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"Inventory Model for Perishable Products with Time Dependent Production and Demand under Shortage Conditions"

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Abstract

This study develops an enhanced economic production quantity (EPQ) model for decaying items, incorporating time-dependent production and demand rates while allowing for shortages. The model assumes that items lifetime follow an exponential distribution. A comprehensive total cost function is formulated, accounting for relevant cost components. By minimizing this function, optimal production schedules and quantities are determined. Additionally, there is an increase in costs and key parameters.

Key words: Economic Production Quantity, deteriorating items, traditional models, cost production, demand

1.1 INTRODUCTION

This study presents an inventory model for deteriorating items, where the production rate varies as a function of time and the demand rate is constant, suitable for scenarios with stable demand. However, in many practical contexts, such as food processing industries, demand is time-dependent. Several researchers have explored inventory models with time-varying demand. Datta and Pal (1992) analyzed an inventory model with demand as a function of time, while Goyal and Giri (2001) and Ruxian et al. (2010) provided comprehensive reviews of inventory models for deteriorating items. Ritchie (1984) derived an exact solution for linearly increasing demand, and Raafat (1991) conducted a survey of literature on inventory models for deteriorating items. Dave (1981) developed a model for deteriorating items with time-proportional demand. Srinivasa Rao et al. (2011) proposed a production inventory system where the demand rate depends on production quantity. Kaliraman, Raj, Chandra, and Chaudhry (2015) and Ardak and Borade (2017) investigated an EPQ model for deteriorating items with Weibull deterioration and stock-dependent demand. Sujata Saha and Chakrabarti (2018) studied an EPQ model for deteriorating items with probabilistic demand and variable production rates. Khurana, Tayal, and Singh (2018) developed an EPQ model for deteriorating items with variable demand rates and allowable shortages. Majumder, Bera, and Maiti (2019) formulated an EPQ model under a trade credit policy. Aruna Kumari (2017) proposed a model for deteriorating items with time-dependent production and production-quantity-dependent demand. Janardan Rao et al. (2020) developed an economic lot size model with Weibull deterioration and on-hand inventory demand, incorporating allowable payment delays,. Dari and Sani (2020) introduced an EPQ model for delayed deteriorating items with quadratic demand and linear holding costs. Malumfashi (2021) proposed an EPQ model for delayed deteriorating items with variable production rates, two-phase demand, and shortages. Malumfashi (2022) further explored deteriorating items with a two-phase production period. This study builds on these foundations, addressing time-dependent production and demand with shortages for deteriorating items.Exponential Demand Rate and Linear Holding Cost. S. Sindhuja1 (2023) studies an inventory model for deteriorating products under preservation technology with time- dependent quality demand. Jitendra Kaushik (2024) developed an inventory model for deteriorating items with ramp type demand pattern: a stock- dependent approach. Marija Cvetković (2025) is developed inventory model with price and trade credit demand for deteriorating items with shortages and preservation technology.

Very little work has been reported in literature regarding inventory models for deteriorating items with demand as a function of time and production rate is time dependent. The time dependent demand can be well characterized by a power pattern demand which is of the form $\phi(t)=\frac{ft^{\frac{1}{q}-1}}{qT^{\frac{1}{q}}}$, where f is the total demand, $\phi(t)$ is demand rate at time t, m is the index parameter, and T is the cycle length. For different values of m this function includes various patterns of demand. If m = 1, this reduces to the constant rate of demand. This also includes increasing or decreasing rates of demand. Hence, in this chapter an economic production quantity (EPQ) model for deteriorating item is developed with the assumption that the production and demand rates are functions of time and follows power pattern.

Differential equations are employed to derive and analyze the instantaneous inventory state, accounting for shortages. By incorporating appropriate cost factors, the total cost function is formulated. Optimal production quantity, downtime, and uptime are determined by minimizing this cost function. The model's sensitivity to changes in parameters and costs is evaluated. As a special case, an inventory model with time-dependent production and demand rates, excluding shortages, is also investigated.

1.2 ASSUMPTIONS OF THE MODEL

For developing the model the following assumptions are made.

(i) The demand rate $\phi(t)=\frac{f\,t^{\frac{1}{q}-1}}{qT^{\frac{1}{q}}}$, is a power function of time

where, f is the total demand, m is the index parameter. ϕ

If q =1 then $\phi(t)=\frac{f}{T}$, which gives constant demand.

If q = 0.5 than $\phi(t)=\frac{f\,t}{qT^{2}}$, which is linear function of time

For different values of m it includes various patterns of demand.

(ii) The rate of production R (t) is time dependent and follows a power pattern.

$R(t)=\frac{rt^{\frac{1}{p}-1}}{pT^{\frac{1}{p}}}$, where, r is the total production and n is the index parameter

If p =1, $R(t)=\frac{r}{T}$, which includes the finite rate of production

If p≠ 1, it includes increasing or decreasing rates of production.

