Bi-Parabolic temperature variation effect on free vibration of visco–elastic parallelogram plate with linear thickness variation in one dimensions

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ABSTRACT-

This study analyzes the vibration modes of a non-homogeneous isotropic parallelogram plate with one-dimensional thickness variation, focusing on CSCS edge conditions, where "C" and "S" denote clamped and simply supported edges, respectively.. Thermal Induced vibration of these plates has been taken as bi-parabolic temperature distributions in both directions.. The time period corresponding to the first two modes of vibration has been calculated for a parallelogram plate under CSCS boundary conditions for various values of aspect ratio, thermal constant and taper constant.

Keywords- Vibration, visco-elastic, non-homogeneous, parallelogram plate, bi-Parabolic temperature, thermal gradient.

1. Introduction

The analysis of vibrational modes in various plate configurations, including tapered plates, is crucial in engineering due to their diverse practical applications. Numerous studies have significantly contributed to this area of research. Tapered plates—whether of uniform or non-uniform thickness and subjected to varying temperatures—are commonly used in the automotive sector, aerospace industry, power plants, and marine structures. Vibration is an inherent characteristic of machines, and every machine exhibits a certain vibration level, which may be considered normal or intrinsic. However, excessive or abnormal vibrations can be caused by factors such as imbalance, misalignment, looseness, or faulty bearings. Therefore, it is essential to determine the exact vibration levels in machinery. Numerous studies have investigated the vibration behavior of plates with varying geometries—homogeneous or non-homogeneous, isotropic or orthotropic—considering or ignoring thickness variations and thermal effects.

N. Lather, R. Bhardwaj, etal (2024) the time period of a tapered parallelogram-shaped plate with an exponential profile in Young's modulus is determined by solving the frequency equation of the system, typically using the Rayleigh-Ritz method. This involves considering the plate's geometry (parallelogram shape and tapering), the exponential variation in Young's modulus, and the chosen boundary conditions (e.g., clamped, simply supported). The solution provides the natural frequencies of vibration, and the time period is the inverse of the frequency [1]. T A., Kaushal ,M.K.Dhiman, & P. Prashar(2024)studied the vibration of a visco-elastic square plate with CCCC (clamped-clamped-clamped-clamped) boundary conditions, circular thickness variation, and bi-linear temperature variation, you would need to formulate and solve the governing equations of motion. This involves considering the plate's material properties (visco-elasticity), geometry (square shape, circular thickness variation), and thermal environment (bi-linear temperature variation) [2]. A., Kaushal ,M.K.Dhiman, & P.Prashar2024 A bi-linear temperature variation and a circular thickness variation in an isotropic viscoelastic square plate significantly impact its vibration characteristics. The temperature variation affects the material properties, which in turn influences the natural frequencies and mode shapes of the plate. Similarly, the circular thickness variation, where the thickness changes with radial distance, alters the stiffness of the plate, leading to changes in its vibration behavior [3]. A.K. Sharma and V. Verma(2023) , In the present paper, the author investigates the free vibration behavior of an isotropic parallelogram plate subjected to bi-linear thickness variation and bi-parabolic temperature distribution along both in-plane directions. The analysis focuses on determining the natural frequencies corresponding to the primary two modes of vibration for a simply supported parallelogram plate. The influence of key parameters—including aspect ratio, skew angle, thermal gradient, and taper constants—on the frequency characteristics is thoroughly examined [4]. A.K. Sharma and V. Verma(2023) This research focuses on analyzing the vibration behavior of a non-homogeneous, parallelogram-shaped plate with spatially varying thickness and a non-uniform temperature field. The thickness variation is modeled using a circular distribution, while the temperature variation follows a specified field that varies along both in-plane directions. The study incorporates critical factors such as the skew angle of the plate, material non-homogeneity, and thermal gradients to understand their combined effects on the dynamic response of the structure [5]. A. K. Sharma and M. K. Dhiman (2023) In the present paper, the authors propose a simplified model to study the effect of material non-homogeneity on the vibration behavior of a parallelogram-shaped plate with bi-linear thickness variation. The thermal-induced vibration is modeled using a bi-linear temperature distribution across the plate surface. To represent material non-homogeneity, the density of the plate is assumed to vary linearly along a specified direction. The governing differential equation for the plate's vibration is derived based on classical plate theory and solved using **the** method of separation of variables**,** allowing for analytical insight into the influence of thickness variation, thermal gradients, and material non-homogeneity on the natural frequencies of the plate[6]. A. Sharma (2023) In this paper, the author investigates the natural vibration behavior of a non-homogeneous tapered parallelogram plate under varying temperature conditions. The plate's tapering and material non-homogeneity are modeled using a two-dimensional linear variation in thickness and a one-dimensional circular variation in density. Additionally, a two-dimensional linear temperature distribution is assumed across the plate. To determine the vibrational frequencies, the Rayleigh–Ritz method is employed, considering clamped boundary conditions [9].

