AUM block geometric mean labeling for tadpole, kayak paddle and bull graphs

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Abstract:

Graph labeling is an interesting area of mathematics with many real-world uses, such as in medical coding, networking, transportation, disease diagnosis, and cryptography. The AUM block geometric mean labeling is a recently introduced labeling. In this paper we obtain AUM block geometric mean labeling for tadpole, kayak paddle, and bull graphs. Suitable examples are given.

Keywords:

AUM block Geometric Mean labeling, tadpole graph, kayak paddle graphs, bull graph.

AMS classification: 05C78.

1. Introduction:

Graph labelling [1],[3],[4], an important concept in discrete mathematics, was first introduced by Rosa [5] in 1967. Since then, it has wide developments, with a comprehensive review later provided by Gallian. Over the years, various labeling techniques have been introduced, each labeling technique covers specific properties of graphs and enhance their applicability across different fields. One recent development in this area is the introduction of block labeling techniques. These labeling include methods such as AUM block sum labelling, AUM block labelling, AUM block mean labeling, AUM block prime distance labeling and AUM block geometric mean labeling [8],[9],[10],[11],[12],[13],[14],[15],[16],[17],[18],[19]. These labeling methods are not just theoretical exercises, they have practical applications in fields such medical coding and cryptography. The ability of these labeling techniques to real world system had made them more valuable.

In this paper, we extent the introduced new labeling technique AUM block geometric mean labeling[19] for tadpole graph [2], kayak paddle graphs[6] and bull graph[7]. By introducing this new labeling, we aim to enhance our understanding of these graphs and increasing their potential for real-world use and helping advance the study of graph theory.

2.Preliminaries

Definition: AUM block geometric mean labeling [19]

Let G be a graph with p vertices, q edges and b blocks, p, q, b ≥1.

Let $V(G) = \{v_1, v_2, \dots, v_p\}$, $E(G) = \{e_1, e_2, \dots, e_q\}$, $B(G) = \{B_1, B_2, \dots, B_b\}$ denote the vertex set, edge set and the block set of G respectively.

We say the graph G admits AUM block geometric mean labeling if there exists a bijection

Ψ: V$(G) \rightarrow Z^+$ and Ψ^*: $E(G) \rightarrow Z^+$ by $\Psi^*\ (uv)\ = \lfloor\sqrt{f(u)f(v)}\rfloor$ or $\lceil\sqrt{f(u)f(v)}\ \rceil$ induced from Ψ and Ψ^{**}: $B(G) \rightarrow Z^+$ defined as follows,

Let B$_k$ be incident with the vertices $v_{k_1}, v_{k_2}, \dots v_{k_n}$, $1 \leq\ k_n \leq p$ and the edges $e_{k_1}, e_{k_2}, \dots e_{k_m}$, $1 \leq\ k_m \leq q$ then$\Psi^{**}(B_k) =\ \left\lfloor\sqrt[s]{\prod_{i=1}^{n}\Psi(\ v_{k_i})\prod_{i=1}^{n}\Psi^*\left(e_{k_i}\right)}\right\rfloor$ or $\left\lceil\sqrt[s]{\prod_{i=1}^{n}\Psi(\ v_{k_i})\prod_{i=1}^{n}\Psi^*\left(e_{k_i}\right)}\ \right\rceil$ where, s = sum of the incident number of vertices and edges of B_k and $\Psi^{**}(B_k) \neq \Psi^{**}(B_z)$ for $1 \leq\ k\ , z \leq\ b, k\ \neq z$.

Definition: Tadpole $\boldsymbol{T(m,n)}$ [2]

Tadpole graph represents a cycle graph on m vertices and a path on n vertices connected with a bridge.

Definition: Kayak Paddles Graph [6]

Kayak paddles graph *KP*(*l*,*m*,*n*) is a graph made up of two cycles C_l and C_m having size *l* and m joined by a path of length *n.*

Definition: Bull Graph [7]

The Bull Graph is a planar undirected graph with 5 vertices and 5 edges, in the form of a triangle with two disjoint pendant edges.

3.Mains Results

In this section, we derive an AUM block geometric mean labeling for tadpole graph for two cases, kayak paddle for 4 variant cases and bull graph as well. Appropriate illustrations are given to prove the theorems.

Theorem 3.1: A tadpole $T(m,n)$ permits an AUM block geometric mean labeling

When $m = 3,4$.

Proof: Consider the dragon graph as G with $m = 3$.

Let $V(G) = \{v_1, v_2 \ldots v_{m+n}\}$, $E(G) = \{ e_1, e_2, \ldots e_{m+n}\}$, $B(G)= \{b_1, b_2, \ldots b_{m+1}\}$ in G, respectively.

