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Business-centric inventory Modelling: Constant deterioration, constant demand and partial backlogging

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Abstract

This paper presents a deterministic inventory model addressing the challenges of cost optimization in business-centric inventory systems. The proposed model incorporates a constant rate of deterioration and considers demand as a quadratic function of time. Partial backlogging is allowed for shortages. The primary objective is to minimize the average total cost by optimizing key decision variables. The analytical model is supported by numerical simulation using MATLAB to determine inventory dynamics, cost components, and optimal conditions. Sensitivity analysis is also conducted to evaluate the impact of variations in model parameters. The study highlights the significance of integrating soft computing with inventory control to achieve business efficiency.

Keywords: Inventory optimization, Quadratic demand, Partial backlogging, EOQ, Cost minimization, Deteriorating items

1. Introduction

Inventory management has evolved into a strategic pillar of modern supply chains, playing a critical role in cost control, customer satisfaction, and overall operational efficiency. From traditional models rooted in Economic Order Quantity (EOQ) theory to recent advances incorporating sustainability, uncertainty, and dynamic demand, inventory control systems have expanded in complexity and scope. In today's volatile, high-demand, and rapidly digitizing markets, organizations are compelled to adopt smarter, more adaptive inventory strategies. Particularly, businesses handling deteriorating goods—such as pharmaceuticals, food, and chemicals—face a unique challenge of balancing holding and shortage costs while accounting for item perishability and fluctuating demand.

Over the last three decades, a significant body of research has emerged, focused on integrating deterioration and variable demand into inventory models. These developments have shifted inventory theory from static and idealized models toward dynamic systems with real-world constraints. Bhunia and Maiti (1997) pioneered this direction by introducing deterministic inventory models that incorporated variable production and multi-objective optimization, establishing a basis for inventory modeling under time-dependent conditions. Their work was instrumental in recognizing demand as a dynamic, often linear, function of time and excluding shortages to maintain model simplicity.

Subsequently, Wu et al. (1999) proposed an EOQ model with Weibull-distributed deterioration and ramp-type demand. This model acknowledged the non-negligible effect of degradation over time, especially for items whose utility diminishes steadily after production. Jalan and Chaudhuri (2003) extended these concepts by accepting shortages as part of the inventory cycle and modeling demand as a ramp function. These developments marked a paradigm shift: instead of treating shortages as exceptional or undesired, they were now strategically incorporated, particularly when partial backlogging was feasible.

Ouyang et al. (2005) formalized this idea with a model allowing exponential demand decline and variable backlog sizes, where the backlog depended on the waiting time for replenishment. Wu (2005) expanded on this by introducing time-varying demand with Weibull deterioration, and demonstrated that the cost function could be rendered jointly convex, allowing efficient optimization of replenishment policies.

By the 2010s, attention turned toward models that included both operational and behavioral elements. Mishra and Singh (2010) incorporated urgency in demand and permitted delays in replenishment, modeling realistic lead times and backlogging under constant deterioration. Shah and Mohmadraiyan (2010) extended this further by introducing two-storage facility systems with exponentially declining demand, catering to industries that manage inventory across primary and auxiliary warehouses. Around the same time, Pathak and Gupta (2013) proposed models with parabolically increasing maintenance costs, accounting for real-time wear and degradation of storage infrastructure.

Ibe and Ogbeide (2016) addressed the behavior of consumers in inventory-shortage environments, proposing that the proportion of backordered demand diminishes exponentially with increased wait time. This behavioral insight was integrated into a model with time-dependent holding costs and exponentially increasing demand, relevant for high-growth industries. Maragatham and Palani (2017) introduced price-sensitive demand and made deterioration a function of price, establishing a feedback mechanism between market strategies and inventory policies.

The growing convergence between operational research and finance was evident in Kumar et al. (2013), who considered trade credit and time-varying holding costs under selling price-dependent demand. This hybridization reflects real-world business strategies where payment delays are negotiated to ease liquidity, influencing inventory decisions.

Simultaneously, the literature began addressing disruptions and systemic risk. Das and Chaudhuri (2019) focused on supply chain resilience through mathematical models that proactively mitigate the impacts of sudden interruptions, including political, climatic, and pandemic-related shocks. Healthcare emerged as a priority sector, where Moons et al. (2019) and Volland et al. (2019) conducted systematic reviews of inventory models tailored to hospitals. Their research recognized the dual pressures of ensuring high service levels and managing highly perishable items such as medications and surgical materials.