(iii) Lead time is zero

(iv) Cycle length is known and fixed say T

(v) A deteriorated unit is lost, and there is no repair on replacement of the deteriorated unit
(vi) The life time of the commodity is a random variable and follows an exponential distribution. Then the instantaneous rate of deterioration is θ

1.3 PRODUCTION LEVEL INVENTORY MODEL WITH DEFICIENCY

In this model the stock level is zero at time t = 0. The stock level increases during the period (0, t_1) due to excess production after fulfilling the demand and deterioration. The production stops at time t_1 when the stock level reaches S. The inventory decreases gradually due to demand and deterioration in the interval (t_1, t_2). At time t_2 the inventory reaches zero and back orders accumulate during the period (t_2, t_3). At time t_3 the production again starts and fulfils the backlog after satisfying the demand during (t_3, T), the inventory level increasing.

Let I (t) be inventory level of the system at time t ($0 \le t \le T$).
The differential equations governing the instantaneous state of inventory over the cycle length T are

$$\frac{d}{dt}I(t)+\theta I(t)=\frac{r\,t^{\frac{1}{p}-1}}{T^{\frac{1}{n}}p}-\frac{f\,t^{\frac{1}{q}-1}}{qT^{\frac{1}{q}}}\,, \qquad 0\le t\le t_1 \qquad (1)$$

$$\frac{d}{dt}I(t)+\theta I(t)=-\frac{f\,t^{\frac{1}{q}-1}}{qT^{\frac{1}{q}}}\,, \qquad t_1\le t\le t_2 \qquad (2)$$

$$\frac{d}{dt}I(t)=-\frac{f\,t^{\frac{1}{q}-1}}{qT^{\frac{1}{q}}}\,, \qquad t_2\le t\le t_3 \qquad (3)$$

$$\frac{d}{dt}I(t)=\frac{r\,t^{\frac{1}{p}-1}}{T^{\frac{1}{n}}p}-\frac{f\,t^{\frac{1}{q}-1}}{qT^{\frac{1}{q}}}\,, \qquad t_3\le t\le T \qquad (4)$$

with the initial conditions
$I(t_1) = S$, $I(t_2) = 0$, and $I(T) = 0$
Solving the differential equations (1) to (4) we get the on hand inventory at time t as

$$I(t)=e^{-\theta t}\left[Se^{\theta t_1}-\int_t^{t_1}\left(\frac{r\,u^{\frac{1}{p}-1}}{pT^{\frac{1}{p}}}-\frac{f\,u^{\frac{1}{q}-1}}{qT^{\frac{1}{q}}}\right)e^{\theta u}du\right], \qquad 0\le t\le t_1 \qquad (5)$$

$$I(t)=e^{-\theta t}\left[Se^{\theta t_1}-\frac{f}{qT^{\frac{1}{q}}}\int_t^{t_1}u^{\frac{1}{q}-1}e^{\theta u}du\right], \qquad t_1\le t\le t_2 \qquad (6)$$

$$I(t)=\frac{f}{T^{\frac{1}{q}}}\left(t_2^{\frac{1}{q}}-t^{\frac{1}{q}}\right), \qquad t_2\le t\le t_3 \qquad (7)$$

$$I(t)=f\left(1-\frac{t^{\frac{1}{q}}}{T^{\frac{1}{q}}}\right)+\frac{r}{T^{\frac{1}{p}}}\left(t^{\frac{1}{p}}-T^{\frac{1}{p}}\right), \qquad t_3\le t\le T \qquad (8)$$

The stock loss due to deterioration in the interval (0, t) is

$$L(t)=\int_0^t R(t)dt-\int_0^t \frac{f\,t^{\frac{1}{q}-1}}{qT^{\frac{1}{q}}}dt-I(t)$$

This implies

$$L(t)\;=\;\frac{r\,t^{\frac{1}{p}}}{T^{\frac{1}{p}}}-\frac{f\,t^{\frac{1}{q}}}{T^{\frac{1}{q}}}-e^{-\theta\,t}[Se^{\theta\,t_1}-\int_t^{t_1}(\frac{r\,u^{\frac{1}{p}-1}}{T^{\frac{1}{n}}p}-\frac{fu^{\frac{1}{q}-1}}{qT^{\frac{1}{q}}})e^{\theta u}du]\;,\qquad 0\le t\le t_1$$

$$=\;\frac{r\,t_1^{\frac{1}{p}}}{T^{\frac{1}{p}}}-\frac{f\,t^{\frac{1}{q}}}{T^{\frac{1}{q}}}-e^{-\theta\,t}(Se^{\theta\,t_1}-\frac{f}{qT^{\frac{1}{q}}}\int_{t_1}^{t}u^{\frac{1}{q}-1}e^{\theta\,u}du)\;,\qquad t_1\le t\le t_2$$