AUM block geometric mean labeling is defined as follows,

Case 1: When $m = 3$.

Define $\Psi: V(G) \to Z^+$ by $\Psi(v_i) = 2\text{i} - 1,\ for\ 1 \le i \le m + 3$.

Ψ^* is defined as $E(G) \to Z^+$

$\Psi^*(v_1 v_{i+1}) = i, for\ i = 1,2$.

$\Psi^*(v_i v_{i+1}) = 2i - 1, for\ i = 2 \le i \le m + 2$.

Define $\Psi^{**}: B(G) \to Z^+$ by $\Psi^{**}(B_i) = 2i, for\ i = 1$.

$\Psi^{**}(B_{i+1}) = 2i + 3;\ 1 \le i \le m$.

Case 2: When $m = 4$.

Define $\Psi: V(G) \to Z^+$ by $\Psi(v_i) = 2\text{i} - 1,\ for\ 1 \le i \le m + 4$.

Ψ^* is defined as $E(G) \to Z^+$

$\Psi^*(v_1 v_{2i}) = i, for\ i = 1,2$.

$\Psi^*(v_i v_{i+1}) = 2i - 1, for\ i = 2 \le i \le m + 3$.

Define $\Psi^{**}: B(G) \to Z^+$ by $\Psi^{**}(B_i) = 2i, for\ i = 1$..

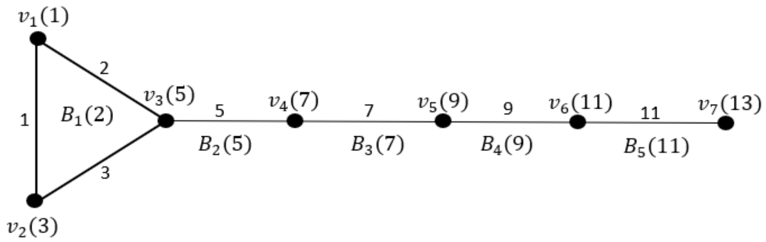
$\Psi^{**}(B_{i+1}) = 2i + 5;\ 1 \le i \le m$.

For $i \neq k$, $\Psi^{**}(B_i) \neq \Psi^{**}(B_k)$.

This shows that block labels are distinct.

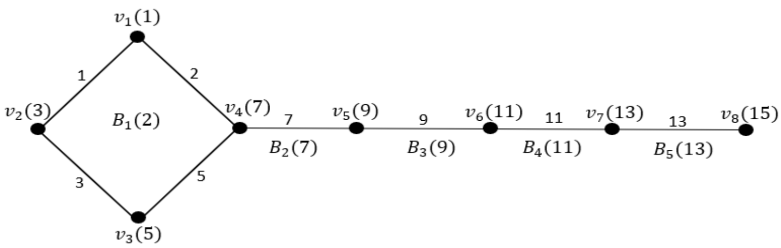
Hence, AUM block geometric mean labeling admits tadpole $T(3,n)$ *and* $T(4,n)$.

Example 3.2: $T(3,4)$



AUM block geometric mean labeling for tadpole graph $T(3,4)$

Example 3.3: $T(4,4)$



AUM block geometric mean labeling for tadpole $T(4,4)$

Note: A tadpole graph $T(m,n)$ does not satisfy AUM block geometric mean labeling when $m > 4$, as the edges are not distinct.

Theorem 3.4: A kayak paddle graph *KP* (*3,4, n*) admits AUM block geometric mean labeling.

Proof: Consider the kayak paddle graph *KP*(*l,m,n*) as G where l is the length of C_l and m is the length of C_m and n is the length of path.

Denote $S = l + m + n$.

Let $V(G) = \{v_1, v_2 \ldots v_S\}$, $E(G) = \{e_1, e_2, \ldots e_{S+1}\}$, $B(G) = \{b_1, b_2, \ldots b_{n+3}\}$ in G, respectively.

AUM block geometric mean labeling is defined as follows,

Define $\Psi: V(G) \rightarrow Z^+$ by $\Psi(v_i) = 2\text{i} - 1,\ for\ 1 \leq i \leq S$.

Ψ^* is defined as $E(G) \rightarrow Z^+$,

$\Psi^*(v_1 v_{i+1}) = i, for\ i = 1$.

$\Psi^*(v_i v_{i+1}) = 2i - 1, for\ 1 \leq i \leq s - 1$.

