, 3 \\ v_{i,j}; i = 7,9,11,\ldots,n; j = m, m-1, m-2, \ldots, \left\lceil\frac{n+7}{2}\right\rceil \\ v_{i,j}v_{i,j+1}; i = 8,10,12,\ldots,n-1; j = m-1, m-2, m-3 \ldots, \left\lceil\frac{n+7}{2}\right\rceil \end{matrix}\right\}$$

…

$$T_{n-1} = \left\{ \begin{matrix} (v_0, v_{n-3,1,}) \\ v_{i,j}; i = 2,4,6, \dots, n-1 \,; j = \left[\frac{n-1}{2}\right], \left[\frac{n-3}{2}\right], \dots, 1 \\ v_{i,j}v_{i,j+1}; i = 1,3,5, \dots, n \,; j = \left[\frac{n-1}{2}\right], \left[\frac{n-3}{2}\right], \dots, 1 \\ v_{i,j}; i = 3,5,7, \dots, n-1; j = m, m-1, \dots, \left[\frac{n+3}{2}\right] \\ v_{i,j}v_{i,j+1}; i = 4,6,8, \dots, n-2; j = m-1, m-2, m-3, \dots, \left[\frac{n+3}{2}\right] \end{matrix} \right\}$$

$$T_{n} = \left\{ \begin{matrix} (v_0, v_{n-2,1}) \\ v_{i,j}; i = 1,3,5, \dots n \,; j = \left[\frac{n+1}{2}\right], \left[\frac{n-1}{2}\right], \dots, 1 \\ v_{i,j}v_{i,j+1}; i = 2,4,6, \dots, n-1 \,; j = \left[\frac{n-1}{2}\right], \left[\frac{n-3}{2}\right], \dots, 1 \\ v_{i,j}; i = 3,5,7, \dots, n; j = m, m-1, m-2, \dots, \left[\frac{n+3}{2}\right] \\ v_{i,j}v_{i,j+1}; i = 4,6,8, \dots, n-1; j = m-1, m-2, m-3, \dots, \left[\frac{n+3}{2}\right] \end{matrix} \right\}$$

$$T_{n+1} = \left\{ \begin{matrix} (v_0) \\ v_{i,j}; i = 2,4,6 \dots, n-1 \,; j = \left[\frac{n+1}{2}\right], \left[\frac{n-1}{2}\right], \left[\frac{n-3}{2}\right], \dots, 2 \\ v_{i,j}v_{i,j+1}; i = 1,3,5, \dots, n \,; j = \left[\frac{n+1}{2}\right], \left[\frac{n-1}{2}\right], \left[\frac{n-1}{2}\right], \dots, 1 \\ v_{i,j}; i = 4,6,8, \dots, n-1; j = m, m-1, m-2, \dots, \left[\frac{n+5}{2}\right] \\ v_{i,j}v_{i,j+1}; i = 5,7,9, \dots, n; j = m-1, m-2, m-3, \dots, \left[\frac{n+3}{2}\right] \end{matrix} \right\}$$

It is clearly that the color classes T_1, T_2, T_3, T_4 ,…,$T_{\Delta+1}$are independent sets of χ_{et} ($S_{n,m}$) T_1=T_2 =$2\Delta - 2$, $T_3 = T_4 = \cdots = T_\Delta = 2\Delta - 1$ are evidents of satisfies the inequality $||T_i - T_j|| \le 1$

for every i & j this implies χ_{et} ($S_{n,m}$) ≤ Δ+1.

Sub-Case:2 n is even

$$T_1 = \left\{ \begin{array}{c} (v_{1,1}), (v_{0\,,}v_{n-1,1}) \\ v_{i,j};\, i = 2,4,6, \dots, n\, ;\, j = \left[\frac{n+2}{2}\right], \left[\frac{n}{2}\right], \left[\frac{n-2}{2}\right], \dots, 2 \\ v_{i,j}v_{i,j+1};\, i = 1,3,5, \dots, n-1\, ;\, j = \left[\frac{n+3}{2}\right], \left[\frac{n+1}{2}\right], \left[\frac{n-1}{2}\right], \dots, 2 \\ v_{i,j};\, i = 5,7,9,11 \dots, n-1;\, j = m, m-1, m-2, \dots, \left[\frac{n+6}{2}\right] \\ v_{i,j}v_{i,j+1};\, i = 6,8,10,12, \dots, n;\, j = m-1, m-2, m-3, \dots, \left[\frac{n+4}{2}\right] \end{array} \right\}$$

$$T_2 = \left\{ \begin{array}{c} (v_{0,}v_{n,1,}), (v_{1,1})(v_{1,2}), (v_{2,1}) \\ v_{i,j};\, i = 1,3,5, \dots, n-1\, ;\, j = \left[\frac{n+4}{2}\right], \left[\frac{n+2}{2}\right], \left[\frac{n}{2}\right], \dots, 3 \\ v_{i,j}v_{i,j+1};\, i = 2,4,6, \dots, n\, ;\, j = \left[\frac{n+2}{2}\right], \left[\frac{n}{2}\right], \left[\frac{n-2}{2}\right], \dots, 2 \\ v_{i,j};\, i = 6,8,10, \dots, n;\, j = m, m-1, m-2, \dots, \left[\frac{n+6}{2}\right] \\ v_{i,j}v_{i,j+1};\, i = 7,9,11, \dots, n-1;\, j = m-1, m-2, m-3, \dots, \left[\frac{n+6}{2}\right] \end{array} \right\}$$