Therefore the stock loss deterioration in the cycle length T is

$$L(T)=\frac{r\,t_1^{\frac{1}{p}}}{T^{\frac{1}{p}}}-\frac{f\,t_2^{\frac{1}{q}}}{T^{\frac{1}{q}}}\quad.$$

,

The production quantity Q in the cycle of length T is

$$Q=\int_0^{t_1}R(t)dt+\int_{t_3}^{T}R(t)dt$$

$$=\frac{r}{T^{\frac{1}{p}}}[t_1^{\frac{1}{p}}+T^{\frac{1}{p}}-t_3^{\frac{1}{p}}]\;. \tag{9}$$

From the equation (5) and using the initial condition I (0) = 0, we get the value of S as

$$S=e^{-\theta\,t_1}\int_t^{t_1}\left(\frac{r\,u^{\frac{1}{p}-1}}{nT^{\frac{1}{p}}}-\frac{f\,u^{\frac{1}{q}-1}}{qT^{\frac{1}{q}}}\right)e^{\theta\,u}du\quad.$$

Using the Taylor's series expansion for small values of θ and ignoring higher order terms of θ, we get

$$S=e^{-\theta t_1}\left[\frac{r}{T^{\frac{1}{p}}}(t_1^{\frac{1}{p}}+\frac{\theta\,t_1^{\frac{1}{p}+1}}{p+1})-\frac{f}{T^{\frac{1}{q}}}(t_1^{\frac{1}{q}}+\frac{\theta\,t_1^{\frac{1}{q}+1}}{q+1})\right]\quad. \tag{10}$$

From equation (6) and using the condition I (t_2) = 0, implies

$$Se^{\theta\,t_1}-\frac{f}{qT^{\frac{1}{q}}}\int_{t_1}^{t_2}u^{\frac{1}{q}-1}e^{\theta\,u}du=0.$$

$$t_2\left(1+\frac{\theta\,t_2}{q+1}\right)^q=\left(\frac{T^{\frac{1}{q}}}{f}Se^{\theta\,t_1}+t_1^{\frac{1}{q}}+\frac{\theta\,t_1^{\frac{1}{q}+1}}{q+1}\right)^q \tag{11}$$

Substituting the value of S from equation (10) in equation (11) we get t_2 from the equation

$$t_2\left(1+\frac{\theta\, t_2}{p+1}\right)^q=\left[\frac{rT^{\frac{1}{q}}}{fT^{\frac{1}{p}}}\left(t_1^{\frac{1}{p}}+\frac{\theta\, t_1^{\frac{1}{p}+1}}{p+1`}\right)\right]^q$$

$$t_2\left(1+\frac{\theta\, t_2}{q+1}\right)^q=[X(t_1)]^q$$

where, $$X(t_1)=\frac{r\,T^{\frac{1}{q}}}{T^{\frac{1}{p}}f}\left(t_1^{\frac{1}{p}}+\frac{\theta\, t_1^{\frac{1}{p}+1}}{p+1}\right). \qquad (12)$$

Considering the bionamial expansion of $\left(1+\frac{\theta\, t_2}{q+1}\right)^q$ and ignoring the higher order terms of θ, This

$$t_2\left(1+\frac{q\,\theta\, t_2}{q+1}\right)=\left[X(t_1)\right]^q$$

implies

$$t_2=\frac{q+1}{2q\theta}\left[\sqrt{1+\frac{4q\theta}{q+1}B^q(t_1)}-1\right]=X(t_1)\ \text{(say)} \quad . \qquad (13)$$

Taking t = t3, in the equations (7) and (8) and equating these we get

$$t_3=\left[\frac{f}{r}T^{\frac{1}{p}}\left(\frac{t_2^{\frac{1}{q}}}{T^{\frac{1}{q}}}-1\right)+T^{\frac{1}{p}}\right]^p \quad . \qquad (14)$$

Substituting the value of t2 in equation (14) we get

$$t_3=\left[\frac{f}{r}T^{\frac{1}{p}}\left(\frac{[Y(t_1)]^{\frac{1}{q}}}{T^{\frac{1}{q}}}-1\right)+T^{\frac{1}{p}}\right]^p \quad . \qquad (15)$$

This implies $t_3=(Z\,(t_1))^p$ (16)

where, $$H(t_1)=\frac{f\,T^{\frac{1}{p}}}{r}\left(\frac{[Y(t_1)]^{\frac{1}{q}}}{T^{\frac{1}{q}}}-1\right)+T^{\frac{1}{p}} \quad . \qquad (17)$$