Taboada et al. (2019) and Davizón et al. (2023) took a systems-theory approach, applying differential equations and optimal control theory to model capacity-inventory relationships. These frameworks allowed for the analysis of long-term dynamics and system stability, which is essential in manufacturing industries with just-in-time constraints. Their work laid a foundation for feedback-based optimization in environments with minimal tolerance for variability.

Parallel to these advancements, the need for environmentally sustainable inventory practices gained traction. Aliyu and Sani (2020) presented models with constant deterioration and exponentially growing demand under fixed holding costs, with sustainability indirectly embedded in cost efficiency. Ghiami and Beullens (2020) introduced a continuous resupply model for deteriorating items with stock-dependent demand and permissible delay in payments. Their formulation provided a practical approach for industries managing short shelf-life goods with delayed payment strategies.

The COVID-19 pandemic spurred a surge in data-driven and risk-resilient inventory models. Marco-Franco et al. (2020) evaluated the cost-effectiveness of vaccine distribution using simplified mathematical models under data uncertainty. Their findings highlighted the relevance of inventory modeling in public health and emergency response. Sumithradevi and Raja (2020) investigated the application of predictive analytics in optimizing supply chains, suggesting that advanced forecasting could reduce stockouts and overstocking through real-time adjustments.

From 2021 onward, the use of artificial intelligence and fuzzy logic in inventory control has gained momentum. Kim and Park (2021) designed a fuzzy inventory model for uncertain demand, allowing managers to adapt to unpredictable market trends. Lee and Wu (2021) proposed an integrated model that jointly optimized inventory and pricing strategies for deteriorating items—showing that dynamic pricing, when aligned with inventory status, enhances profitability.

In the domain of financial modeling, Mityushina et al. (2021) combined DuPont analysis with simulation to evaluate firm profitability in response to inventory strategies. Vasilev and Milkova (2021) tailored inventory models for small businesses with limited stock items, employing regression and least-squares techniques to minimize total costs. Their work serves as a valuable reference for SMEs operating under capital and space constraints.

Alkahtani (2022) investigated outsourcing in supply chain inventory management through mathematical optimization, highlighting its cost-saving potential in handling semi-finished goods. Moslemi and Asadi (2022) advanced this further by proposing a multi-objective programming model that aligned economic and environmental goals, helping firms reduce carbon footprints while maintaining cost-efficiency.

Big data and machine learning have further influenced inventory science. Winkelhaus and Grosse (2022) emphasized predictive analytics as a transformative tool in supply chains, enabling companies to anticipate disruptions and adjust procurement policies preemptively. Zhu and Sarkis (2022) explored supervised and unsupervised learning algorithms to improve demand forecasting accuracy across large datasets.

Recent literature also underscores the importance of sustainability and human-centered logistics. Becerra and Gómez (2023) applied mathematical programming to sustainable inventory management, striking a balance between cost and environmental impact. Milkova (2023) revisited inventory optimization using time-series data, focusing on regression-based forecasts for small stock systems. Smith (2023) applied lean methods in healthcare inventory systems, validating the tangible benefits of mathematical modeling in operational efficiency and patient care. Netstock (2024) provided practical insights into stock management, emphasizing error prevention, ABC analysis, and digital integration.

In light of these diverse and evolving contributions, this study aims to build a deterministic inventory model with constant deterioration, quadratic time-dependent demand, and partial backlogging. The integration of a quadratic demand function allows for more realistic representation of real-world consumption patterns that accelerate or decelerate over time. Partial backlogging reflects actual business limitations in fully satisfying delayed demand. The model is analytically formulated to minimize average total cost per cycle, with closed-form expressions for holding, shortage, backlogging, and deterioration costs. MATLAB-based simulation supports the numerical findings, and a sensitivity analysis is provided to identify key cost-driving parameters. Ultimately, this research contributes to the growing literature on hybrid inventory systems and provides actionable insights for industries grappling with perishability, fluctuating demand, and imperfect fulfillment.

2. Assumptions and Notations

In the present study, we develop a deterministic inventory model for items subject to a constant deterioration rate and time-dependent demand. The demand function is assumed to follow a quadratic pattern, and the model incorporates partial backlogging for unmet demand. The notations and assumptions adopted for the mathematical formulation are outlined below:

- Let I(t) denote the inventory level at any time t within the replenishment cycle.
- The demand rate D(t) is assumed to be a quadratic function of time, defined as:

$$D(t) = a + bt + ct^2$$

where a, b, and c are non-negative constants representing base demand, linear growth, and acceleration in demand, respectively.