$$T_3 = \left\{ \begin{array}{c} (v_{3,1}), (v_{0,}v_{1,1,}), (v_{2,1})(v_{2,2}), (v_{1,2}) \\ v_{i,j};\, i = 2,4,6, \dots, n\, ;\, j = \left[\frac{n+4}{2}\right], \left[\frac{n+2}{2}\right], \left[\frac{n}{2}\right], \left[\frac{n-1}{2}\right], \dots, 3 \\ v_{i,j}v_{i,j+1};\, i = 1,3,5, \dots, n-1\, ;\, j = \left[\frac{n+4}{2}\right], \left[\frac{n+2}{2}\right], \left[\frac{n}{2}\right], \dots, 3 \\ v_{i,j};\, i = 7,9,11, \dots, n-1;\, j = m, m-1, m-2, \dots, \left[\frac{n+8}{2}\right] \\ v_{i,j}v_{i,j+1};\, i = 8,10,12, \dots, n;\, j = m-1, m-2, m-3, \dots, \left[\frac{n+6}{2}\right] \end{array} \right\}$$

$$T_4 = \left\{ \begin{array}{c} (v_{4,1}), (v_{0,}v_{2,1,}), (v_{3,1})(v_{3,2}), (v_{2,2}), (v_{1,2})(v_{1,3}) \\ v_{i,j};\, i = 1,3,5, \dots, n-1\, ;\, j = \left[\frac{n+6}{2}\right], \left[\frac{n+4}{2}\right], \left[\frac{n+2}{2}\right], \dots, 4 \\ v_{i,j}v_{i,j+1};\, i = 2,4,6, \dots, n\, ;\, j = \left[\frac{n+4}{2}\right], \left[\frac{n+2}{2}\right], \left[\frac{n}{2}\right], \dots, 3 \\ v_{i,j};\, i = 8,10,12, \dots, n;\, j = m, m-1, m-2, \dots, \left[\frac{n+8}{2}\right] \\ v_{i,j}v_{i,j+1};\, i = 9,11,13, \dots, n-1;\, j = m-1, m-2, m-3, \dots, \left[\frac{n+8}{2}\right] \end{array} \right\}$$

…

$$T_{n-1}=\left\{\begin{matrix}(v_{0,}v_{n-3,1})\\ v_{i,j};i=1,3,5,\dots,n-1\,;j=\left[\frac{n}{2}\right],\left[\frac{n-2}{2}\right],\left[\frac{n-4}{2}\right],\dots,1\\ v_{i,j}v_{i,j+1};i=2,4,6,\dots,n-2\,;j=\left[\frac{n-2}{2}\right],\left[\frac{n-4}{2}\right],\left[\frac{n-6}{2}\right],\dots,1\\ v_{i,j};i=2,4,6,\dots,n;j=m,m-1,m-2,\dots,\left[\frac{n+2}{2}\right]\\ v_{i,j}v_{i,j+1};i=1,3,5,\dots,n-1;j=m-1,m-2,m-3,\dots,\left[\frac{n+2}{2}\right]\end{matrix}\right\}$$

$$T_{n}=\left\{\begin{matrix}(v_{0,}v_{n-2,1})\\ v_{i,j};i=2,4,6,\dots,n\,;j=\left[\frac{n}{2}\right],\left[\frac{n-2}{2}\right],\left[\frac{n-4}{2}\right],\dots,1\\ v_{i,j}v_{i,j+1};i=1,3,5,\dots,n-1\,;j=\left[\frac{n}{2}\right],\left[\frac{n-2}{2}\right],\left[\frac{n-4}{2}\right],\dots,1\\ v_{i,j};i=3,5,7,\dots,n-1;j=m,m-1,m-2,\dots,\left[\frac{n+4}{2}\right]\\ v_{i,j}v_{i,j+1};i=4,6,8,\dots,n;j=m-1,m-2,m-3,\dots,\left[\frac{n+2}{2}\right]\end{matrix}\right\}$$

$$T_{n+1}=\left\{\begin{matrix}(v_{0})\\ v_{i,j};i=1,3,5,\dots,n-1\,;j=\left[\frac{n+2}{2}\right],\left[\frac{n}{2}\right],\left[\frac{n-2}{2}\right],\dots,2\\ v_{i,j}v_{i,j+1};i=2,4,6,\dots,n\,;j=1,2,3,\dots,\left[\frac{n}{2}\right],\left[\frac{n-2}{2}\right],\left[\frac{n-4}{2}\right],\dots,1\\ v_{i,j};i=4,6,8,..,n;j=m,m-1,m-2,\dots,\left[\frac{n+4}{2}\right]\\ v_{i,j}v_{i,j+1};i=5,7,9,\dots n-1;j=m-1,m-2,m-3,\dots,\left[\frac{n+4}{2}\right]\end{matrix}\right\}$$

It is clearly that the color classes T_1, T_2, T_3, T_4 ,…,$T_{\Delta+1}$are independent sets of χ_{et} ($S_{n,m}$) T_1=T_2= T_3=$T_{\Delta+1}$=$2\Delta-1$, T_4 = T_5 =… = $T_{\Delta}=2\Delta-2$ are evidents of satisfies the inequality || T_i−T_j || ≤ 1 for i & j this implies χ_{et} ($S_{n,m}$) ≤ Δ+1.