Let K (t1, t2, t3) be the total cost per unit time. Since the total cost is sum of the setup cost, cost of the units, the inventory holding cost and shortage cost, the

K (t1, t2, t3) can be obtained as

$$K(t_1,t_2,t_3)=\frac{A}{T}+\frac{CQ}{T}+\frac{h}{T}\left[\int_0^{t_1}I(t)dt+\int_{t_1}^{t_2}I(t)dt\right]+\frac{\pi}{T}\left[\int_{t_2}^{t_3}-I(t)dt+\int_{t_3}^{T}-I(t)dt\right] \quad . \qquad (18)$$

Substituting the values of I (t) and Q from equations (5), (.6), (7), (8) and

(9) in equation (18), we get

$$K(t_1,t_2,t_3)=\frac{A}{T}+\frac{C\,r}{T^{\frac{p+1}{p}}}\left(T^{\frac{1}{p}}+t_1^{\frac{1}{p}}-t_3^{\frac{1}{p}}\right)+\frac{h}{T}\left\{\int_0^{t_1}e^{-\theta t}\left[Se^{\theta t_1}-\int_t^{t_1}\left(\frac{r}{pT^{\frac{1}{p}}}u^{\frac{1}{p}-1}-\frac{f\,u^{\frac{1}{q}-1}}{qT^{\frac{1}{q}}}\right)e^{\theta u}du\right]dt\right.$$

$$\left.+\int_{t_1}^{t_2}e^{-\theta t}\left[Se^{\theta t_1}-\frac{f}{qT^{\frac{1}{q}}}\int_{t_1}^{t}u^{\frac{1}{q}-1}e^{\theta u}du\right]dt\right\}+\left\{\frac{\pi}{T}\int_{t_2}^{t_3}\frac{f}{T^{\frac{1}{q}}}\left(t^{\frac{1}{q}}-t_2^{\frac{1}{q}}\right)dt\right.$$

$$\left.+\int_{t_3}^{T}\left[f\left(\frac{t^{\frac{1}{q}}}{T^{\frac{1}{q}}}-1\right)+\frac{r}{T^{\frac{1}{p}}}\left(T^{\frac{1}{p}}-t^{\frac{1}{p}}\right)\right]dt\right\}. \qquad (19)$$

Substituting the values of S, t₂, and t₃ from equations (10), (13) and (16) in equation (19), K (t₁, t₂, t₃)

becomes K (t₁),

$$K(t_1)=\frac{A}{T}+\frac{C\,r}{T^{\frac{p+1}{p}}}\left(T^{\frac{1}{p}}+t_1^{\frac{1}{p}}-Z(t_1)\right)+\frac{h}{T}\left\{\frac{r}{T^{\frac{1}{p}}\theta}\left(t_1^{\frac{1}{q}}+\frac{\theta\,t_1^{\frac{1}{q}+1}}{q+1}\right)(e^{-\theta t_1}-e^{-\theta D(t_1)})+\frac{r\,p}{T^{\frac{1}{p}}(p+1)}\right.$$

$$\left.\left(t_1^{\frac{1}{p}+1}-\frac{n\,\theta\,t_1^{\frac{1}{p}+2}}{(2p+1)}-\frac{\theta^2\,t_1^{\frac{1}{p}+3}}{(3p+1)}\right)-\frac{qf}{(q+1)T^{\frac{1}{q}}}\left((Y(t_1))^{\frac{1}{q}+1}-\frac{q\,\theta\,(Y(t_1))^{\frac{1}{q}+2}}{(2q+1)}-\frac{\theta^2\,(Y(t_1))^{\frac{1}{q}+3}}{(3q+1)}\right)\right\}$$

$$+\frac{\pi}{T}\left\{\frac{f}{(q+1)T^{\frac{1}{q}}}\left((Y(t_1))^{\frac{1}{q}+1}-(q+1)(Y(t_1))^{\frac{1}{q}}(H(t_1))^{p}\right)+f\left((Z(t_1))^{p}-\frac{T}{q+1}\right)\right.$$

$$\left.+\frac{r\,p}{T^{\frac{1}{p}}(p+1)}\left(\frac{1}{p}T^{\frac{1}{p}+1}+(Z(t_1))^{p+1}-(\frac{p+1}{p})T^{\frac{1}{p}}(Z(t_1))^{p}\right)\right\}. \qquad (20)$$

1.4 OPTIMAL POLICIES OF THE MODEL

In this section the optimal policies of the inventory system are derived. To find the optimal value of t₁, we minimize the total cost per unit time with respect to t₁. The conditions for optimal value of t₁ are

$$\frac{\partial K(t_1)}{\partial t_1}=0 \text{ and } \frac{\partial^2 K(t_1)}{\partial t_1^{\,2}}>0$$

