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DOMINATION ON SOME SPECIAL GRAPH

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Abstract

A set of vertices S is said to dominate the graph G, if for every vertex$v \notin S$, there is a vertex $u \in s$ with v adjacent to$u$. The minimum cardinality of any dominating set is called the domination number of G and is denoted by $\gamma(G)$. In this article domination in the soft graph of some types of graphs which valid like null, complete, cycle, complete bipartite, star graph, and wheel graph are been determined

1 Introduction

Consider a graph $G = (V, E)$ where V is denoted to the vertex set and E to the edge set of a graph G. One of the most important concepts that used in the graph theory as its starting point is the concept of domination. The concept of domination took a wide scope in most life and scientific applications through the use of the tools of graph theory, which depend on the vertex set and the edge set, and participated in finding solutions for most sciences such as chemistry, physics, biology, economics and others.

The first initiation of this concept is in [1]. After that, it started appearing in various sciences such as computers, engineering, chemistry, medicine, and others. In mathematics this term is dealt with in various fields to find solutions to its life problems, and these fields include general graph [2] - [5], fuzzy graph [6] - [9], topological graph [9] - [11], labelling graph [12], topological indices [13] - [14], and others.

A set D of vertices of G is a dominating set if for each vertex of the set V − D there is a vertex in the set D. A minimal dominating set in graph G is a dominating set that contains no proper dominating subset. For more details about other terms and notions not provided in this paper for the domination in graphs, the reader can refer [15] - [17]. Molodtsov has introduced the soft set theory in (1999) [18] as defined the set W be an initial universe and the set of parameters E with respect to the set W. Suppose that P(W) denotes the power set of W and $A \in E$. Now, let $F : A \to P(U)$, then the pair (F, A) is called a soft set over U [7], and Maji, P. K. defined and studied many processes on the soft set [19]. Thumbakara, R. K. et al. have been introduced soft graphs [20] and investigated some of their properties. The pair (F, A) is said to be a soft graph of G if the sub-graph induced by F(x) in G is connected sub-graph of G for all $x \in A$. The set of all soft graph of G is denoted by S. In this work, the domination in the soft graph of some types of graphs which valid like null, complete, cycle, complete bipartite, star graph, and wheel graph are been determined. Moreover, to obtain the results in this research, we need to address the following proposition and theorems.

Theorem 5.1.1. [1] Let $\mathrm{N_n}$ be a null graph then (F, A) is a soft graph if $\mathrm{F(x)} = \{\mathrm{y} \in \mathrm{V} \mid \mathrm{xRy} \iff \mathrm{d(x,y)} = 0\}$.

Theorem 5.1.2. [1] Let $\mathrm{P_n}$ be path graph, then (F, A) is soft graph if $\mathrm{F(x)} = \{\mathrm{y} \in \mathrm{V} \mid \mathrm{xRy} \iff \mathrm{d(x,y)} \leq \mathrm{k}\}$. where either $\mathrm{k} \leq \mathrm{rad(G)}$ or $\mathrm{k} = \mathrm{diam(G)}$.

Theorem 5.1.3. [1] Let $\mathrm{C_n}$ be a cycle graph, then (F, A) is soft graph if $\mathrm{F(x)} = \{\mathrm{xRy} \iff \mathrm{d(x,y)} = \left\lfloor \frac{\mathrm{n}}{2} \right\rfloor$ for every $\mathrm{n} \in \mathrm{N}\}$.

Theorem 5.1.4. [1] Let $\mathrm{K_n}$ be complete graph then (F, A) is soft graph if $\mathrm{F(x)} = \{\mathrm{y} \in \mathrm{V} \mid \mathrm{xRy} \iff \mathrm{d(x,y)} \leq 1\}$.

Theorem 5.1.5. [1] Let $\mathrm{G} \cong \mathrm{K_{n,m}}$ be a complete bipartite graph of order mn and $(\mathrm{n}, \mathrm{m}) \geq 1$. Then (F, A) is a soft graph if $\mathrm{F(x)} = \{\mathrm{xRy} \iff \mathrm{d(x,y)} \leq \mathrm{k}\}$ where $\mathrm{k} = 1, 2$.