The aim of the present study is to investigate the effect of bi-parabolic temperature variation on the vibration characteristics of non-homogeneous parallelogram plates with linearly varying thickness in one direction. The plates are considered to have clamped-simply supported-clamped-simply supported (CSCS) boundary conditions on all four edges. Due to the imposed temperature gradient, it is assumed that the non-homogeneity is introduced through variation in the modulus of elasticity. The natural frequencies corresponding to the first two vibration modes are computed for various values of the tapering constant, material non-homogeneity, thermal gradient, and aspect ratio. The results are presented in the form of tables and graphs for better visualization and comparison.

2. Geometry and Analytic Approach

The parallelogram plate R with skew angle θ and sides a,b be shown in figure 1. Since it is a special case of rectangular plate, we take $\theta = 0^{o}$. The plate is taken to be orthotropic and non-uniform.

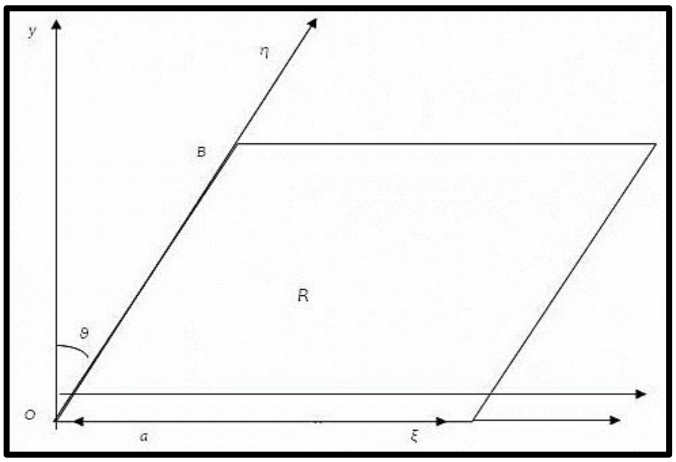

Figure1 (The parallelogram plate R)

.Here

$$\xi = x - y\tan\theta,\ \eta = y\sec\theta \quad (1)$$

Displacement $W(\xi, \eta, t)$ for free vibration of the parallelogram plate is given by

$$W(\xi, \eta, t) = W(\xi, \eta)T(t) \quad (2)$$

Here $W(\xi, \eta)$ is the maximum displacement at time t and T (t) is the time function.

2.1 Temperature Variation

The plate considered here is subjected to parabolic temperature distribution along ξ- and η-directions, then

$$\tau = \tau_0\left(1 - \frac{\xi^2}{a^2}\right)\left(1 - \frac{\eta^2}{b^2}\right) \quad (3)$$

where 'a' represents length, ' b' represents breadth and τ_0 is temperature at origin of the plate. For isotropic material, the temperature dependent modulus of elasticity is taken as

$$E_\xi(\tau) = E_1\,(1 - \gamma\,\tau),\; E_\eta(\tau) = E_2\,(1 - \gamma\,\tau),\; G_{\xi\eta}(\tau) = G_0\,(1 - \gamma\,\tau) \qquad (4)$$

where E_ξ and E_η are Young's moduli in ξ- and η- directions respectively, $G_{\xi\eta}$ is shear modulus and γ is taken as slope variation of moduli with temperature. Using eqn. (3) in eqn. (4) one has

$$E_\xi(\tau) = E_1\left[1 - \alpha(1 - \frac{\xi^2}{a^2})(1 - \frac{\eta^2}{b^2})\right],$$

$$E_\eta(\tau) = E_2\left[1 - \alpha(1 - \frac{\xi^2}{a^2})(1 - \frac{\eta^2}{b^2})\right],$$

$$G_{\xi\eta}(\tau) = G_0\left[1 - \alpha(1 - \frac{\xi^2}{a^2})(1 - \frac{\eta^2}{b^2})\right] \qquad (5)$$

where $\alpha = \gamma\,\tau_0$, ($0 \le \alpha < 1$) is thermal gradient.