Case:2 Both n and m are even

Sub-Case:1 n < m

$$T_{1}=\left\{\begin{matrix}(v_{1,1}),(v_{0,}v_{n-1,1})\\ v_{i,j};i=2,4,6,\dots,n\,;j=\left[\frac{n+2}{2}\right],\left[\frac{n}{2}\right],\left[\frac{n-1}{2}\right],\dots,2\\ v_{i,j}v_{i,j+1};i=1,3,5,\dots,n-1\,;j=\left[\frac{n+3}{2}\right],\left[\frac{n}{2}\right],\left[\frac{n-1}{2}\right],\dots,2\\ v_{i,j};i=1,3,5,\dots,n-1;j=m,m-1,m-2,\dots,\left[\frac{n+6}{2}\right]\\ v_{i,j}v_{i,j+1};i=2,4,6,\dots,n;j=m-1,m-2,m-3,\dots,\left[\frac{n+4}{2}\right]\end{matrix}\right\}$$

$$T_2 = \begin{cases} (v_{0,}v_{n,1,}), (v_{1,1})(v_{1,2}), (v_{2,1}) \\ v_{i,j}; i = 1,3,5, \dots, n-1\ ; j = \left[\frac{n+4}{2}\right], \left[\frac{n+2}{2}\right], \left[\frac{n}{2}\right], \dots, 2 \\ v_{i,j}v_{i,j+1}; i = 2,4,6, \dots, n\ ; j = \left[\frac{n+2}{2}\right], \left[\frac{n}{2}\right], \left[\frac{n-2}{2}\right], \dots, 2 \\ v_{i,j}; i = 2,4,6, \dots, n; j = m, m-1, m-2, \dots, \left[\frac{n+6}{2}\right] \\ v_{i,j}v_{i,j+1}; i = 1,3,5, \dots, n-1; j = m-1, m-2, m-3, \dots, \left[\frac{n+6}{2}\right] \end{cases}$$

$$T_3 = \begin{cases} (v_{3,1}), (v_{0,}v_{1,1,}), (v_{2,1})(v_{2,2}), (v_{1,2}) \\ v_{i,j}; i = 2,4,6, \dots, n\ ; j = \left[\frac{n+4}{2}\right], \left[\frac{n+2}{2}\right], \left[\frac{n}{2}\right], \left[\frac{n-1}{2}\right], \dots, 3 \\ v_{i,j}v_{i,j+1}; i = 1,3,5, \dots, n-1\ ; j = \left[\frac{n+4}{2}\right], \left[\frac{n+2}{2}\right], \left[\frac{n}{2}\right], \dots, 3 \\ v_{i,j}; i = 3,5,7, \dots, n-1; j = m, m-1, m-2, \dots, \left[\frac{n+8}{2}\right] \\ v_{i,j}v_{i,j+1}; i = 4,6,8, \dots, n; j = m-1, m-2, m-3, \dots, \left[\frac{n+6}{2}\right] \end{cases}$$

$$T_4 = \begin{cases} (v_{4,1}), (v_{0,}v_{2,1,}), (v_{3,1})(v_{3,2}), (v_{2,2}), (v_{1,2})(v_{1,3}) \\ v_{i,j}; i = 1,3,5, \dots, n-1\ ; j = \left[\frac{n+6}{2}\right], \left[\frac{n+4}{2}\right], \left[\frac{n+2}{2}\right], \dots, 4 \\ v_{i,j}v_{i,j+1}; i = 2,4,6, \dots, n\ ; j = \left[\frac{n+4}{2}\right], \left[\frac{n+2}{2}\right], \left[\frac{n}{2}\right], \dots, 3 \\ v_{i,j}; i = 4,6,8, \dots, n; j = m, m-1, m-2, \dots, \left[\frac{n+8}{2}\right] \\ v_{i,j}v_{i,j+1}; i = 5,7,9, \dots, n-1; j = m-1, m-2, m-3, \dots, \left[\frac{n+8}{2}\right] \end{cases}$$

…

$$T_{n-1} = \begin{cases} (v_{0,}v_{n-3,1,}); v_{i,j}; i = n-1, j = m; v_{i,j}v_{i,j+1}; i = n; j = m-1 \\ v_{i,j}; i = 1,3,5, \dots, n-1; j = \left[\frac{n}{2}\right], \left[\frac{n-2}{2}\right], \left[\frac{n-4}{2}\right], \dots, 1 \\ v_{i,j}v_{i,j+1}; i = 2,4,6, \dots, n-2; j = \left[\frac{n-2}{2}\right], \left[\frac{n-4}{2}\right], \left[\frac{n-6}{2}\right], \dots, 1 \\ v_{i,j}; i = 2,4,6, \dots, n; j = m-3, m-4, m-5, \dots, \left[\frac{n+2}{2}\right] \\ v_{i,j}v_{i,j+1}; i = 1,3,5, \dots, n-1; j = m-2, m-3, m-4, \dots, \left[\frac{n+2}{2}\right] \end{cases}$$

$$T_n = \begin{cases} (v_{0,}v_{n-2,1}) \\ v_{i,j}; i = 2,4,6,\dots,n\ ; j = \left[\frac{n}{2}\right], \left[\frac{n-2}{2}\right], \left[\frac{n-4}{2}\right],\dots,1 \\ v_{i,j}v_{i,j+1}; i = 1,3,5,\dots,n-1\ ; j = \left[\frac{n}{2}\right], \left[\frac{n-2}{2}\right], \left[\frac{n-4}{2}\right],\dots,1 \\ v_{i,j}; i = n; j = m \\ v_{i,j}; i = 1,3,5,\dots,n-1; j = m-1, m-2, m-3,\dots,\left[\frac{n+4}{2}\right] \\ v_{i,j}v_{i,j+1}; i = 2,4,6,\dots,n; j = m-2, m-3, m-4,\dots,\left[\frac{n+2}{2}\right] \end{cases}$$