Proposition 5.1.6. [7] For a graph on n vertices, we have

- $\gamma(K_n) = \gamma(W_n) = 1$.
- $\gamma(P_n) = \gamma(C_n) = \left\lceil \frac{n}{3} \right\rceil$.
- $\gamma(K_{m,n}) = min\{m, n\}$.

5.2 Main results

In this section, we shall consider the concept of domination in soft graphs. We also prove some properties and theorems by giving illustrative examples.

Definition 5.2.1. Let G be a simple graph and D^*be a set of vertices of G. The setD^* is a dominating set of a soft graph (F, A) if every vertex in $\mathrm{S} - \mathrm{D}^*$is adjacent to at least one vertex in D^* where $S = \{F(x) : x \in A\}$ *and* $D^* \subseteq S$.

Definition 5.2.2. *A* minimal dominating set in the soft graph (F, A) is a dominating set if it contains no proper dominating set.

Definition 5.2.3. The cardinality of the minimum dominating set of G is called the domination number of the soft graph (F, A) and is denoted by $\gamma_S(S)$.

Example 5.2.4. Consider the simple graph $G = (V, E)$ *as shown in Figure 1. Let* $A = \{v_1, v_2\}$ *and* $F(x) = \{y \in V \mid xRy \iff d(x,y) \leq 1\}$. *Then* $F(v_1) = \{v_4, v_2, v_1\}$, $F(v_2) = \{v_3, v_2, v_1\}$ *and* $S = \{v_4, v_3, v_2, v_1\}$.

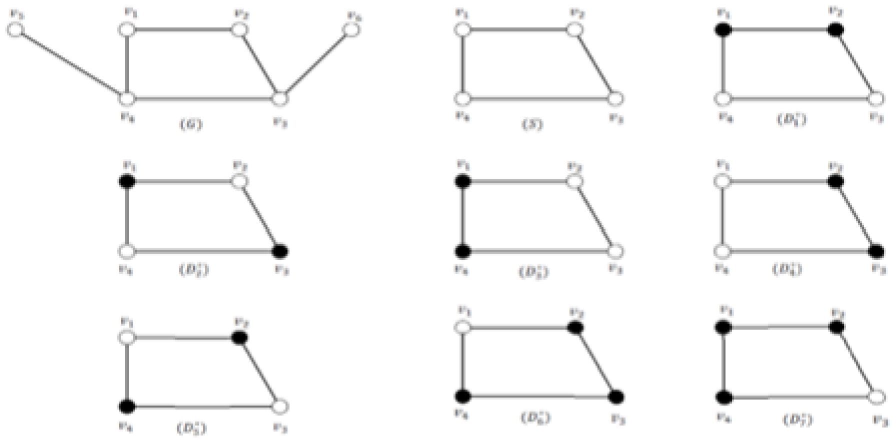


Figure 5.1: Domination in soft Graph

The possible dominating sets are: $D_1 = \{v_2, v_1\}$, $D_2 = \{v_3, v_1\}$, $D_3 = \{v_4, v_1\}$, $D_4 = \{v_3, v_2\}$, $D_5 = \{v_4, v_2\}$, $D_6 = \{v_4, v_3, v_2\}$, $D_1 = \{v_4, v_2, v_1\}$. Therefore, $\gamma_s(S) = 2$.

Proposition 5.2.5. *Let* N_n be null graph of order $n \geq 1$ *and* (F, A) is soft graph, then $\gamma_s(N_n) = |A| = |S|$.

Proof. Let $\mathrm{A} = \{\mathrm{v_i}\}$ for some i. Since (F, A) is a soft graph, by the Theorem 5.1.1, we have $F(x) = \{xRy \Longleftrightarrow d(x, y) = 0\}$. Thus $F(v_i) = \{v_i\} and\ S = \{v_i\}$, $\forall v_i \in A$. Since Nn is a null graph, each vertex $v_i\ of\ N_n$ is independent and hence $|A| = |S|$. Further S will be an independent set, therefore $\mathrm{D} = \{\mathrm{v_i}\}$, $\forall\, \mathrm{v_i} \in \mathrm{A}$. Hence, $\mathrm{A} = \mathrm{S} = \mathrm{D}$, which implies that $\gamma_s(S) = |A| = |S|$.