2.2 Thickness Variation

The plate's thickness variation for the present study is to be assumed linearly in ξ- directions which is represented by

$$h = h_0\left[\left(1 + \beta\,\frac{\xi}{a}\right)\right] \qquad (6)$$

Here β is known as tapering constant in ξ- direction respectively and $h = h_0$ at $\xi = 0$.

The flexural rigidities (D_ξ , D_η) and torsional rigidity ($D_{\xi\eta}$) of the plate are taken as

$$D_\xi = \frac{E_\xi h^3}{12(1 - v_\xi\, v_\eta)}, \qquad D_\eta = \frac{E_\eta h^3}{12(1 - v_\xi\, v_\eta)},\; D_{\xi\eta} = \frac{G_{\xi\eta} h^3}{12},$$

$$D_1 = v_\xi\, D_\eta = v_\eta\, D_\xi,\; H = D_1 + 2\,D_{\xi\eta} \qquad (7)$$

where v_ξ , v_η are Poisson's ratio. Using eqns. (3.5) and (3.6) in eqn. (3.7), we have

$$\left.\begin{aligned} D_\xi &= \frac{E_1 {h_0}^3}{12(1-v_\xi v_\eta)}\left[\left\{1-\alpha\left(1-\frac{\xi^2}{a^2}\right)\left(1-\frac{\eta^2}{b^2}\right)\right\}\left\{\left(1+\beta\frac{\xi}{a}\right)\right\}^3\right], \\ D_\eta &= \frac{E_2 {h_0}^3}{12(1-v_\xi v_\eta)}\left[\left\{1-\alpha\left(1-\frac{\xi^2}{a^2}\right)\left(1-\frac{\eta^2}{b^2}\right)\right\}\left\{\left(1+\beta\frac{\xi}{a}\right)\right\}^3\right], \\ D_{\xi\eta} &= \frac{G_0 {h_0}^3}{12}\left[\left\{1-\alpha\left(1-\frac{\xi^2}{a^2}\right)\left(1-\frac{\eta^2}{b^2}\right)\right\}\left\{\left(1+\beta\frac{\xi}{a}\right)\right\}^3\right] \end{aligned}\right\} \quad (8)$$

For non-homogeneous material, linear variation taken in density is

$$\rho = \rho_0\left(1 - c_1\frac{\xi}{a}\right) \quad (9)$$

where c_1 ($0 \le c_1 < 1$) is non − homogeneity constant.

2.3 <u>Boundary conditions and frequency equation</u>

Boundary conditions for a non-homogeneous isotropic (CSCS) parallelogram plate are taken as

$$\left.\begin{aligned} W = W_{,\xi\xi} = 0 \text{ at } \xi = 0, a \\ W = W_{,\eta\eta} = 0 \text{ at } \eta = 0, b \end{aligned}\right\} \quad (10)$$

Two-term deflection function to satisfy the boundary conditions, can be taken as

$$W = \left[\left(\frac{\xi}{a}\right)^2\left(\frac{\eta}{b}\right)\left(1-\frac{\xi}{a}\right)^2\left(1-\frac{\eta}{b}\right)\right]\left[A_1 + A_2\left(\frac{\xi}{a}\right)\left(\frac{\eta}{b}\right)\left(1-\frac{\xi}{a}\right)\left(1-\frac{\eta}{b}\right)\right] \quad (11)$$

where A_1, A_2 are constants to satisfy boundary conditions.

Now, unit less non dimensional variables taken for our convince as

$$E_1^* = \frac{E_1}{1-v_\xi v_\eta}, \; E_2^* = \frac{E_2}{1-v_\xi v_\eta}, \; E^* = v_\xi E_2^* = v_\eta E_1^* \quad (12)$$

Components of E_1^*, E_2^* , E^* and G_0 are E_1^*, $E_2^*\sec\theta$, $E^*\sec\theta$ and $G_0\sec\theta$ respectively

in ξ- and η- directions.