$$T_{n+1} = \begin{cases} (v_0) \\ v_{i,j}; i = 1,3,5,\dots,n-1\ ; j = \left[\frac{n+2}{2}\right], \left[\frac{n}{2}\right], \left[\frac{n-2}{2}\right],\dots,2 \\ v_{i,j}v_{i,j+1}; i = 2,4,6,\dots,n; j = 1,2,3,\dots,\left[\frac{n}{2}\right], \left[\frac{n-2}{2}\right], \left[\frac{n-4}{2}\right],\dots,1 \\ v_{i,j}; i = 2,4,6,\dots,n; j = m-1, m-2, m-3,\dots,\left[\frac{n+4}{2}\right] \\ v_{i,j}v_{i,j+1}; i = 1,3,5,\dots,n-1; j = m-1, m-2, m-3,\dots,\left[\frac{n+4}{2}\right] \end{cases}$$

It is clearly that the color classes T_1, T_2, T_3, T_4 ,…,$T_{\Delta+1}$ are independent sets of χ_{et} $(S_{n,m})$ $T_1=T_2= T_3=T_4,\dots,T_{\Delta}=2\Delta+2$, $T_{\Delta+1}=2\Delta+1$ are evidents of satisfies the inequality $|| T_i - T_j || \leq 1$ for i & j this implies χ_{et} $(S_{n,m}) \leq \Delta+1$.

Sub-Case:2 n >m

$$T_1 = \begin{cases} (v_{1,1}), (v_{0,}v_{n-1,1}) \\ v_{i,j}; i = 5,6,7,\dots,n\ ; j = m.m-1, m-2,\dots,3 \\ v_{i,j}v_{i,j+1}; i = 4,3,2,1\ ; j = m-1, m-2, m-3, m-4 \\ v_{i,j}v_{i,j+1}; i = 8,9,10,11\dots,n; j = 1,2,3,\dots,m \\ i \neq 10,12,14,\dots,n; j \neq 3,4,5,\dots,m-1 \end{cases}$$

$$T_2 = \begin{cases} (v_{0,}v_{n,1,}), (v_{1,2}), (v_{2,1}) \\ v_{i,j}; i = 6,7,8,\dots,n\ ; j = m, m-1, m-3,\dots,4 \\ v_{i,j}v_{i,j+1}; i = 5,4,3,2,1\ ; j = m-1, m-2, m-3, m-4, m-5 \\ v_{i,j}v_{i,j+1}; i = 9,10,11,\dots,n; j = 1,2,3,\dots,m-1 \\ i \neq 12,14,16,\dots,n; j \neq 4,5,\dots,m-1 \end{cases}$$

$$T_3 = \begin{cases} (v_{3,1}), (v_{0,}v_{1,1,}), (v_{2,2}), (v_{1,3}) \\ v_{i,j}; i = 7,8,9,\dots,n\ ; j = m, m-1, m-2,\dots,5 \\ v_{i,j}v_{i,j+1}; i = 6,5,4,3,2,1\ ; j = m-1, m-2, m-3, m-4, m-5, m-6 \\ v_{i,j}v_{i,j+1}; i = 10,11,12,\dots,n; j = 1,2,3,\dots,m-1 \\ i \neq 14,16,18,\dots,n; j \neq 5,6,7,\dots,m-1 \end{cases}$$

$$T_4 = \left\{\begin{matrix} (v_{4,1}),(v_{0,}v_{2,1,}),(v_{3,2}),(v_{2,3}),(v_{1,4}) \\ v_{i,j}; i = 8,9,10,\dots n\ ; j = m-1, m-2,\dots,6 \\ v_{i,j}v_{i,j+1}; i = 7,6,5,4,3,2,1,..n\ ; j = m-1, m-2, m-3,\dots, m-6 \\ v_{i,j}v_{i,j+1}; i = 11,12,13,\dots,n; j = 1,2,3,\dots,m-1 \\ i \neq 16,18,20,\dots,n; j \neq 6,7,8,\dots,m-1 \end{matrix}\right\}$$

$$T_5 = \left\{\begin{matrix} (v_{5,1}),(v_{0,}v_{3,1,}),(v_{4,2}),(v_{3,3}),(v_{2,4}) \\ v_{i,j}; i = 9,10,11,\dots,n\ ; j = m, m-1, m-2,\dots,7 \\ v_{i,j}v_{i,j+1}; i = 8,7,6,5,4,3,2,1\ ; j = m-1, m-2, m-3,\dots, m-6 \\ v_{i,j}v_{i,j+1}; i = 12,13,14,\dots,n\ ; j = 1,23,\dots,m-1 \\ i \neq 18,20,22,\dots,n; j \neq 7,8,9,\dots,m-1 \end{matrix}\right\}$$

$$T_6 = \left\{\begin{matrix} (v_{6,1}),(v_{0,}v_{4,1,}),(v_{5,2}),(v_{4,3}),(v_{3,4}),(v_{2,5}),(v_{1,6}) \\ v_{i,j}; i = 10,11,12,\dots,n\ ; j = m, m-1, m-2,\dots,10 \\ v_{i,j}v_{i,j+1}; i = 9,8,7,6,5,4,3,2,1\ ; j = m-1, m-2, m-3,\dots, m-8 \\ v_{i,j}v_{i,j+1}; i = 13,14,15,\dots,n\ ; j = 1,23,\dots,m-1 \\ i \neq 20,22,23,\dots,n; j \neq 8,9,10,\dots,m-1 \end{matrix}\right\}$$