Theorem 5.2.6. *Let G =* P_n be a path graph and (F, A) is a soft graph, then

$$\gamma_s(P_n) = \begin{cases} \left\lceil \frac{n}{3} \right\rceil, & if\ S \equiv G \\ \left\lceil \frac{n-m}{3} \right\rceil, & if\ A = \{v_j\}\ where\ m = min\{d(v_i, v_j)\}\ and\ v_i \in cent(G). \end{cases}$$

Proof. Since (F, A) is a soft graph, by the Theorem 5.1.1, $F(x) = \{y \in V\ |xRy \Longleftrightarrow d(x, y) \leq k\}$, where $either\ k \leq rad(G)\ or\ k = diam(G)$. Let $A = \{v_i\}\ for\ some\ i$, so two cases are distinguished as follows:

Case (1): $If\ k = rad(G)$, then again there are two sub-cases depending on A, as follows:

Sub - Case (i): $If\ A \cap cent(G) \neq \phi, then\ F(v_i) = \{v_i\ ,\ i = 1, \ldots, n\}\ and\ S = \{v_i\ ,\ i = 1, \ldots, n\}$. Therefore, $S \equiv G$. Hence $D^* \equiv D$ implying that $\gamma_s(P_n) = \gamma(G)$. Thus, by proposition 5.1.6, $\gamma_s(P_n) = \left\lceil \frac{n}{3} \right\rceil$.

Sub - Case (ii): $If\ A \cap cent(G) = \phi$, then two cases are possible as follows:

(a) $If\ v_{j_1},\ v_{j_2} \in A\ and\ \ v_i \in cent(G)$. Then there exists $j_1 < i < j_2\ and\ S = \{v_j\ ,\ i = 1,\ldots,n\} \equiv G$. Therefore $D^* \equiv D \Longrightarrow \gamma_s(P_n) = \gamma(G)$ and so by proposition 5.1.6, $\gamma_s(P_n) = \left\lceil\frac{n}{3}\right\rceil$.

(b) $If\ v_j \in A$, then either $v_j < v_i\ or\ v_j > v_i$ where $v_i \in cent(G)$. Then $F(v_j) = \{v_j\ ,j = 1,2,\ldots,n-m\}$ where $m = min\, d(v_i\ ,v_j\ )$. So $S = \{v_j\ ,j = 1,2,\ldots,n-m\} \not\equiv G$ and since $|S| = n - m$, we have $\gamma_s(P_n) = \left\lceil\frac{n-m}{3}\right\rceil$.

Case (2): Suppose $k = diam(G)\ and\ A = \{v_i\}$ then $F(v_i) = \{v_i\ ,i = 1,2,\ldots,n\}$ and $S = \{v_i\ ,i = 1,2,\ldots,n\} \equiv G$. Therefore $D^* \equiv D$ which implies that $\gamma_s(P_n) = \gamma(G)$. Now, by proposition 5.1.6, we have $\gamma_s(P_n) = \left\lceil\frac{n}{3}\right\rceil$.

Example 5.2.7. *Let* $G \equiv P_7$ *be a path graph on seven vertices and* (F, A) *be a soft graph. Then* $F(x) = \{y \in V\ /\ xRy \Longleftrightarrow d(x,y) \leq k\}$, *where either* $k \leq rad(G)\ or\ k = daim(G)$. *Now, we have two possible cases here:*

Case (1): *Suppose* $k \leq rad(G)$. *We have again two possible sub-cases here:*