The expressions for strain energy (V_E) and kinetic energy (T_E) are taken as

$$V_E = \frac{1}{2}\int_0^a\int_0^b \Big[D_\xi\big(W_{,\xi\xi}\big)^2 + D_\eta\big(W_{,\xi\xi}\tan^2\theta - 2W_{,\xi\eta}\sec\theta\tan\theta + W_{,\eta\eta}\sec^2\theta\big)^2 + 2D_1 W_{,\xi\xi}(W_{,\xi\xi}\tan^2\theta + 2W_{,\xi\eta}\sec\theta\tan\theta + W_{,\eta\eta}\sec^2\theta) + 4D_{\xi\eta}\big(-W_{,\xi\xi}\tan\theta + W_{,\xi\eta}\sec\theta\big)^2\Big]\cos\theta\, d\eta d\xi \quad (13)$$

and

$$T_E = \frac{1}{2}\, p^2 \int_0^a\int_0^b (\rho h W^2 \cos\theta)\, d\eta d\xi \quad (14)$$

2.5 <u>Solution by Rayleigh-Ritz Method</u>

The Rayleigh–Ritz technique is employed to determine the approximate natural frequencies of vibration. This method is based on the principle that the maximum strain energy (denoted as V_E) must be equal to the maximum kinetic energy (denoted as T_E) during free vibration. Using this approach, the following general form of the frequency equation is obtained

$$\delta(V_E - T_E) = 0 \quad (15)$$

Using eqns. (8), (12) in eqn. (13) and (14), then substituting the values of V_E and T_E in eqn. (15), we obtained

$$\delta\big(V_E^* - \lambda^2 T_E^*\big) = 0 \quad (16)$$

Here,

$$V_E^* = \int_0^a\int_0^b \left[\left\{1 - \alpha\left(1 - \frac{\xi^2}{a^2}\right)\left(1 - \frac{\eta^2}{b^2}\right)\right\}\left\{\left(1 + \beta\,\frac{\xi}{a}\right)\right\}^3\right]\Big[\Big\{\cos^4\theta + \frac{E_2^*}{E_1^*}\sin^4\theta + 2\frac{E^*}{E_1^*}\sin^2\theta\cos^2\theta + 4\frac{G_0}{E_1^*}\sin^2\theta\cos^2\theta\Big\}W^2{}_{,\xi\xi} + \frac{E_2^*}{E_1^*}W^2{}_{,\eta\eta} + 4\Big\{\frac{E_2^*}{E_1^*}\sin^2\theta + \frac{G_0}{E_1^*}\cos^2\theta\Big\}W^2{}_{,\xi\eta} + 2\Big\{\frac{E_2^*}{E_1^*}\sin^2\theta + \frac{E^*}{E_1^*}\cos^2\theta\Big\}W_{,\xi\xi}W_{,\eta\eta} - 4\Big\{\frac{E_2^*}{E_1^*}\sin^3\theta + 2\frac{E^*}{E_1^*}\sin\theta\cos^2\theta + 2\frac{G_0}{E_1^*}\sin\theta\cos^2\theta\Big\}W_{,\xi\xi}W_{,\xi\eta} - 4\Big\{\frac{E_2^*}{E_1^*}\sin\theta\Big\}W_{,\eta\eta}W_{,\xi\eta}\Big]\, d\eta d\xi \quad (17)$$

$$T_E^* = \int_0^a \int_0^b [\left(1 - c_1 \frac{\xi}{a}\right)\left\{\left(1 + \beta \frac{\xi}{a}\right)\right\}] W^2 \, d\eta d\xi \qquad (18)$$

and frequency $\lambda^2 = \frac{12p^2 \rho_0 a^2 \cos^5\theta}{E_1^* h_0^2}$

Now, the value of A_1 & A_2 is to be determined from (16) as

$$\frac{\partial(V_E^* - \lambda^2 T_E^*)}{\partial A_s} = 0, \qquad \text{for } s = 1, 2 \qquad (19)$$

On solving equation (19), we have

$$m_{s1}A_1 + m_{s2}A_2 = 0, \quad \text{for } s = 1, 2 \qquad (20)$$

Here m_{s1}, m_{s2} (s = 1,2) comprises parametric constant and the frequency parameter.