,

Differentiating K(t_1) with respect to t_1 and equate to zero, we get

$$\frac{C\,r}{pT^{\frac{1}{p}}}\left[t_1^{\frac{1}{p}-1}-(Z(t_1))^{1-p}\,W(t_1)\right]+h\left\{\frac{r}{T^{\frac{1}{p}}}\left(t_1^{\frac{1}{p}}+\frac{\theta\, t_1^{\frac{1}{p}+1}}{(p+1)}\right)\left(e^{-\theta\,(D\,(t_1))}V(t_1)-e^{-\theta\, t_1}\right)+\frac{r}{pT^{\frac{1}{p}}}\left(e^{-\theta\, t_1}-e^{-\theta\, D\,(t_1)}\right)\right.$$

$$\left.\left(\frac{t_1^{\frac{1}{p}-1}}{\theta}+t_1^{\frac{1}{p}}\right)+\frac{r}{T^{\frac{1}{n}}}\left(t_1^{\frac{1}{p}}-\frac{\theta\, p}{(p+1)}t^{\frac{1}{p}+1}-\frac{\theta^2}{(p+1)}t_1^{\frac{1}{p}+2}\right)-\frac{f\,V\,(t_1)}{T^{\frac{1}{p}}}\left((Y\;(t_1))^{\frac{1}{q}}-\frac{q\,\theta}{(q+1)}(Y(t_1))^{\frac{1}{q}+1}-\frac{\theta^2}{(q+1)}(Y(t_1))^{\frac{1}{q}+2}\right)\right\}$$

$$+\;\frac{\pi}{T}\left\{\frac{f}{T^{\frac{1}{q}}}[\frac{1}{q}(Y(t_1))^{\frac{1}{q}}\;V(t_1)-(Y\;(t_1))^{\frac{1}{q}}\;W(t_1)-q(Z(t_1))^{p}\;\;(Y(t_1))^{\frac{1}{q}-1}\;V(t_1)]\right.$$

$$\left.+\;f\,W(t_1)+\frac{r}{T^{\frac{1}{p}}}W(t_1)\left[Z(t_1)-T^{\frac{1}{p}}\right]\right\}=0\;\;. \tag{21}$$

where, $$V(t_1)=\frac{dt_2}{dt_1}=\frac{q\,r}{T^{\frac{1}{p}}p\,d}T^{\frac{1}{m}}\left(1+\frac{4\,q\,\theta}{q+1}X(t_1)\right)^{-\frac{1}{2}}X^{q-1}(t_1)\left(t_1^{\frac{1}{p}-1}+\theta\, t_1^{\frac{1}{p}}\right)$$

and $$W(t_1)=\frac{dt_3}{dt_1}=\frac{p\,f\,T^{\frac{1}{p}}}{r\,q\,T^{\frac{1}{q}}}(Z(t_1))^{p-1}(Y(t_1))^{\frac{1}{q}-1}V(t_1)\quad ,$$

X(t_1), Y(t_1) and Z(t_1) are as given in equations (12), (13) and (17) respectively.

Solving the equation (21) for t_1 using numerical methods, the optimal value of production downtime t_1^* of t_1 can be obtained. The optimum time, $t_3^*$of t_3 at which the production uptime is obtained by substituting the optimal value of t_1 in equation (16).

Therefore, the optimal time at which the production to be started is

$$t_3^{*}=\left[\frac{fT^{\frac{1}{p}}}{r}\left(\left(\frac{q+1}{2q\theta T}\left[\sqrt{1+\frac{4q\theta T}{q+1}(\frac{r}{T^{\frac{1}{p}}f})^{q}(t_1^{*\frac{1}{p}}+\frac{\theta t_1^{\frac{1}{p}+1}}{p+1})}-1\right]\right)^{\frac{1}{q}}-1\right)+T^{\frac{1}{p}}\right]^{n}\;. \tag{22}$$

The optimum production quantity Q^* of Q in the cycle of length T is obtained by substituting the optimal values of t_1 and t_3 in equation (9)

Therefore the optimal production quantity is

$$Q^{*}=\frac{r}{T^{\frac{1}{p}}}\left[t_1^{*\frac{1}{p}}-\frac{fT^{\frac{1}{p}}}{r}\left[\left(\frac{q+1}{2q\theta T}\left(1+\frac{4q\theta T}{q+1}\left(\frac{r}{T^{\frac{1}{p}}f}\right)^{m}\left(t_1^{*\frac{1}{p}}+\frac{\theta t_1^{*\frac{1}{p}+1}}{(p+1)}\right)=1\right)\right)^{\frac{1}{q}}-1\right]\right]. \tag{23}$$