Sub - Case (i): $If\ A \cap cent(G) \neq \phi \implies v_4 \in cent(G)$. *Then* $A = \{v_4\}$ *and* $rad(G) = 3$ *implies that* $k = 3$. *Therefore* $F(v_4) = \{v_i\ ,i = 1,\ldots,7\}$ *and* $S = \{v_i\ ,i = 1,\ldots,7\} \equiv G$. *Hence* $D^* = \{v_2, v_5, v_7\}$. *Now,* $D^* \equiv D$ *and* $\gamma_s(P_7) = \left\lceil\frac{n}{3}\right\rceil$. *This implies that* $\gamma_s(P_7) = \left\lceil\frac{7}{3}\right\rceil$ *Hence,* $\gamma_s(P_7) = 3$.

Sub - Case (ii): If $A \cap\ cent(G) = \phi$, then there are two cases as follows:
(a): Suppose $A = \{v_2, v_5\}$. Then $F(v_2) = \{v_i\ ,i = 1,\ldots,5\}$, and $F(v_5) = \{v_i\ ,i = 2,\ldots,7\}$. Therefore, $S = \{v_i\ ,i = 1,\ldots,7\} \equiv G$. Hence $D^* \equiv D$. Now, by proposition 5.1.6, we have $\gamma_s(P_7) = \left\lceil\frac{n}{3}\right\rceil$. Thus $\gamma_s(P_7) = \left\lceil\frac{7}{3}\right\rceil]$. Hence $\gamma_s(P_7) = 3$.
(b): Assume that $A = \{v_2\}$. Then $F(v_2) = \{v_i,\ i = 1,\ldots,5\}$ and $S = \{v_i\ ,i = 1,\ldots,5 \not\equiv G$ so $m = 2$. Then $\gamma_s(P_7) = \left\lceil\frac{7-2}{3}\right\rceil$ that implying that $\gamma_s(P_7) = \left\lceil\frac{5}{3}\right\rceil$. Therefore, $\gamma_s(P_7) = 2$
Case (2): Suppose $k = diam(G)$ That is $diam(G) = 6$. Then $F(x) = \{xRy \Longleftrightarrow d(x, y) \leq 6\}$ and $F(v_i) = \{v_i\ ,\ i = 1,\ldots,n\}$. Let $A = \{v_1\}$, then $F(v_1) = \{v_i\ ,i = 1,\ldots,7\}$ and $S = \{v_i, i = 1,\ldots,7\} \equiv G$. Hence $D^* \equiv D$, then by proposition 5.1.6 then $\gamma_s(P_7) = \left\lceil\frac{n}{3}\right\rceil$. Thus $\gamma_s(P_7) = \left\lceil\frac{7}{3}\right\rceil$. This implies that $\gamma_s(P_7) = 3$.

Proposition 5.2.8. $Let\ G \cong C_n\ be\ a\ cycle\ graph\ of\ order\ n\ and\ (F, A)\ is\ soft\ graph,$ *then*

$$\gamma_s(C_n) = \begin{cases} |A|, & if\ A\ is\ independent\ and\ A \cap N(vi) \neq \phi. \\ \left\lceil \dfrac{|A| \times 2}{3} \right\rceil, & if\ A\ is\ independent\ and\ n\ is\ odd\ with\ A \cap N(v_i) \neq \phi. \\ \sum \left\lceil \dfrac{|A|_i}{3} \right\rceil, & if\ A\ is\ non-independent\ and\ n\ is\ even. \\ \sum \left\lceil \dfrac{|A_i|+1}{3} \right\rceil, & if\ A\ is\ non-independent\ and\ n\ is\ odd. \end{cases}$$

Proof. Let $G \cong C_n$ be a cycle graph of order n. Since (F, A) is soft graph, then by Theorem 5.1.3, we have $F(x) = \{y \in V / xRy \Leftrightarrow d(x,y) = \lfloor \frac{n}{2} \rfloor\}$. Let $A = \{v_i\}$ for some i. Three cases are distinguished as follows:

Case (1): Assume n is even. Then two cases are distinguished as follows:

(i): Suppose A is an independent set. Then, let $A = \{v_i\}$ for some i. Now $F(v_i) = \{v_{i+\frac{n}{2}}\}$ and $S = \{v_{i+\frac{n}{2}}\}$ for all $v_i \in A$. So $|S| = |A|\ and\ S$ is an independent set. Then $S \equiv D^*$ and $\gamma_s(C_n) = |A|$.