The determinant of the co-efficient of equation (20) must be zero, for non-trivial solution, we get the equation of frequency as follows

$$\begin{vmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{vmatrix} = 0 \qquad (21)$$

With the help of equation (21), we get quadratic equation in λ^2. We can obtain two roots of λ^2 from this equation. These roots give the first (λ_1) and second (λ_2) modes of vibration of frequency for various parameters of tapering constants, thermal gradient and aspect ratio for a simply supported plate.

2.6 Numerical Results and Discussion

The frequency (λ) for 1st and 2nd mode of vibration of an isotropic (simply supported) parallelogram plate has been calculated for various values of thermal constant (α), tapering constant (β), aspect ratio (a/b) and non-homogeneity constant (c_1). All the results are obtained by using MATLAB / MAPLE software. Following parameters are used for these calculations:

$\frac{E_2^*}{E_1^*} = 0.01$, $\frac{E^*}{E_1^*} = 0.3$, $\frac{G_0}{E_1^*} = 0.0333$, $\frac{E_1^*}{\rho} = 3.0 \times 10^5$, $h_0 = 0.01\text{m}$ and $\rho_0 = 0.345$

The results are given in tables [1 - 4] and figures [1 - 4].

Table-1 represents thermal gradient (α) versus frequency (λ) with fixed value of aspect ratio (a/b = 1) and different values of taper constants and non-homogeneity constant ($\beta = c_1 = 0, 0.4, 0.8$). It is apparent from Table-1 that as estimation of thermal gradient (α) increments from 0 to 0.8 corresponding frequency value (λ) for 1st and 2nd mode of vibration decreases. It is also clear from Table.1 that the values of frequency increases for both modes as skew angle (θ) varies from 30^o to 60^o.

Table-1 Thermal Gradient (α) Vs Frequency (λ)

α	$\beta = c_1=0, \theta = 30^o$		$\beta= c_1=0.4, \theta = 45^o$		$\beta = c_1=0.8, \theta = 60^o$	
	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0	4.32	14.06	15.13	49.09	41.16	134.53
0.2	4.12	13.38	14.58	47.36	39.79	130.49
0.4	3.91	12.67	13.99	45.57	38.35	126.32
0.6	3.68	11.91	13.37	43.71	36.81	122.02
0.8	3.44	11.11	12.71	41.77	35.17	117.58