1.5 NUMERICAL ILLUSTRATION

In this section, consider the case of deriving the optimal production quantity, production downtime and production uptime of an industry. Here it is assumed that the product is of deteriorating nature and shortages are allowed and fully backlogged. For demonstrating the solution procedure of the model the deteriorating parameter θ is considered to vary as 0.2, 0.3, 0.4, 0.5 and 0.6. The values of other parameters and costs associated with the model are

A = 1000, 1200, 1400, 1600, 1800; C = 25, 26, 27, 28, 29
h = 2.2, 2.5, 2.8, 3.1, 3.4 ; π = 3, 4, 5, 6, 7
p = 1.5. 1.6, 1.7, 1.8, 1.9 ; r = 125, 130, 135, 140, 145
θ = 0.2, 0.3, 0.4, 0.5, 0.6 ; q = 1.8.1.9, 2.0, 2.1, 2.2
f = 100, 101, 102, 103, 104 ; T = 12

Substituting these values the optimal production quantity Q^* , production downtime t_1*, production uptime t_3* and optimal cost of production K are computed and presented in Table 1.1. From Table 1.1 it is observed that the deterioration parameter and production parameters have a tremendous influence on the optimal values of the other parameters. As the deterioration parameter θ varies from 0.2 to 0.6, then the optimal production quantity Q^* decreases from 129.397 to 112.291, the optimal value of production down time t_1* decreases from 3.186 to 2.516, the optimal values of production uptime t_3* decreases from 8.616 to 6.622 and the total cost of production per unit time K increases from 318.414 to 321.254 units.

As the production parameter r increases from 150 units to 170 units, the optimal production quantity Q^* increases from 131.397 to 140.175 units, this increase is marginal. The optimal values of production downtime t_1^* increases from 3.986 to 4.262, the production uptime t_3^* also increases from 8.660 to 10.163, this increase is marginal. The total cost for a production per unit time K increases from 328.404 to 402.360 units. The indexing parameter of the production rate n increases from 2.0 to 2.8 then the optimal value of production quantity increases from 131.397 to 135.286. The optimal production downtime t_1^* decreases from 3.986 to 3.738 and the optimal value of production uptime t_3^* decreases from 8.660 to 8.121, the optimal value of production cost per unit time K increases from 318.414 t0 321.254.

The demand parameter g increases from 100 to 104 units then the optimal values of production downtime t_1* decreases from 3.986 to 3.895 and production uptime t_3^* decreases from 8.660 to 8.155 and production quantity Q^* increases from 131.397 to 134.126, this increase is marginal. The cost parameter C increases from 20 units to 24 units, the optimal values of production quantity Q^* increases from 131.397 to 132.918 units, this increase is marginal. The optimal values of production down time t_1^* increases from 3.986 to 4.205, the production uptime t_3^* increases from 8.660 to 8.945, the total cost of production per unit time K increases from 318.412 to 364.170 units.

Table-1.1

OPTIMAL VALUES OF t_1^*, t_3^*, Q^*, AND K

C	R	p	h	θ	f	q	π	A	T	t_1*	t_3*	Q*	K
25	150	1.5	2.2	0.2	100	1.8	3	1000	12	3.186	8.616	129.397	318.414
26									12	4.043	8.734	131.797	329.405
27									12	4.098	8.806	132.180	341.209
28									12	4.152	8.876	132.553	352.657
29									12	4.205	8.945	132.918	364.170
	155								12	4.062	9.072	133.554	343.514
	160								12	4.133	9.459	135.736	360.744
	165								12	4.199	9.821	137.940	380.255
	170								12	4.262	10.163	140.175	402.360
		1.6							12	3.800	8.274	133.916	351.577
		1.7							12	3.768	8.197	134.384	355.504
		1.8							12	3.738	8.121	134.842	359.255
		1.9							12	3.709	8.045	135.286	362.805
			2.5						12	3.768	8.375	129.845	332.525
			2.8						12	3.667	8.242	129.111	335.034
			3.1						12	3.580	8.128	128.472	337.476
			3.4						12	3.506	8.030	127.923	339.961
				0.3					12	3.422	7.887	127.101	328.478
				0.4					12	3.121	7.152	125.509	327.185
				0.5					12	2.532	6.851	120.715	325.215
				0.6					12	2.516	6.622	112.291	321.254
					101				12	3.964	8.534	132.081	326.765
					102				12	3.941	8.408	132.760	325.247
					103				12	3.918	8.281	133.442	323.881
					104				12	3.895	8.155	134.126	322.657
						1.9			12	3.917	8.728	131.008	324.687
						2.0			12	3.850	8.786	130.601	320.969
						2.1			12	3.784	8.833	130.174	317.228
						2.2			12	3.718	8.870	129.726	313.448
							4		12	3.971	8.640	131.292	329.313
							5		12	3.955	8.619	131.179	330.228
							6		12	3.939	8.599	131.066	331.187
							7		12	3.923	8.578	130.953	332.191
								1200	12	3.986	8.660	131.397	370.070
								1400	12	3.986	8.660	131.397	411.737
								1600	12	3.986	8.660	131.397	453.404
								1800	12	3.986	8.660	131.397	495.070