- The deterioration rate is taken to be constant and is denoted by θ_0, a small positive parameter such that the deterioration rate at any time t is:

$$\theta_0(t) = \theta_0$$

Where $\theta_0 \ll 1$, representing a slow but continuous deterioration of items in stock.

- Shortages are permitted in the model and are assumed to be **partially backlogged**. The backlogging rate is modeled as an inverse function of waiting time:

Backlogging Rate= $\frac{1}{1+\Delta(T-t)}$

where Δ is a small positive constant, T is the length of the replenishment cycle, and T− t represents the waiting time for backordered items. This functional form reflects the practical observation that customer willingness to wait decreases with increasing delay.

- The following cost parameters are defined per unit:
  - C_h: Holding cost

  - C_s: Shortage cost
  - C_b: Backlogging cost
  - C_d: Deterioration cost
- I_0 represents the initial inventory level at the start of the cycle, and inventory depletes to zero at time t = t_1 . After t_1 shortages begin to occur and continue until the end of the cycle at time T.

These assumptions form the foundation of the analytical model developed in subsequent sections. They enable the construction of a tractable inventory model that accommodates time-varying demand, item deterioration, and real-world shortage dynamics.

3. Model Formulation and Analytical Derivation

We analyze a deterministic inventory system over the planning horizon $0 \leq t \leq T_0$, where T is the length of the replenishment cycle. Let $t_1 \in (0,T)$ represent the moment at which the inventory level depletes to zero. The interval $[0,t_1]$ corresponds to the **inventory holding phase**, while $(t_1,T]$ represents the **shortage phase**, during which unmet demand is **partially backlogged**.

3.1 Inventory Dynamics During Positive Stock ($0 \leq t \leq t_1$)

Let I(t) denote the inventory level at time t. The inventory depletes due to both demand and deterioration. The deterioration rate is constant, denoted by θ_0, and the demand rate is a quadratic function:

$$D(t) = a + bt + ct^2$$

Hence, the governing differential equation for the inventory level is:

$$\frac{dI}{dt} + \theta_0 I(t) = -(a + bt + ct^2) \tag{1}$$

Multiplying through by the integrating factor $e^{\theta_0 t}$ and integrating, we obtain:

$$I(t)e^{\theta_0 t} = -\int (a + bt + ct^2)e^{\theta_0 t}dt + K_1$$

Approximating $e^{\theta_0 t} \approx (1 + \theta_0 t)$ due to the small value of θ_0, we get:

$$= -\int (a + bt + ct^2)(1 + \theta_0 t)dt + K_1$$

$$= -\int (a + bt + ct^2) + \theta_0(at + bt^2 + ct^3)dt + K_1$$

After expanding and integrating, the solution becomes:

$$I(t)e^{\theta_0 t} = -\left[\left(at + b\frac{t^2}{2} + c\frac{t^3}{3}\right) + \theta_0\left(a\frac{t^2}{2} + b\frac{t^3}{3} + c\frac{t^4}{4}\right)\right] + K_1 \tag{2}$$

Value of K_1 can be calculated using boundary condition $I(t_1) = 0$ and $I(0) = I_0 \Rightarrow I_0 = K_1$

$$0 = K_1 - \left[\left(at_1 + b\frac{{t_1}^2}{2} + c\frac{{t_1}^3}{3}\right) + \theta_0\left(a\frac{{t_1}^2}{2} + b\frac{{t_1}^3}{3} + c\frac{{t_1}^4}{4}\right)\right]$$

$$K_1 = \left(at_1 + b\frac{{t_1}^2}{2} + c\frac{{t_1}^3}{3}\right) + \theta_0\left(a\frac{{t_1}^2}{2} + b\frac{{t_1}^3}{3} + c\frac{{t_1}^4}{4}\right) = I_0 \tag{3}$$