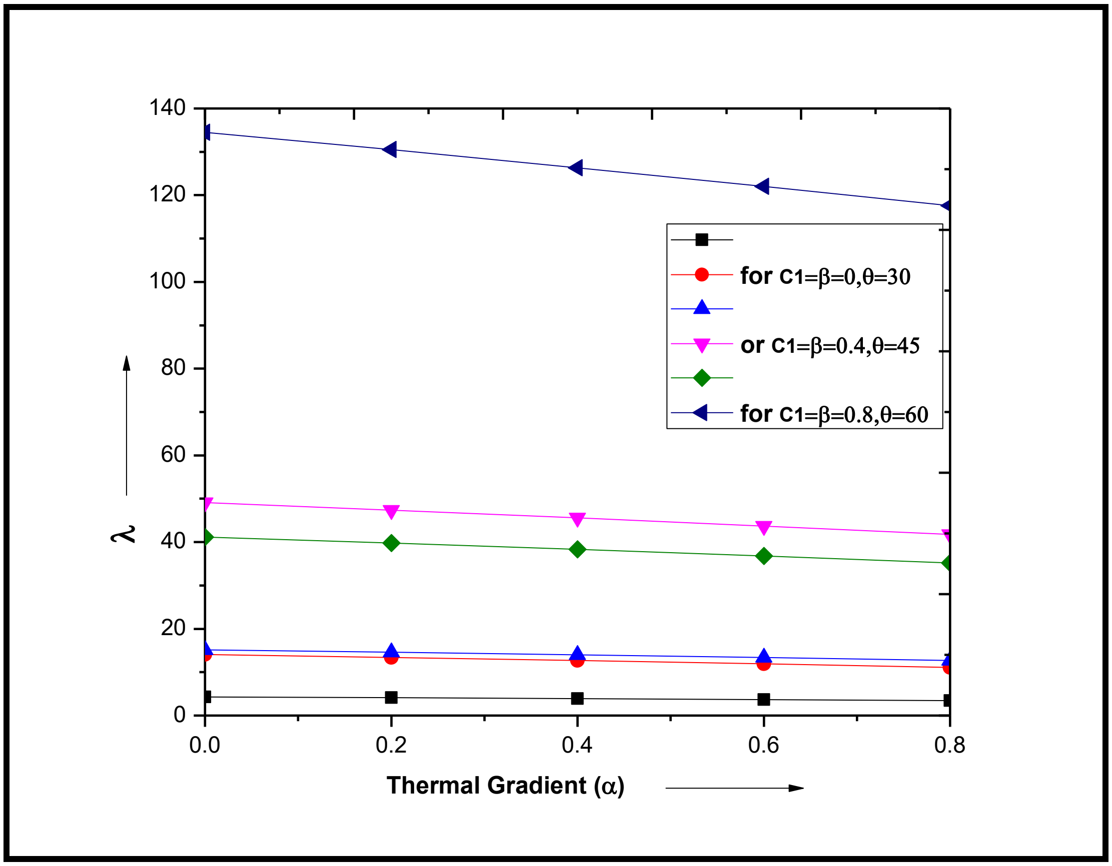

Figure-.1 Thermal Gradient Vs Frequency

<u>Table-2</u> represents taper constant (β) versus Frequency (λ) with fixed value of aspect ratio (a/b = 1) and different values of thermal gradient, tapering constant and non-homogeneity ($\alpha = c_1 = 0$, 0.4, 0.8). From Table-2 it is clear that as value of tapering constant (β) varies from 0 to 0.8 corresponding frequency value (λ) also increases for 1st and 2nd mode of vibration.

Table-2 Taper Constant (β) Vs Frequency (λ)

β	$\alpha = c_1 = 0, \theta = 30^o$		$\alpha = c_1 = 0.4, \theta = 45^o$		$\alpha = c_1 = 0.8, \theta = 60^o$	
	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0	4.32	14.06	11.23	36.44	22	71.25
0.2	4.78	15.55	12.55	40.79	25.07	81.83
0.4	5.28	17.19	13.99	45.57	28.33	93.22
0.6	5.81	18.93	15.51	50.65	31.70	105.19
0.8	6.37	20.74	17.08	55.96	35.17	117.58

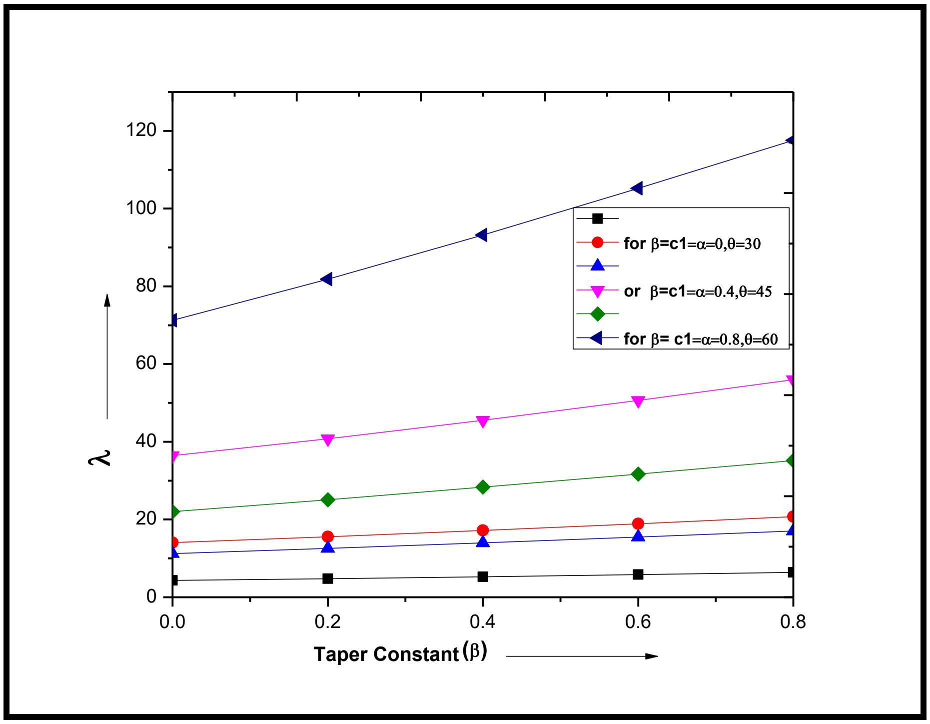

Figure-2 Taper Constant vs Frequency

Table-3 represents non-homogeneity constant (c_1) versus frequency (λ) with fixed value of aspect ratio (a/b = 1) and different values of tapering constants and thermal constant ($\beta = \alpha = 0$, 0.4, 0.8). It is evident from Table-3 that as value of non-homogeneity constant (c_1) varies from 0 to 0.8 corresponding value of frequency (λ) also increases for 1st and 2nd mode of vibration.