…

$$T_{n-1} = \left\{\begin{matrix} (v_{0,}v_{n-1,1}) \\ v_{i,j}; i = 2,3,4,5,\dots,n-1\ ; j = m, m-1, m-2,\dots,1 \\ v_{i,j}v_{i,j+1}; i = 3,2,1\ ; j = m-1, m-2, m-3 \\ v_{i,j}v_{i,j+1}; i = 5,6,7,\dots,n; j = 1,2,3,\dots,m-1 \\ i \neq 12,14,16,\dots,n; j \neq 4,5,6,\dots,m-1 \end{matrix}\right\}$$

$$T_n = \left\{\begin{matrix} (v_{0,}v_{n-2,1}) \\ v_{i,j}; i = 3,4,5,\dots,n\ ; j = m, m-1, m-2,\dots,1 \\ v_{i,j}v_{i,j+1}; i = 2,1\ ; j = m-1, m-2 \\ v_{i,j}; i = 6,7,8,\dots,n; j = 1,2,3,\dots,m \\ i \neq 10,12,14,\dots,n; j \neq n-2, n-3, n-4,\dots,m \end{matrix}\right\}$$

$$T_{n+1} = \left\{\begin{matrix} (v_0) \\ v_{i,j}; i = 4,5,6,\dots,n\ ; j = m, m-1, m=2,\dots,2 \\ v_{i,j}v_{i,j+1}; i = 3,2,1\ ; j = m-1, m-2, m-3 \\ v_{i,j}; i = 7,8,9,\dots,n; j = 1,2,3,\dots,m \\ i \neq 10,12,14,\dots,n; j \neq n-1, n-2, n-3,\dots,m-5 \end{matrix}\right\}$$

It is clearly that the color classes T_1, T_2, T_3, T_4 ,…,$T_{\Delta+1}$are independent sets of χ_{et} ($S_{n,m}$) T_1=T_2= T_3=$T_{\Delta+1}$=$2\Delta-1$, T_4 = T_5 =…=$T_\Delta = 2\Delta-2$ are evidents of satisfies the inequality || T_i−T_j || ≤ 1 for i & j this implies χ_{et} ($S_{n,m}$) ≤ Δ+1.

Case:3 Both n and m are odd (n < m)

$$T_1 = \begin{Bmatrix} (v_{1,1}),(v_{0,}v_{n-1,1}) \\ v_{i,j}; i = 1,3,5,\dots,n\,; j = \left[\frac{n+3}{2}\right],\left[\frac{n+1}{2}\right],\left[\frac{n-1}{2}\right],\dots,2 \\ v_{i,j}v_{i,j+1}; i = 2,4,6,\dots,n-1\,; j = \left[\frac{n+1}{2}\right],\left[\frac{n-1}{2}\right],\left[\frac{n}{2}\right],\dots,2 \\ v_{i,j}; i = 1,3,5,\dots,n; j = m, m-1, m-2,\dots,\left[\frac{n+5}{2}\right] \\ v_{i,j}v_{i,j+1}; i = 2,4,6,\dots,n-1; j = m-1, m-2, m-3,\dots,\left[\frac{n+5}{2}\right] \end{Bmatrix}$$

$$T_2 = \begin{Bmatrix} (v_{0,}v_{n,1}),(v_{1,1})(v_{1,2}),(v_{2,1}) \\ v_{i,j}; i = 2,4,6,\dots,n-1\,; j = \left[\frac{n+3}{2}\right],\left[\frac{n+1}{2}\right],\left[\frac{n-1}{2}\right],\dots,2 \\ v_{i,j}v_{i,j+1}; i = 3,5,7,\dots,n\,; j = \left[\frac{n+3}{2}\right],\left[\frac{n+1}{2}\right],\left[\frac{n-1}{2}\right],\dots,2 \\ v_{i,j}; i = 2,4,6,\dots,n-1; j = m, m-1, m-2,\dots,\left[\frac{n+7}{2}\right] \\ v_{i,j}v_{i,j+1}; i = 1,3,5,\dots,n; j = m-1, m-2, m-3,\dots,\left[\frac{n+5}{2}\right] \end{Bmatrix}$$

$$T_3 = \begin{Bmatrix} (v_{3,1}),(v_{0,}v_{1,1}),(v_{2,1})(v_{2,2}),(v_{1,2}) \\ v_{i,j}; i = 1,3,5,\dots,n\,; j = \left[\frac{n+5}{2}\right],\left[\frac{n+3}{2}\right],\left[\frac{n+1}{2}\right],\left[\frac{n-1}{2}\right],\dots,3 \\ v_{i,j}v_{i,j+1}; i = 2,4,6,\dots,n-1\,; j = \left[\frac{n+4}{2}\right],\left[\frac{n+2}{2}\right],\left[\frac{n}{2}\right],\dots,3 \\ v_{i,j}; i = 3,5,7,\dots n; j = m, m-1, m-2,\dots,\left[\frac{n+7}{2}\right] \\ v_{i,j}v_{i,j+1}; i = 4,6,8,\dots,n-1; j = m-1, m-2, m-3,\dots,\left[\frac{n+7}{2}\right] \end{Bmatrix}$$