(ii): Assume that A is not an independent set. Let $A_i \subseteq A$ be the largest connected sub-graph of the cycle such that $\cup Ai = A$. Then $\gamma_s(C_n) = \Sigma\lceil \frac{|A_i|}{3} \rceil$.

Case (2): Suppose n is an odd integer. There are three possible cases here:

(i): Suppose A is an independent set. Then $A \cap N(v_i) \neq \phi$ and $F(v_i) = \{v_{i+\frac{n}{2}}, v_{i-\frac{n}{2}}\}$. We have $S = \{v_{i+\frac{n}{2}}, v_{i-\frac{n}{2}}\}$ for all $v_i \in$ A, and so $|S| = 2 \times |A|$. So S is non-independent set. Therefore $\gamma_s(C_n) = |A|$.

(ii): Suppose A is an independent set such that $A \cap N(v_i) \neq \phi$. $Then\ F(v_i) = \{v_{i+\frac{n}{2}}, v_{i-\frac{n}{2}}\}$ and $S = \left\{v_{i+\frac{n}{2}}, v_{i-\frac{n}{2}}\right\}$ $\forall\ v_i \in A$. Then $|S| = 2 \times |A|$, so S is connected sub-graph. Therefore $\gamma_s(C_n) = \left\lceil \frac{2 \times |A_i|}{3} \right\rceil$.

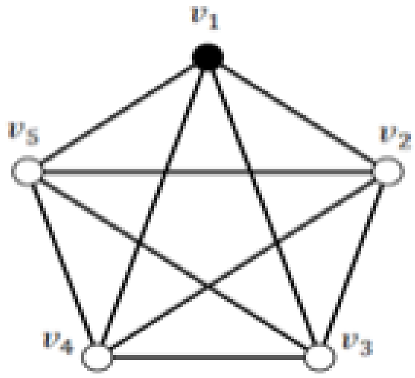
(iii): If A is not an independent set. Let $A_i \subseteq A$ is largest connected sub-graph of the cycle such that $A_i = A$ Then $\gamma_s(C_n) = \left\lceil \frac{|A_i|}{3} \right\rceil$.

Example 5.2.9. Let $G \cong C_n$ and (F, A) is a soft graph, then $F(x) = \{y \in V/xRy \Leftrightarrow d(x,y) = \frac{n}{2}\}$. We have two possible cases here:

Case (1): Suppose n is an even integer. For instance, let us take $n = 8, then\ d(x,y) = \frac{8}{2}$. Thus, $d(x,y) = 4$. If A is an independent set and let $A = \{v_1, v_4, v_7\}$, then $F(v_1) = \{v_5\}$, $F(v_4) = \{v_8\}$, $F(v_7) = \{v_3\}$ and so $S = \{v_3, v_5, v_3\}$. Thus, $D^* = \{v_3, v_5, v_3\}$ and So $\gamma_S(S) = 3 = |A|$. Next, assume that A is non-independent and let $A = A_1 \cup A_2$ such that $A_1 = \{v_1, v_2, v_3\}$ and $A_2 = \{v_6, v_7\}$. Then $F(v_1) = \{v_5\}$, $F(v_2) = \{v_6\}$, $F(v_3) = \{v_7\}$, $F(v_6) = \{v_1\}$, $F(v_7) = \{v_2\}$ and $S = \{v_1, v_2, v_5, v_6, v_7,\}$. So $D^* = \{v_1, v_6\}$. Therefore, $\gamma_S(C_8) = \lceil \frac{|A_1|}{3} \rceil + \lceil \frac{|A_2|}{3} \rceil = \lceil \frac{3}{3} \rceil + \lceil \frac{2}{3} \rceil = 1 + 1 = 2$.