Table-3.3 Non-homogeneity constant (c_1) Vs Frequency (λ)

c_1	$\alpha = \beta = 0, \theta = 30^o$		$\alpha = \beta = 0.4, \theta = 45^o$		$\alpha = \beta = 0.8, \theta = 60^o$	
	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0	4.32	14.06	12.49	40.64	27.03	89.79
0.2	4.55	14.82	13.18	42.89	28.53	94.87
0.4	4.83	15.72	13.99	45.57	30.31	100.92
0.6	5.16	16.81	14.98	48.82	32.47	108.30
0.8	5.58	18.15	16.21	52.88	35.17	117.58

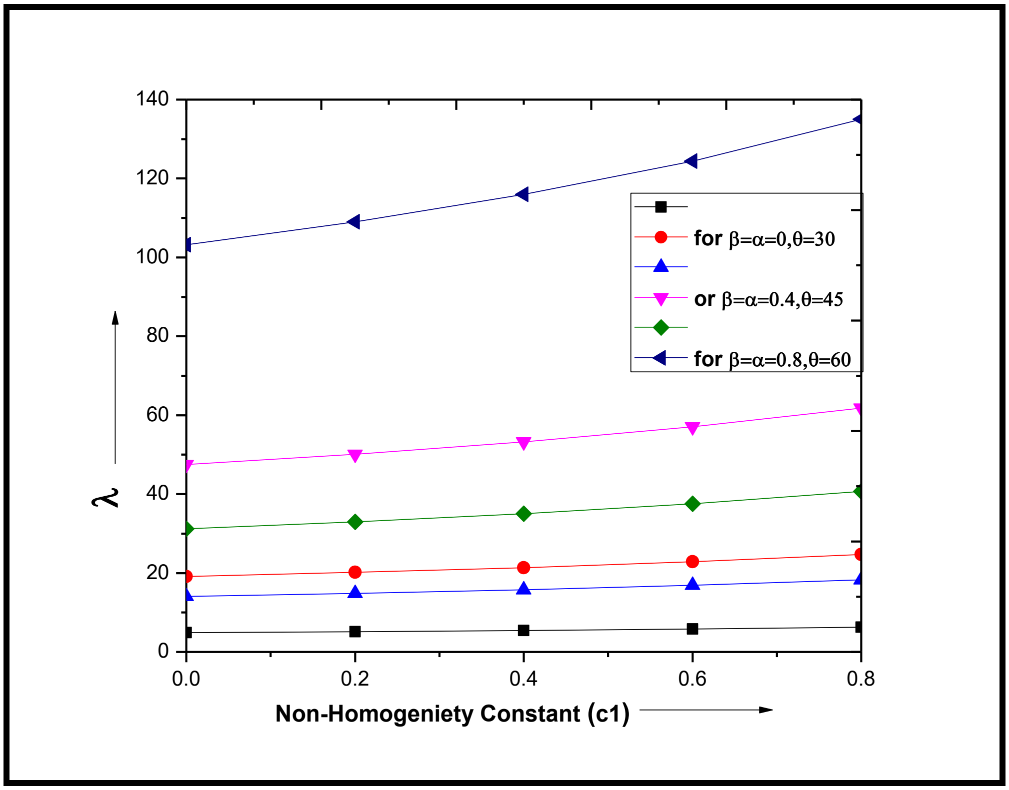

Figure-3 Non-homogeneity constant Vs Frequency

Table-4 represents aspect ratio (a/b) versus frequency (λ) with various values of tapering constants, thermal constant and non-homogeneity ($\beta = \alpha = c_1 = 0, 0.4, 0.8$). It is evident from Table-4 that as the value of aspect ratio increases from 0 to 1 corresponding value of frequency (λ) for 1^{st} and 2^{nd} mode of vibration decreases for both modes as skew angle (θ) varies from 30^o to 60^o.