…

$$T_{n-1} = \begin{Bmatrix} (v_{0,}v_{n-3,1}), v_{i,j}; i = n-1, j = m, v_{i,j}v_{i,j+1}; i = n, j = m-1 \\ v_{i,j}; i = 2,4,6,\dots,n-1\,; j = \left[\frac{n-1}{2}\right],\left[\frac{n}{2}\right],\dots,1 \\ v_{i,j}v_{i,j+1}; i = 1,3,5,\dots,n-2\,; j = \left[\frac{n-1}{2}\right],\left[\frac{n}{2}\right],\left[\frac{n+1}{2}\right],\dots,1 \\ v_{i,j}; i = 2,4,6,\dots,n-1;\ j = m-3, m-4, m-5,\dots,\left[\frac{n+3}{2}\right] \\ v_{i,j}v_{i,j+1}; i = 1,3,5,\dots,n;\ j = m-2, m-3, m-4,\dots,\left[\frac{n+1}{2}\right] \end{Bmatrix}$$

$$T_n = \left\{ \begin{array}{c} (v_{0,} v_{n-2}) \\ v_{i,j}; i = 1,3,5, \dots, n \,; j = \left[\frac{n+1}{2}\right], \left[\frac{n-1}{2}\right], \left[\frac{n-3}{2}\right], \dots 1 \\ v_{i,j}v_{i,j+1}; i = 2,4,6, \dots, n-1 \,; j = \left[\frac{n+1}{2}\right], \left[\frac{n-1}{2}\right], \left[\frac{n-3}{2}\right], \dots, 1 \\ v_{i,j}; i = n; j = m \\ v_{i,j}; i = 1,3,5, \dots, n-1; j = m-1, m-2, m-3, \dots, \left[\frac{n+3}{2}\right] \\ v_{i,j}v_{i,j+1}; i = 2,4,6, \dots, n; j = m-2, m-3, m-4, \dots, \left[\frac{n+3}{2}\right] \end{array} \right\}$$

$$T_{n+1} = \left\{ \begin{array}{c} (v_0) \\ v_{i,j}; i = 2,4,6, \dots, n-1 \,; j = \left[\frac{n+1}{2}\right], \left[\frac{n-1}{2}\right], \left[\frac{n-3}{2}\right], \dots, 2 \\ v_{i,j}v_{i,j+1}; i = 1,3,5, \dots, n; j = \left[\frac{n+1}{2}\right], \left[\frac{n-1}{2}\right], \left[\frac{n-3}{2}\right], \dots, 1 \\ v_{i,j}; i = 2,4,6, \dots, n-1; j = m-1, m-2, m-3, \dots, \left[\frac{n+5}{2}\right] \\ v_{i,j}v_{i,j+1}; i = 1,3,5, \dots, n; j = m-1, m-2, m-3, \dots, \left[\frac{n+3}{2}\right] \end{array} \right\}$$

It is clearly that the color classes T_1, T_2, T_3, T_4 ,…,$T_{\Delta+1}$are independent sets of χ_{et} ($S_{n,m}$) T_1=T_2= T_3,…,$T_{\Delta+1}$=2Δ , $T_\Delta = 2\Delta + 1$ are evidents of satisfies the inequality || T_i−T_j || ≤ 1 for i & j this implies χ_{et} ($S_{n,m}$) ≤ Δ+1.

Case:4 n is Odd and m is Even positive integers (n<m)

$$T_1 = \left\{ \begin{array}{c} (v_{1,1}), (v_{0,} v_{n-1}) \\ v_{i,j}; i = 1,3,5, \dots, n \,; j = \left[\frac{n+3}{2}\right], \left[\frac{n+1}{2}\right], \left[\frac{n-1}{2}\right], \dots, 2 \\ v_{i,j}v_{i,j+1}; i = 2,4,6, \dots, n-1 \,; j = \left[\frac{n+1}{2}\right], \left[\frac{n-1}{2}\right], \left[\frac{n-3}{2}\right], \dots, 2 \\ v_{i,j}; i = 3,5,7, \dots, n; j = m, m-1, m-2, \dots, \left[\frac{n+5}{2}\right] \\ v_{i,j}v_{i,j+1}; i = 4,6,8, \dots, n-1; j = m-1, m-2, m-3, \dots, \left[\frac{n+5}{2}\right] \end{array} \right\}$$

$$T_2 = \left\{ \begin{matrix} (v_{0,}v_{n,1}), (v_{1,1})(v_{1,2}), (v_{2,1}), \\ v_{i,j}; i = 2,4,6, \dots, n-1 \,; j = \left[\frac{n+3}{2}\right], \left[\frac{n+1}{2}\right], \left[\frac{n-1}{2}\right], \dots, 3 \\ v_{i,j}v_{i,j+1}; i = 1,3,5, \dots, n \,; j = \left[\frac{n+3}{2}\right], \left[\frac{n+1}{2}\right], \left[\frac{n-1}{2}\right], \dots, 2 \\ v_{i,j}; i = 4,6,8, \dots, n-1; j = m, m-1, m-2, \dots, \left[\frac{n+7}{2}\right] \\ v_{i,j}v_{i,j+1}; i = 3,5,9, \dots, n; j = m-1, m-2, m-3, \dots, \left[\frac{n+5}{2}\right] \end{matrix} \right\}$$

$$T_3 = \left\{ \begin{matrix} (v_{3,1}), (v_{0,}v_{1,1}), (v_{2,1})(v_{2,2}), (v_{1,2}) \\ v_{i,j}; i = 1,3,5, \dots, n \,; j = \left[\frac{n+5}{2}\right], \left[\frac{n+3}{2}\right], \left[\frac{n+1}{2}\right], \dots, 3 \\ v_{i,j}v_{i,j+1}; i = 2,4,6, \dots, n-1 \,; j = \left[\frac{n+5}{2}\right], \left[\frac{n+3}{2}\right], \left[\frac{n+1}{2}\right], \dots, 3 \\ v_{i,j}; i = 5,7,9, \dots, n; j = m, m-1, m-2, \dots, \left[\frac{n+7}{2}\right] \\ v_{i,j}v_{i,j+1}; i = 6,8,10, \dots, n-1; j = m-1, m-2, m-3, \dots, \left[\frac{n+7}{2}\right] \end{matrix} \right\}$$