The holding cost h increases from 2.0 to 3.4, the optimal production quantity Q^* decreases from 131.397 to 127.923, the optimal value of production down time t_1^* decreases from 3.986 to 3.506, the optimal value of production uptime t_3^* decreases from 8.66 to 8.03, and the total cost of production per unit time K increases from 328.404 to 339.961 units. The shortage cost π increases from 4 to 8, then the optimal production quantity Q^* decreases from 131.397 to 130.953, the optimal value of the production downtime t_1^* decreases from 3.986 to 3.923, the optimal value of production uptime t_3^* decreases from 8.66 to 8.578 and the total cost of production per unit time K increases from 328.404 to 332.191 units. When as the setup cost A increases from 1000 to 3000 there is no effect of change in optimal production quantity Q^*, production down time t_1^*, and production uptime t_3^*, where as the total cost function K is increasing from 328.404 to 495.07 units, this increase is marginal.

1.6 SENSITIVITY ANALYSIS OF THE MODEL

A sensitivity analysis is carried out to explore the effect of changes in parameters and costs on the optimal policies by varying each parameter (-15%, -10%, -5%, 5%, 10%, 15%) at a time and all parameters together for the model under study. The results are obtained and presented in Table 1.2.The relationship between the parameters, cost on the optimal values of the production schedule are shown in Figure 1.1. From Table 1.2 it is observed that the variation in the deterioration parameter θ and the demand parameter f, m have significant influence on optimal production quantity Q^*.

Table – 1.2

SENSITIVITY ANALYSIS OF THE MODEL WITH RESPECT TO PARAMETERS AND COSTS

Variation in parameters	Optimal policies	Percentage Change in Parameters						
		-15	-10	-5	0	5	10	15
C	t_1^*	3.817	3.859	3.918	3.976	4.023	4.078	4.162
	t_3^*	8.325	8.417	8.484	8.560	8.634	8.706	8.776
	Q^*	131.156	131.559	131.988	132397	132.797	133.184	133.553
	K	295.194	304.680	317.061	328.404	339.805	351.219	362.681
r	t_1^*	3.561	3.626	3.763	3.886	4.198	4.199	4.292
	t_3^*	6.153	7.359	7.998	8.666	9.468	9.921	10.926
	Q^*	123.072	126.117	129.224	133.297	134.141	136.194	141.321
	K	281.137	296.87	319.122	329.414	371.147	381.265	414.454
p	t_1^*	4.034	4.071	4.012	3.786	3.744	3.805	3.061
	t_3^*	8.413	8.219	8.140	8.060	8.782	8.604	8.527
	Q^*	310.432	316.381	323.043	328.404	333.531	338.412	343.032
	Q	127.937	129.256	130.179	131.397	131.918	132.434	132.939
h	t_1^*	4.181	4.109	4.060	3.886	3.737	3.690	3.547
	t_3^*	8.887	8.806	8.730	8.660	8.596	8.535	8.478
	Q^*	132.615	132.18	131.776	131.397	131.052	130.719	130.412
	K	325.99	326.792	327.603	328.404	329.255	330.051	330.888
θ	t_1^*	4.253	4.190	4.089	3.886	3.782	3.685	3.677
	t_3^*	9.247	8.976	8.817	8.760	8.532	8.382	8.173
	Q^*	134.516	133.767	132.152	131.197	130.698	130.219	129.715
	K	329.001	328.716	328.462	328.424	328.374	328.303	328.261
f	t_1^*	4.331	4.215	4.101	3.986	3.872	3.757	3.642
	t_3^*	10.566	9.931	9.295	8.66	8.029	7.404	6.788
	Q^*	121.587	124.776	128.055	131.397	134.812	138.279	141.801
	K	381.641	356.554	339.583	328.404	321.568	317.858	316.45
q	t_1^*	4.222	4.136	4.059	3.986	3.917	3.850	3.784
	t_3^*	8.400	8.495	8.583	8.660	8.728	8.786	8.833
	Q^*	132.47	132.127	131.774	131.397	131.008	130.601	130.174
	K	339.918	335.961	332.172	328.404	324.687	320.969	317.228
π	t_1^*	3.996	3.993	3.990	3.986	3.983	3.980	3.977
	t_3^*	8.673	8.669	8.665	8.660	8.656	8.652	8.648
	Q^*	131.468	131.447	131.426	131.397	131.376	131.355	131.334
	K	327.918	328.092	328.267	328.404	328.582	328.763	328.944
A	t_1^*	3.986	3.986	3.986	3.986	3.986	3.986	3.986
	t_3^*	8.660	8.660	8.660	8.660	8.660	8.660	8.660
	Q	131.397	131.397	131.397	131.397	131.397	131.397	131.397
	K	315.904	320.07	324.237	328.404	332.57	336.737	340.904