Case (2): Suppose n is odd. For instance, let us take $n = 9$, then $d(x,y) = \frac{9}{2}$ which implies that $d(x,y) = 4$. Now, suppose A is an independent set such that $A \cap N(v_i) = \phi$ and let $A = \{v_1, v_4, v_7\}$. Then $F(v_1) = \{v_5, v_6\}$, $F(v_4) = \{v_8, v_9\}$, $F(v_7) = \{v_2, v_3\}$ and $S = \{v_2, v_3, v_5, v_6, v_8, v_9\}$ is non-independent. Thus, $D^* = \{v_3, v_5, v_8\}$ and hence $\gamma_S(C_9) = 3 = |A|$.

Assume that A is an independent set such that $A \cap N(v_i) \neq \phi$. Let $A = \{v_1, v_3, v_5\}$. Then $F(v_1) = \{v_5, v_6\}$, $F(v_3) = \{v_7, v_8\}$, $F(v_5) = \{v_9, v_1\}$ and $S = \{v_1, v_5, v_6, v_7, v_8, v_9\}$ is connected sub-graph, thus $D^* = \{v_6, v_9\}$ and $\gamma_S(C_9) = \lceil \frac{2\times|A|}{3} \rceil = \lceil \frac{2\times 3}{3} \rceil = 2$.



$Figure$ 5.2: $Domination\ of\ soft\ graph\ in\ K_5$

If A is non-independent and $A = A_1 \cup A_2$ such that $A_1 = \{v_1, v_2, v_3\}$ and $A_2 = \{v_7, v_8\}$. Then $F(v_1) = \{v_5, v_6\}$, $F(v_2) = \{v_6, v_7\}$, $F(v_3) = \{v_7, v_8\}$, $F(v_7) = \{v_2, v_3\}$, $F(v_8) = \{v_3, v_4\}$ and $S = \{v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$. Thus $D^* = \{v_3, v_6, v_8\}$ and So $\gamma_S(C_9) = \left\lceil \frac{|A_1|+1}{3} \right\rceil + \left\lceil \frac{|A_2|+1}{3} \right\rceil = \left\lceil \frac{3+1}{3} \right\rceil + \left\lceil \frac{2+1}{3} \right\rceil = 2 + 1 = 3$.

Theorem 5.2.10. Let $G \cong K_n$ be a complete graph of order n and (F, A) is soft graph. Then $\gamma_S(K_n) = 1$.

Proof. Let $G \cong K_n$ be a complete graph of order n. Since (F, A) is soft graph, by the Theorem 5.1.4, we have $F(x) = \{y \in V \,|xRy \Longleftrightarrow d(x,y) \leq 1\}$. Let $A = \{v_i\}\ for\ some\ i$, then $F(v_i) = \{v_i,\ i = 1, \ldots, n\}$ and $S = \{v_i,\ i = 1, \ldots, n\} \equiv G$. Therefore $D^* \cong D$. Hence by proposition 5.1.6, we have $\gamma_S(S) = 1$.

Example 5.2.11. Let $G \cong K_5$, as shown in Figure 5.2. Let $A = \{v_2, v_1\}$ and (F, A) is soft graph. So, $F(x) = \{xRy \Longleftrightarrow d(x,y) \leq 1\}$, and $F(v_1) = \{v_5, v_2, v_4, v_3, v_1\}$ and $F(v_2) = \{v_5, v_4, v_3, v_2, v_1\}$. Then $S = \{v_5, v_4, v_3, v_2, v_1\} \equiv G$. Thus $D^* \equiv D$ and $D^* = \{v_1\}$. Therefore by Proposition 5.1.6, we have$\gamma_S(K_5) = 1$.