…

$$T_{n-1} = \left\{ \begin{matrix} (v_{0,}v_{n-3,1}) \\ v_{i,j}; i = 2,4,6, \dots, n-1 \,; j = \left[\frac{n-1}{2}\right], \left[\frac{n-3}{2}\right], \left[\frac{n-5}{2}\right], \dots, 1 \\ v_{i,j}v_{i,j+1}; i = 1,3,5, \dots, n-2 \,; j = \left[\frac{n-1}{2}\right], \left[\frac{n-3}{2}\right], \left[\frac{n-5}{2}\right], \dots, 1 \\ v_{i,j}; i = 1,3,5, \dots, n; j = m, m-1, m-2, \dots, \left[\frac{n+3}{2}\right] \\ v_{i,j}v_{i,j+1}; i = 7,9,11, \dots, n-1; j = m-1, m-2, m-3, \dots, \left[\frac{n+3}{2}\right] \end{matrix} \right\}$$

$$T_n = \left\{ \begin{matrix} (v_{0,}v_{n-2,1}) \\ v_{i,j}; i = 1,3,5, \dots, n \,; j = \left[\frac{n+1}{2}\right], \left[\frac{n-1}{2}\right], \left[\frac{n-3}{2}\right], \dots, 1 \\ v_{i,j}v_{i,j+1} i = 2,4.6, \dots, n-1 \,; j = \left[\frac{n-1}{2}\right], \left[\frac{n-3}{2}\right], \left[\frac{n-5}{2}\right], \dots, 1 \\ v_{i,j}; i = 1,3,5, \dots, n; j = m, m-1, m-2, \dots, \left[\frac{n+3}{2}\right] \\ v_{i,j}v_{i,j+1}; i = 2,4,6, \dots, n-1; j = m-1, m-2, m-3, \dots, \left[\frac{n+3}{2}\right] \end{matrix} \right\}$$

$$T_{n+1} = \left\{ \begin{array}{c} (v_0) \\ v_{i,j}; i = 2,4,6, \dots, n-1 \,; j = \left[\frac{n+1}{2}\right], \left[\frac{n-1}{2}\right], \left[\frac{n-3}{2}\right], \dots, 2 \\ v_{i,j}v_{i,j+1}; i = 1,3,5, \dots, n \,; j = \left[\frac{n+1}{2}\right], \left[\frac{n-1}{2}\right], \left[\frac{n-3}{2}\right], \dots, 1 \\ v_{i,j}; i = 2,4,6, \dots, n-1; j = m, m-1, m-2, \dots, \left[\frac{n+5}{2}\right] \\ v_{i,j}v_{i,j+1}; i = 3,5,7, \dots, n; j = m-1, m-2, m-3, \dots, \left[\frac{n+3}{2}\right] \end{array} \right\}$$

It is clearly that the color classes T1, T2, T3, T4 ,…,$T_{\Delta+1}$are independent sets of χ_{et} ($S_{n,m}$) T1=T2= T3,…,$T_{\Delta+1}$=2Δ , $T_{\Delta} = 2\Delta + 1$ are evidents of satisfies the inequality || $T_i - T_j$ || ≤ 1 for i & j this implies χ_{et} ($S_{n,m}$) ≤ Δ+1.

Case: 5 n is even and m is odd positive integer (n<m)

$$T_1 = \left\{ \begin{array}{c} (v_{1,1}), (v_{0,}v_{n-1,1}) \\ v_{i,j}; i = 2,4,6, \dots, n \,; j = \left[\frac{n+2}{2}\right], \left[\frac{n}{2}\right], \left[\frac{n-2}{2}\right], \dots, 2 \\ v_{i,j}v_{i,j+1}; i = 2,4,6, \dots, n-1 \,; j = \left[\frac{n+2}{2}\right], \left[\frac{n}{2}\right], \left[\frac{n-2}{2}\right], \dots, 2 \\ v_{i,j}; i = 3,5,7, \dots, n-1; j = m, m-1, m-2, \dots, \left[\frac{n+5}{2}\right] \\ v_{i,j}v_{i,j+1}; i = 4,6,8, \dots, n; j = m-1, m-2, m-3, \dots, \left[\frac{n+5}{2}\right] \end{array} \right\}$$

$$T_2 = \left\{ \begin{array}{c} (v_{0,}v_{n,1,}), (v_{1,1})(v_{1,2}), (v_{2,1}) \\ v_{i,j}; i = 1,3,5, \dots, n-1 \,; j = \left[\frac{n+4}{2}\right], \left[\frac{n+2}{2}\right], \left[\frac{n}{2}\right], \dots 3 \\ v_{i,j}v_{i,j+1}; i = 2,4,6, \dots, n \,; j = \left[\frac{n+2}{2}\right], \left[\frac{n}{2}\right], \left[\frac{n-2}{2}\right], \dots, 2 \\ v_{i,j}; i = 4,6,8, \dots, n; j = m, m-1, m-2, \dots, \left[\frac{n+5}{2}\right] \\ v_{i,j}v_{i,j+1}; i = 5,7,9, \dots, n-1; j = m-1, m-2, m-3, \dots, \left[\frac{n+7}{2}\right] \end{array} \right\}$$