As θ increases the production quantity Q* is decreasing and the production down time and production uptime are also decreasing when other parameters remain fixed. As f increases the optimal production quantity Q* is increasing and the production down time and uptime are decreasing. When demand parameter m is increasing the production quantity Q^* and the production downtime t_1^* time is decreasing. This decrease in both Q^* and t_1^* marginal. Whereas the production uptime t_3^* is increasing when m increases.

It is further observed that the costs are having a significant influence on the optimal production quantity and production schedules. As the penalty cost π increases the optimal values of the production quantity, production down time and production uptime are decreasing. This decrease is very low. When the cost per unit time C is increasing the optimal values of t_1^* and t_3^* are increasing. When the holding cost h increases the optimal production quantity Q^*, the optimal production down time and production up time are decreasing. There is no effect of change in set up cost on the optimal values of Q^*, t_1^* and t_3^*. However the total cost is increasing when A increases. The production rate parameters have significant influence on the optimal values of the production quantity Q^* and production down time t_1^* and production uptime t_3^*. As r increases the values of Q^*, t_1^* and t_3^* are increasing. This increase is rapid, as n increases the optimal values of t_1^* and t_3^* are decreasing. This decrease is marginal.

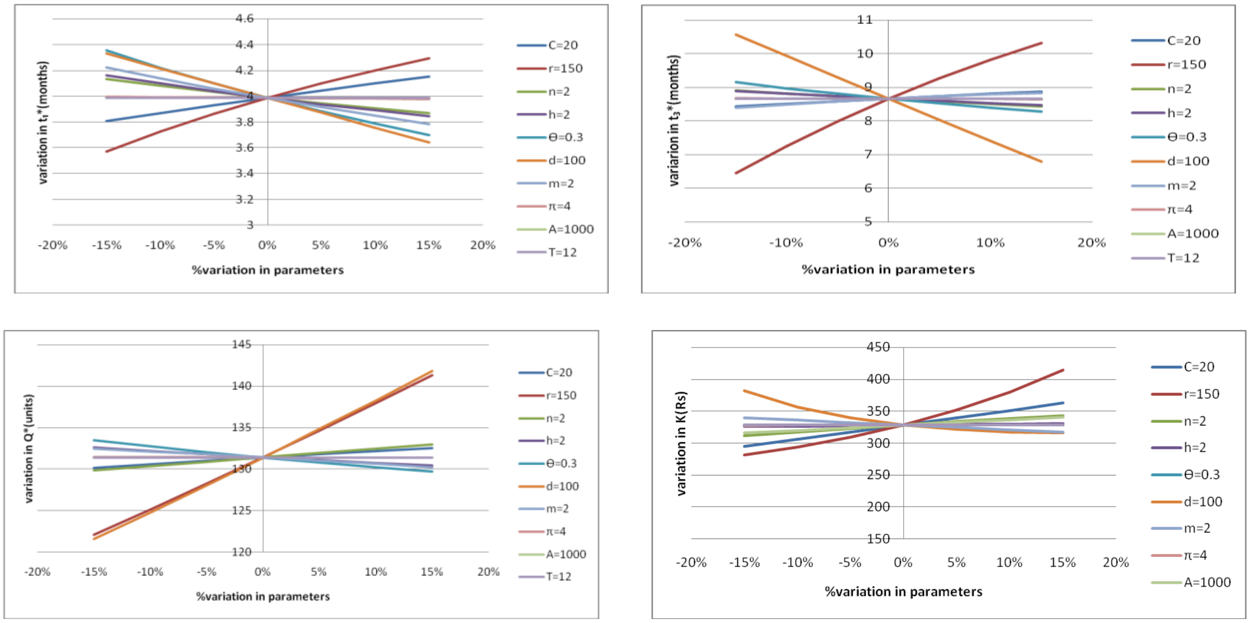

Fig. 1.2 The graphical representation of sensitivity analysis of production and demand – with shortages

1.6. Conclusion

A novel EPQ model featuring a time-dependent production rate is investigated in this study. The production is modeled to follow a power pattern, encompassing constant, increasing, or decreasing rates. The model permits shortages, which are fully backlogged, and derives the instantaneous inventory state accordingly. Numerical analysis indicates that the optimal production schedule and quantity are significantly affected by the deterioration rate, production rate, and demand rate. For specific parameter values, this model encapsulates earlier models as special cases. Future extensions could incorporate demand as a function of time and selling price, to be explored in subsequent research.

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