Table-4 Aspect ratio (a/b) Vs Frequency (λ)

a/b	**$\alpha = \beta = c_1 = 0, \theta = 30^o$**		**$\alpha = \beta = c_1 = 0.4, \theta = 45^o$**		**$\alpha = \beta = c_1 = 0.8, \theta = 60^o$**	
	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0.2	108.09	351.65	349.93	1139.35	879.30	2939.74
0.4	27.02	87.91	87.48	284.83	219.82	734.93
0.6	12.01	39.07	38.88	126.59	97.70	326.63
0.8	6.75	21.97	21.87	71.20	54.95	183.73
1	4.32	14.06	13.99	45.57	35.17	117.58

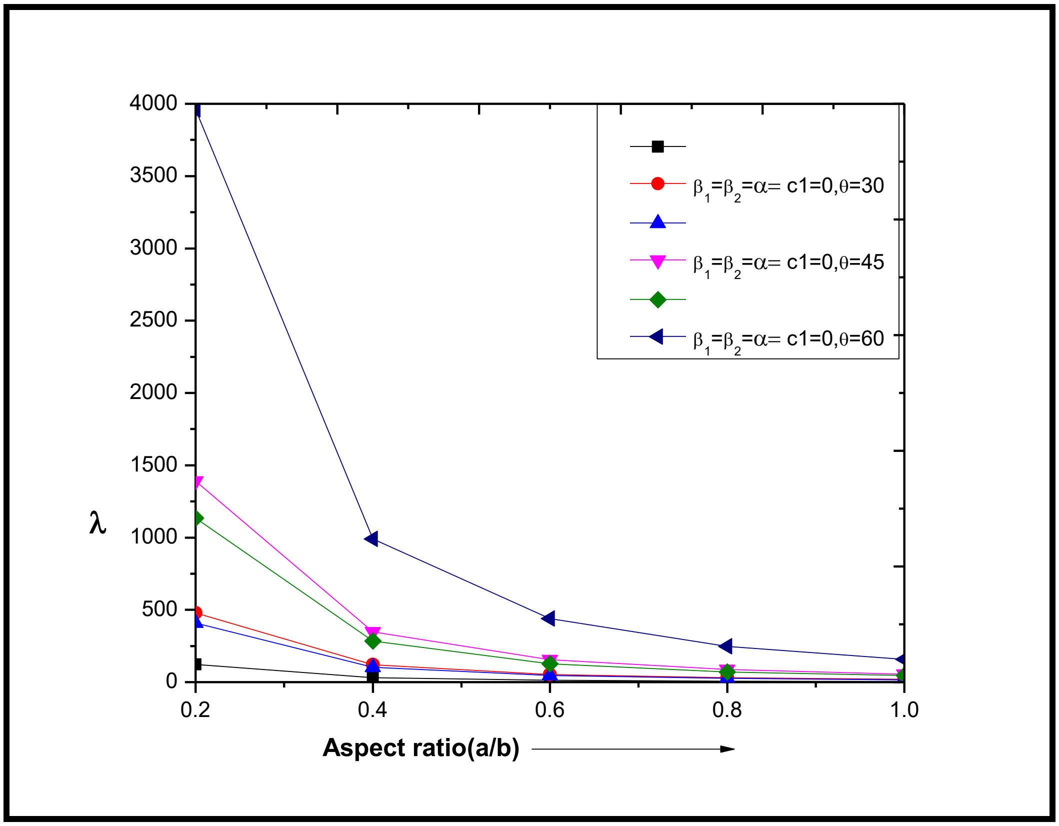

Figure-4 Aspect ratio (a/b) Vs Frequency (λ)

3. Conclusion

In this study, the free vibration behavior of a viscoelastic parallelogram plate with one-dimensional linear thickness variation under bi-parabolic temperature distribution has been investigated using the Rayleigh–Ritz technique. The analysis demonstrates that both the geometric tapering and non-uniform temperature field significantly influence the natural frequencies of the plate. Specifically, the bi-parabolic temperature variation alters the stiffness and damping characteristics of the material, thereby affecting the vibrational response. For this we considered the boundary condition C-S-C-S. The results also reveal that an increase in temperature gradient generally leads to a decrease in natural frequencies due to thermal softening. The adopted method proves to be effective for solving complex vibration problems involving variable material properties and non-uniform geometries. This study provides a useful foundation for the design and analysis of advanced structural components operating under thermal and mechanical loading conditions.

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