$$T_3 = \left\{ \begin{array}{c} (v_{3,1}),(v_{0,}v_{1,1,}),(v_{2,1})(v_{2,2}),(v_{1,2}) \\ v_{i,j};i=2,4,6,8,\dots,n\,;j=\left[\frac{n+4}{2}\right],\left[\frac{n+2}{2}\right],\left[\frac{n}{2}\right],\dots,3 \\ v_{i,j}v_{i,j+1};i=1,3,5,\dots,n-1\,;j=\left[\frac{n+4}{2}\right],\left[\frac{n+2}{2}\right],\left[\frac{n}{2}\right],\dots,3 \\ v_{i,j};i=5,7,9,\dots,n;j=m,m-1,m-2,\dots,\left[\frac{n+7}{2}\right] \\ v_{i,j}v_{i,j+1};i=6,8,10\dots n-1;j=m-1,m-2,m-3,\dots,\left[\frac{n+7}{2}\right] \end{array} \right\}$$

$$T_4 = \left\{ \begin{array}{c} (v_{4,1}),(v_{0,}v_{2,1,}),(v_{3,1})(v_{3,2}),(v_{2,2}),(v_{1,2})(v_{1,3}) \\ v_{i,j};i=1,3,5,\dots,n-1\,;j=\left[\frac{n+6}{2}\right],\left[\frac{n+4}{2}\right],\left[\frac{n+2}{2}\right],\dots,4 \\ v_{i,j}v_{i,j+1};i=2,4,6,\dots,n\,;j=\left[\frac{n+6}{2}\right],\left[\frac{n+4}{2}\right],\left[\frac{n+2}{2}\right],\dots,3 \\ v_{i,j};i=6,8,10\dots,n;j=m,m-1,m-2,\dots,\left[\frac{n+9}{2}\right] \\ v_{i,j}v_{i,j+1};i=7,9,11\dots,n-1;j=m-1,m-2,m-3,\dots,\left[\frac{n+7}{2}\right] \end{array} \right\}$$

…

$$T_{n-1} = \left\{ \begin{array}{c} (v_{0,}v_{n-3,1}) \\ v_{i,j};i=1,3,5,\dots,n-1\,;j=\left[\frac{n}{2}\right],\left[\frac{n-2}{2}\right],\left[\frac{n-4}{2}\right],\dots,1 \\ v_{i,j}v_{i,j+1};i=2,4,6,\dots,n-2;j=\left[\frac{n-2}{2}\right],\left[\frac{n-4}{2}\right],\left[\frac{n-6}{2}\right],\dots,1 \\ v_{i,j};i=2,4,6,\dots,n;j=m-1,m-2,\dots,\left[\frac{n+2}{2}\right] \\ v_{i,j}v_{i,j+1};i=1,3,5,\dots,n-1;j=m-1,m-2,m-3,\dots,\left[\frac{n+2}{2}\right] \end{array} \right\}$$

$$T_n = \left\{ \begin{array}{c} (v_{0,}v_{n-2,1}) \\ v_{i,j};i=2,4,6,\dots,n;j=\left[\frac{n}{2}\right],\left[\frac{n-2}{2}\right],\left[\frac{n-4}{2}\right]\dots,1 \\ v_{i,j}v_{i,j+1};i=1,3,5,\dots,n-1\,;j=\left[\frac{n}{2}\right],\left[\frac{n-2}{2}\right],\left[\frac{n-4}{2}\right]\dots,1 \\ v_{i,j};i=1,3,5,\dots,n-1;j=m,m-1,m-2,\dots,\left[\frac{n+4}{2}\right] \\ v_{i,j}v_{i,j+1};i=2,4,6,\dots n;j=m-1,m-2,m-3,\dots,\left[\frac{n+2}{2}\right] \end{array} \right\}$$

$$T_{n+1} = \begin{Bmatrix} (v_0) \\ v_{i,j}; i = 1,3,5, \dots n-1 \,; j = \left[\frac{n+2}{2}\right], \left[\frac{n}{2}\right], \left[\frac{n-2}{2}\right], \dots, 2 \\ v_{i,j}v_{i,j+1}; i = 2,4,6, \dots n \,; j = \left[\frac{n}{2}\right], \left[\frac{n-2}{2}\right], \left[\frac{n-4}{2}\right], \dots, 1 \\ v_{i,j}; i = 2,4,6, \dots n; j = m, m-1, m-2, \dots, \left[\frac{n+4}{2}\right] \\ v_{i,j}v_{i,j+1}; i = 1,3,5, \dots n-1; j = m-1, m-2, m-3, \dots, \left[\frac{n+4}{2}\right] \end{Bmatrix}$$

It is clearly that the color classes T_1, T_2, T_3, T_4 ,…,$T_{\Delta+1}$are independent sets of χ_{et} ($S_{n,m}$) T_1=T_2= T_3,…,$T_{\Delta+1}$=2Δ , $T_{\Delta} = 2\Delta + 1$ are evidents of satisfies the inequality || T_i−T_j || ≤ 1 for i & j this implies χ_{et} ($S_{n,m}$) ≤ Δ+1.

In below 3 cases we find the solutions of equitable total coloring of χ_{et} ($S_{n,m}$) ≤ Δ+1. But we can't write the generalization because there is no proper sequences.

1.Both n and m are odd (n>m)

2. n is Odd and m is Even positive integers(n>m)

3. n is Even and m is Odd positive integers(n>m)

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