Theorem 5.2.12. Let $G = K_{m,n}$ be a bipartite graph of order $mn\ and\ n, m \geq 1$ and (F, A) is a soft graph, then $\gamma_S(S) = \begin{cases} 1 & \text{if } |X| = 1 \text{ or } |Y| = 1 \\ 2 & otherwise \end{cases}$

Proof. Let $G = K_{m,n}$ be bipartite graph, and since (F, A) is soft graph, then by Theorem 5.1.5, $F(x) = \{xRy \Longleftrightarrow d(x,y) \leq k\}$,where $k = 1,2$ so three cases are distinguished as follows:

Case(1): $If\ |X| = 1\ or\ |Y| = 1,\ let\ |X| = 1$ and $A = \{v\}$ such that $v \in X$, then $F(x) = \{y \in V/xRy \Longleftrightarrow d(x,y) \leq 1\}$, and $F(v) = \{v \cup Y\}$ and $S = \{v \cup Y\}$, So $D^* = \{v\}\ and\ \gamma_S(K_{m,n}) = 1$.

Case(2): $If\ |X| > 1\ and\ |Y| > 1,\ \ let\ A = \{v_i\} for\ some\ i, i = 1, \dots, m$ such that $|A| > 1\ and\ v_i \in X$, then there are two cases as follows:

(i): $If\ F(x) = \{xRy \Longleftrightarrow d(x,y) \leq 1\}$, then $F(vi) = \{vi \cup Y\}, for\ some\ i$ and $vi \in X$, so $S = \{v_i \cup Y\}\ \forall v_i \in A$, and $D^* = \{v_i, u_j\}$ where $v_i \in X,\ u_j \in Y\ for\ one\ of\ i,j\ and\ i = 1, \dots, m,\ j = 1, \dots, n$ therefore $\gamma_S(K_{m,n}) = 2$. .

(ii): $If\ F(x) = \{xRy \Longleftrightarrow d(x,y) \leq 2\}$, then $F(v_i) = \{X \cup Y\}$ and $S = \{X \cup Y\} \equiv G$, so $D^* \equiv D$, then by proposition 5.1.6 $\gamma_S(K_{m,n}) = 2$.

Example 5.2.13. Let $G = K_{m,n}$ be a complete bipartite graph. There are two possible cases here:

Case(1): $If\ m = 1\ and\ n = 3$. Let $A = \{v_1\}$ where $v_1 \in X\ and\ |X| = 1$. Then $F(x) = \{y \in V/xRy \Longleftrightarrow d(x,y) \leq 1\}$ and $F(v_1) = \{v_1, u_1, u_2, u_3\}$ where $u_j \in Y, j = 1, \dots, 3$. Hence $S = \{v_1, u_1, u_2, u_3\}$ and $D^* = \{v_1\}$. Therefore $\gamma_S(K_{1,3}) = 1$.

Case (2): $If\ m > 1. Let\ m = 4, n = 3$ and let $A = \{v_1, v_2\}$. Then we have two possibilities here:

(i): $If\ F(x) = \{xRy \Longleftrightarrow d(x,y) \leq 1\}$. Then $F(v_1) = \{v_1, u_1, u_2, u_3\}$, $F(v_2) = \{v_2, u_1, u_2, u_3\}$ Hence $S = \{v_2, u_1, u_2, u_3\}$ and $D^* = \{v_1, u_1\}$. Therefore $\gamma_S(K_{4,3}) = 2$.

(ii): $If\ F(x) = \{xRy \Longleftrightarrow d(x,y) \leq 2\}$. Then $F(v1) = \{v_1, v_2, v_3, v_4, u_1, u_2, u_3\}$, $F(v_2) = \{v_1, v_2, v_3, v_4, u_1, u_2, u_3\}$. Hence $S = \{v_1, v_2, v_3, v_4, u_1, u_2, u_3\} \equiv G$. Therefore, $D^* \equiv D$ and $D^* = \{v_1, u_1\}.\ So\ \gamma_S(K_{4,3}) = 2$.

Corollary 5.2.14. $Let\ S_n\ be\ star\ graph\ and\ (F, A)\ is\ soft\ graph, then\ \gamma_S(S) = 1$.

5.3 Conclusion

Through the results proved above on the certain graphs mentioned above, the different results for the same graph can be getting by changing the condition in the set F. The theories of each graph are proved with illustrative examples.

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