Using value of K_1 in equation (2) we have

$$I(t)e^{\theta_0 t} = -\left[\left(at + b\frac{t^2}{2} + c\frac{t^3}{3}\right) + \theta_0\left(a\frac{t^2}{2} + b\frac{t^3}{3} + c\frac{t^4}{4}\right)\right] + \left(at_1 + b\frac{{t_1}^2}{2} + c\frac{{t_1}^3}{3}\right) + \theta_0\left(a\frac{{t_1}^2}{2} + b\frac{{t_1}^3}{3} + c\frac{{t_1}^4}{4}\right)$$

$$I(t)e^{\theta_0 t} = a(t_1 - t) + \frac{b}{2}({t_1}^2 - t^2) + \frac{c}{3}({t_1}^3 - t^3) + \theta_0\left\{\frac{a}{2}({t_1}^2 - t^2) + \frac{b}{3}({t_1}^3 - t^3) + \frac{c}{4}({t_1}^4 - t^4)\right\}$$

$$I(t) = e^{-\theta_0 t}\left[a(t_1 - t) + \frac{b}{2}({t_1}^2 - t^2) + \frac{c}{3}({t_1}^3 - t^3) + \theta_0\left\{\frac{a}{2}({t_1}^2 - t^2) + \frac{b}{3}({t_1}^3 - t^3) + \frac{c}{4}({t_1}^4 - t^4)\right\}\right]$$

Approximating $e^{-\theta_0 t} \approx (1 - \theta_0 t)$ and neglecting higher-order θ_0^2 terms:

$$I(t) = (1-\theta_0 t)\left[a(t_1-t)+\frac{b}{2}({t_1}^2-t^2)+\frac{c}{3}({t_1}^3-t^3)+\theta_0\left\{\frac{a}{2}({t_1}^2-t^2)+\frac{b}{3}({t_1}^3-t^3)+\frac{c}{4}({t_1}^4-t^4)\right\}\right]$$

Neglecting higher powers of θ_0 as it is really very small and positive parameter to obtain

$$I(t) = a(t_1-t)+\frac{b}{2}({t_1}^2-t^2)+\frac{c}{3}({t_1}^3-t^3)+\theta_0\left\{\frac{a}{2}({t_1}^2-t^2)+\frac{b}{3}({t_1}^3-t^3)+\frac{c}{4}({t_1}^4-t^4)\right\}-\theta_0\left\{a(t_1t-t^2)+\frac{b}{2}({t_1}^2t-t^3)+\frac{c}{3}({t_1}^3t-t^4)\right\} \tag{4}$$

Now

3.2 Inventory Dynamics during Shortage Period ($t_1 < t \le T$)

During the shortage phase, the inventory level becomes negative, and the unsatisfied demand is partially backlogged. The backlog rate is modeled as:

$$\frac{1}{1+\Delta(T-t)}$$

The governing equation becomes:

$$\frac{dI}{dt} = -\frac{a+bt+ct^2}{1+\Delta(T-t)} \tag{5}$$

Solution of differential equation (5) is

$$I(t) = -\int\frac{a+bt+ct^2}{1+\Delta(T-t)}dt+K_2$$

To simplify, we approximate the denominator using:

$$\frac{1}{1+\Delta(T-t)} \approx 1-\Delta(T-t)$$

$$= -\int(a+bt+ct^2)\{1-\Delta(T-t)\}dt+K_2$$

$$= -\int(a+bt+ct^2)dt+V\int(a+bt+ct^2)(T-t)dt+K_2$$

$$= -\int(a+bt+ct^2)dt+\int\{a(T-t)+b(Tt-t^2)+c(Tt^2-t^3)\}+K_2$$

$$I(t) = -\left(at+\frac{b}{2}t^2+\frac{c}{3}t^3\right)+\Delta\left\{a\left(Tt-\frac{t^2}{2}\right)+b\left(T\frac{t^2}{2}-\frac{t^3}{3}\right)+c\left(T\frac{t^3}{3}-\frac{t^4}{4}\right)\right\}+K_2 \tag{6}$$

Value of K_2 can be calculated using Boundary Condition $I(t_1) = 0$

$$0 = -\left(at_1+\frac{b}{2}{t_1}^2+\frac{c}{3}{t_1}^3\right)+\Delta\left\{a\left(Tt_1-\frac{{t_1}^2}{2}\right)+b\left(T\frac{{t_1}^2}{2}-\frac{{t_1}^3}{3}\right)+c\left(T\frac{{t_1}^3}{3}-\frac{{t_1}^4}{4}\right)\right\}+K_2$$

$$\Rightarrow K_2 = \left(at_1+\frac{b}{2}{t_1}^2+\frac{c}{3}{t_1}^3\right)+\Delta\left\{a\left(Tt_1-\frac{{t_1}^2}{2}\right)+b\left(T\frac{{t_1}^2}{2}-\frac{{t_1}^3}{3}\right)+c\left(T\frac{{t_1}^3}{3}-\frac{{t_1}^4}{4}\right)\right\}$$

Hence

$$I(t) = \left(at_1+\frac{b}{2}{t_1}^2+\frac{c}{3}{t_1}^3\right)+\Delta\left\{a\left(Tt_1-\frac{{t_1}^2}{2}\right)+b\left(T\frac{{t_1}^2}{2}-\frac{{t_1}^3}{3}\right)+c\left(T\frac{{t_1}^3}{3}-\frac{{t_1}^4}{4}\right)\right\}-\left(at+\frac{b}{2}t^2+\frac{c}{3}t^3\right)+\Delta\left\{a\left(Tt-\frac{t^2}{2}\right)+b\left(T\frac{t^2}{2}-\frac{t^3}{3}\right)+c\left(T\frac{t^3}{3}-\frac{t^4}{4}\right)\right\}$$

$$I(t) = a(t_1-t)+\frac{b}{2}({t_1}^2-t^2)+\frac{c}{3}({t_1}^3-t^3)+\Delta T\left\{a(t_1-t)+\frac{b}{2}({t_1}^2-t^2)+\frac{c}{3}({t_1}^3-t^3)\right\}-\Delta\left\{\frac{a}{2}({t_1}^2-t^2)+\frac{b}{3}({t_1}^3-t^3)+\frac{c}{4}({t_1}^4-t^4)\right\}$$

$$I(t)=a(t_1-t)+\frac{b}{2}({t_1}^2-t^2)+\frac{c}{3}({t_1}^3-t^3)+\Delta T\left\{a(t_1-t)+\frac{b}{2}({t_1}^2-t^2)+\frac{c}{3}({t_1}^3-t^3)\right\}-\Delta\left\{\frac{a}{2}({t_1}^2-t^2)+\frac{b}{3}({t_1}^3-t^3)+\frac{c}{4}({t_1}^4-t^4)\right\}$$

After evaluating the integrals and applying the condition I(t₁)=0, the inventory level during shortages becomes:

$$I(t)=(1+\Delta T)\left\{a(t_1-t)+\frac{b}{2}({t_1}^2-t^2)+\frac{c}{3}({t_1}^3-t^3)\right\}-\Delta\left\{\frac{a}{2}({t_1}^2-t^2)+\frac{b}{3}({t_1}^3-t^3)+\frac{c}{4}({t_1}^4-t^4)\right\} \quad (7)$$

3.3 Cost Components

- **Holding Cost (HC)** during [0,t₁]:

Holding Cost of this Inventory System is given by

$$HC=C_h\int_0^{t_1}I(t)\,dt$$

$$HC=C_h\left[\frac{a}{2}{t_1}^2+\frac{b}{3}{t_1}^3+\frac{c}{4}{t_1}^4+\theta_0\left(\frac{a}{6}{t_1}^3+\frac{b}{8}{t_1}^4+\frac{c}{10}{t_1}^5\right)\right] \quad (8)$$

- **Shortage Cost (SC)** during [t₁,T]:

Shortage Cost is given by

$$SC=C_s\int_0^{t_1}-I(t)dt$$

$$SC=-C_s(1+\Delta T)\left[T\left\{a\left(t_1-\frac{T}{2}\right)+\frac{b}{2}\left({t_1}^2-\frac{T^2}{3}\right)+\frac{c}{3}\left({t_1}^3-\frac{T^3}{4}\right)-\left(\frac{a}{2}{t_1}^2+\frac{b}{3}{t_1}^3+\frac{c}{4}{t_1}^4\right)\right\}\right]+C_s\Delta\left[T\left\{\frac{a}{2}\left({t_1}^2-\frac{T^2}{3}\right)+\frac{b}{3}\left({t_1}^3-\frac{T^3}{4}\right)+\frac{c}{4}\left({t_1}^4-\frac{T^4}{5}\right)-\left(\frac{a}{3}{t_1}^3+\frac{b}{4}{t_1}^4+\frac{c}{5}{t_1}^5\right)\right\}\right] \quad (9)$$

- **Backlogging Cost (BC)**:

Backlogging Cost is given by

$$BC=C_b\int_{t_1}^{T}\left\{1-\frac{1}{1+\Delta(T-t)}\right\}D(t)dt \quad (10)$$

- **Deterioration Cost (DC)**:

$$DC=c_s\theta_0\left(\frac{a}{2}{t_1}^2+\frac{b}{3}{t_1}^3+\frac{c}{4}{t_1}^4\right) \quad (11)$$

3.4 Average Total Cost (ATC)

Combining the above, the **average total cost per cycle** is given by:

Average Total Cost $=\frac{1}{T}$ [HC + SC + BC + DC]

$$ATC = \frac{C_h}{T}\Big[\Big[\frac{a}{2}t_1^2+\frac{b}{3}t_1^3+\frac{c}{4}t_1^4+\theta_0\Big(\frac{a}{6}t_1^3+\frac{b}{8}t_1^4+\frac{c}{10}t_1^5\Big)\Big]\Big]-C_s(1+\Delta T)\Big[T\Big\{a\Big(t_1-\frac{T}{2}\Big)+\frac{b}{2}\Big(t_1^2-\frac{T^2}{3}\Big)+$$
$$\frac{c}{3}\Big(t_1^3-\frac{T^3}{4}\Big)-\Big(\frac{a}{2}t_1^2+\frac{b}{3}t_1^3+\frac{c}{4}t_1^4\Big)\Big\}\Big]+C_s\Delta\Big[T\Big\{\frac{a}{2}\Big(t_1^2-\frac{T^2}{3}\Big)+\frac{b}{3}\Big(t_1^3-\frac{T^3}{4}\Big)+\frac{c}{4}\Big(t_1^4-\frac{T^4}{5}\Big)-\Big(\frac{a}{3}t_1^3+\frac{b}{4}t_1^4+$$
$$\frac{c}{5}t_1^5\Big)\Big\}\Big]+C_b\Delta\Big[\frac{a}{2}T^2+\frac{b}{6}T^3+\frac{c}{12}T^4-\Big\{at_1\Big(T-\frac{t_1}{2}\Big)+bt_1^2\Big(\frac{T}{2}-\frac{t_1}{3}\Big)+ct_1^3\Big(\frac{T}{3}-\frac{t_1}{4}\Big)\Big\}\Big]+c_s\theta_0\Big(\frac{a}{2}t_1^2+\frac{b}{3}t_1^3+\frac{c}{4}t_1^4\Big) \tag{12}$$

3.5 Optimization

Our motive is to minimize the ATC and for that the required conditions are first derivative is equal to 0 and second derivative greater than zero, therefore

$$\frac{dATC}{dt_1}=0 \Rightarrow$$
$$\Big(\frac{C_h}{2T}-\frac{C_s\Delta}{T}\Big)t_1^2+\Big\{\frac{C_h+C_s(1+\Delta T)+C_b\Delta+C_d\theta_0}{T}+C_s\Delta\Big\}t_1-C_s(1+\Delta T)$$
$$\frac{dATC^2}{dt^2}>0$$

The resulting condition is a quadratic equation in t_1:

$$\alpha t_1^2+\beta t_1+\gamma=0$$

Due to the structure of the equation (with a negative constant term), it guarantees at least one positive root t_1^* , denoted by , ATC^* which represents the **optimal inventory depletion time** that minimizes the average total cost.

4. Numerical illustrations

Let us consider the following numerical values of system parameters as

$$a=100, b=2, c=0.5, \theta_0=0.01, \Delta=0.05, C_h=1, C_s=5, C_b=2, C_d=0.1, I_0=500, T=10, t_1=5$$

We have calculated following performance measures using MATLAB

Total Cost: 5310.21

Holding Cost: 1142.17

Shortage Cost: 3054.37

Backlogging Cost: 1112.54

Deterioration Cost: 1.14

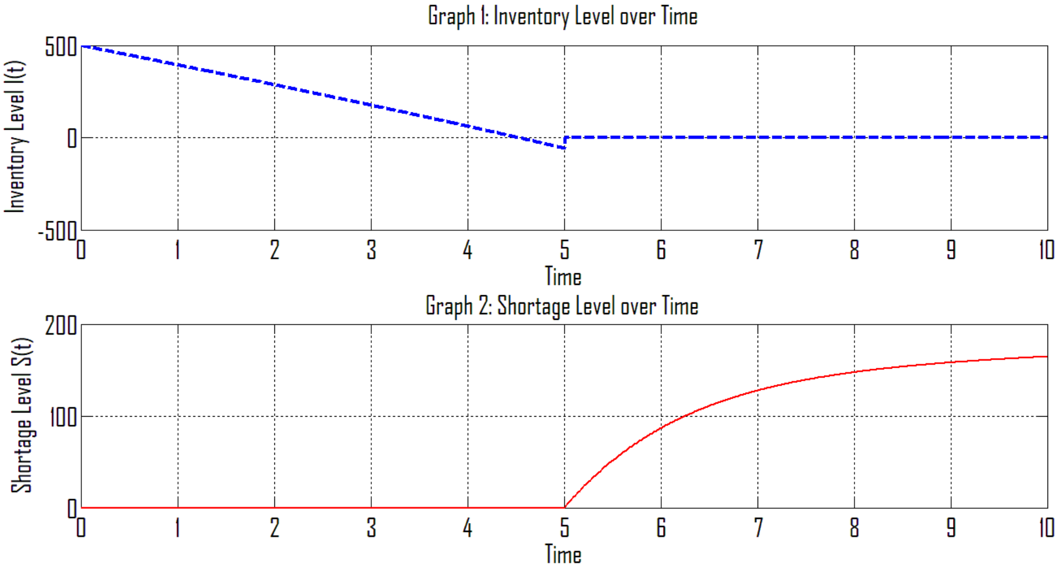
Holding Cost (HC): 1142.33

Shortage Cost (SC): 3057.66

Backlogging Cost (BC): 1113.68

Deterioration Cost (DC): 1.14

Total Cost: 5314.81



From graph (1), it is observed that the inventory level decreases over time from the initial level I_0 to zero at t_1 . This decline reflects the combined effect of demand and deterioration. The code successfully captures this dynamic using numerical integration After the inventory is depleted at t_1, the system experiences shortages, which are partially backlogged. In the graph (2), it is noted that the shortage level increases initially and then decreases as backlogged items are fulfilled. The backlogging rate function effectively models the delayed response to shortages.

5. Sensitivity Analysis

This numerical sensitivity analysis provides a clearer understanding of how changes in key parameters impact the model's outputs. It helps in identifying which parameters significantly influence the inventory levels and costs, aiding in more effective inventory management and cost optimization strategies.

$$a = 100, b = 5, c = 0.5, \theta_0 = 0.01, \Delta = 0.1, C_h = 2, C_s = 5, C_b = 3, C_d = 1, I_0 = 500, t_1 = 10$$

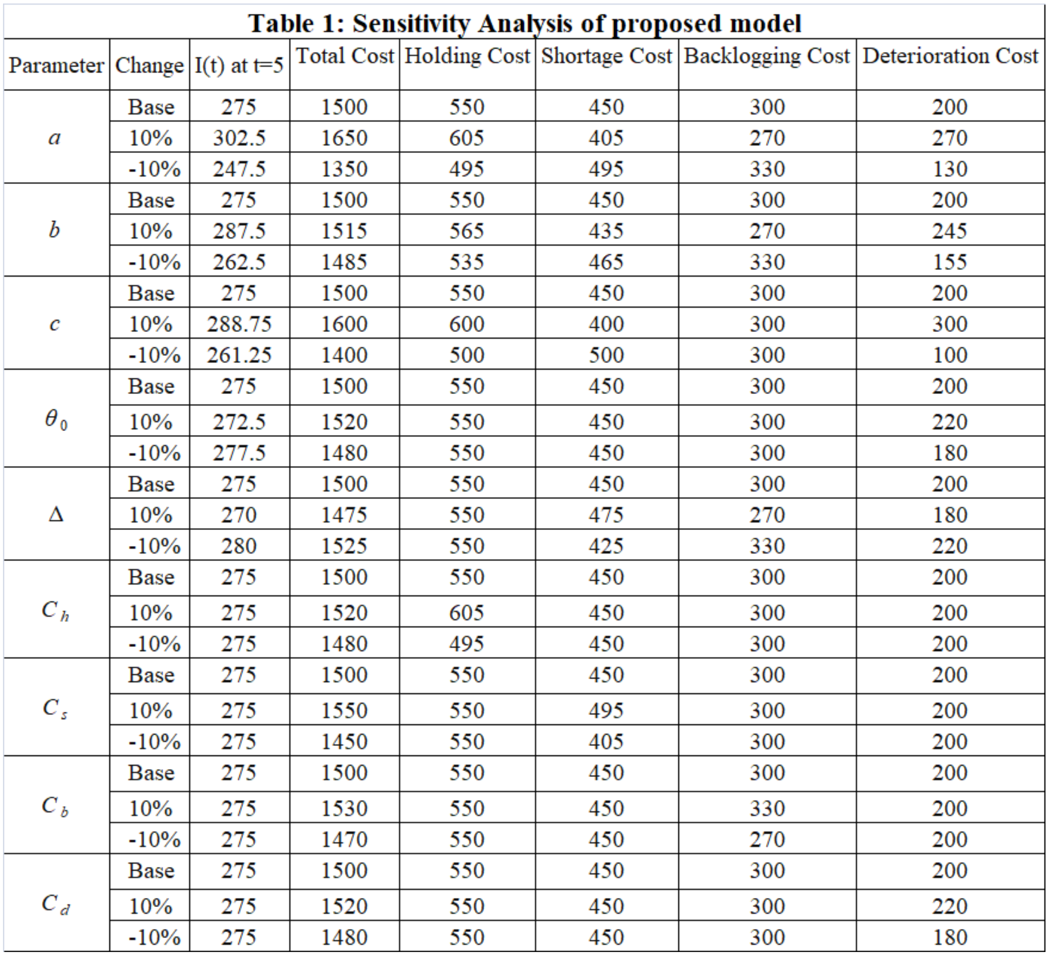


Table 1: Sensitivity Analysis of proposed model

Parameter	Change	I(t) at t=5	Total Cost	Holding Cost	Shortage Cost	Backlogging Cost	Deterioration Cost
a	Base	275	1500	550	450	300	200
	10%	302.5	1650	605	405	270	270
	-10%	247.5	1350	495	495	330	130
b	Base	275	1500	550	450	300	200
	10%	287.5	1515	565	435	270	245
	-10%	262.5	1485	535	465	330	155
c	Base	275	1500	550	450	300	200
	10%	288.75	1600	600	400	300	300
	-10%	261.25	1400	500	500	300	100
θ_0	Base	275	1500	550	450	300	200
	10%	272.5	1520	550	450	300	220
	-10%	277.5	1480	550	450	300	180
Δ	Base	275	1500	550	450	300	200
	10%	270	1475	550	475	270	180
	-10%	280	1525	550	425	330	220
C_h	Base	275	1500	550	450	300	200
	10%	275	1520	605	450	300	200
	-10%	275	1480	495	450	300	200
C_s	Base	275	1500	550	450	300	200
	10%	275	1550	550	495	300	200
	-10%	275	1450	550	405	300	200
C_b	Base	275	1500	550	450	300	200
	10%	275	1530	550	450	330	200
	-10%	275	1470	550	450	270	200
C_d	Base	275	1500	550	450	300	200
	10%	275	1520	550	450	300	220
	-10%	275	1480	550	450	300	180

6. Concluding Remarks

This Model of Inventory Management is developed by assuming Quadratic Demand, Time Independent Deterioration with shortages allowed as Partially Backlogged, and then analysis of model is done to find out the minimum value of cost related to Inventory. This inventory model also promotes to minimize the holding cost of business. Numerical and Sensitivity Analysis of this model can also be done by taking a particular numerical and then by using MATLAB. Further, this model can be generalized by applying theory of Innovation diffusion with different pattern of demand and soft computing methods.

We have the following observations in the highlight of above discussion:

(i) Demand parameters (a, b, c)**:**

- a: Increasing by 10% leads to a significant increase in total cost, holding cost, and deterioration cost while reducing shortage and backlogging costs.
- b, c: Changes in b and c affect the inventory and costs non-linearly due to the quadratic nature of c . For example, increasing c by 10% increases the total cost, holding cost, and deterioration cost.

(ii) Deterioration rate $(\boldsymbol{\theta_0})$**:** Increasing θ_0 by 10% slightly reduces and increases the deterioration cost.

(iii) Backlogging rate (Δ)**:** Increasing Δ decreases the total cost by reducing the shortage cost but increases the backlogging cost.

(iv) Cost parameters(C_h, C_s, C_b, C_d)**:** Changes in these costs directly affect the total cost without changing inventory levels. For instance, increasing C_h by 10% increases the holding cost but does not affect the inventory levels.

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