Ψ^* $(v_{n+4}v_{n+7}) = 2(n+5), for\ all\ n.$

Define Ψ^{**}: $B(G) \rightarrow Z^+$ by Ψ^{**} $(B_i) = 2i, for\ i = 1.$

Ψ^{**} $(B_{i+1}) = 2i + 3;\ 1 \leq i \leq n + 1.$

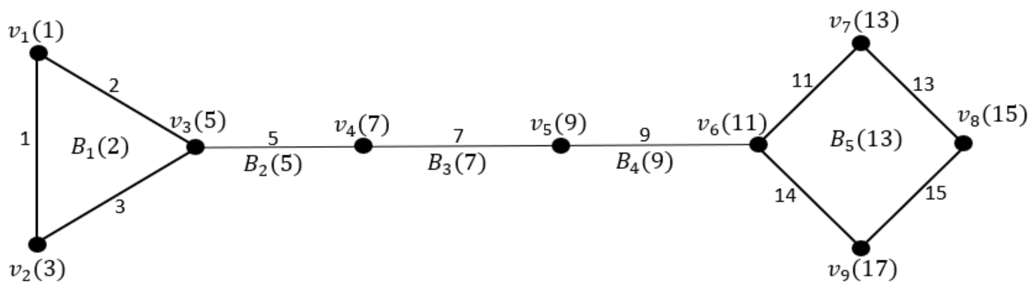
Ψ^{**} $(B_{n+3}) = 2n + 9, for\ all\ n.$

For $i \neq k$, $\Psi^{**}(B_i) \neq \Psi^{**}(B_k)$.

This shows that block labels are distinct.

Hence, A Kayak paddle graphs *KP* (*3,4, n*) admits AUM block geometric mean labeling.

Example 3.5: *KP* (*3,4, n*)



AUM block geometric mean labeling for kayak paddle $K(3,4,2)$

Theorem 3.6: A kayak paddle graph *KP* (*4,3, n*) admits AUM block geometric mean labeling.

Proof: Consider the kayak paddle graph *KP*(*l,m,n*) as G where l is the length of C_l and m is the length of C_m and n is the length of path.

Denote $S = l + m + n.$

Let $V(G) = \{v_1, v_2 \ \dots v_S\}$, $E(G) = \{e_1, e_2, \dots e_{S+1}\}$, $B(G) = \{b_1, b_2, \dots b_{n+3}\}$ in G, respectively.

AUM block geometric mean labeling is defined as follows,

Define Ψ: $V(G) \rightarrow Z^+$ by Ψ $(v_i) = 2\text{i} - 1,\ for\ 1 \leq i \leq S.$

Ψ^* is defined as $E(G) \rightarrow Z^+$,

Ψ^* $(v_1v_{2i}) = i, for\ i = 1.$

Ψ^* $(v_iv_{i+1}) = 2i - 1, for\ 1 \leq i \leq s - 1.$

Ψ^* $(v_{n+5}v_{n+7}) = 2(n+5), for\ all\ n.$

Define $\Psi^{**}: B(G) \to Z^+$ by $\Psi^{**}(B_i) = 2i, for\ i = 1.$

$\Psi^{**}(B_{i+1}) = 2i + 5;\ 1 \le i \le n+1.$

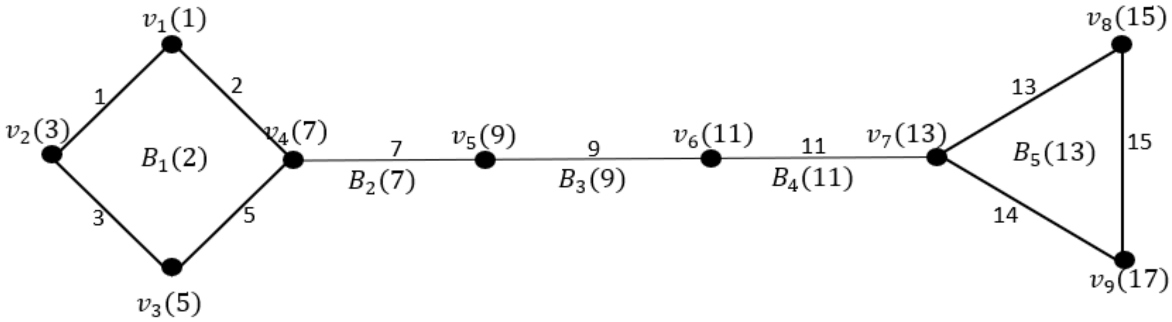
$\Psi^{**}(B_{n+3}) = 2n + 9, for\ all\ n.$

For $i \neq k$, $\Psi^{**}(B_i) \neq \Psi^{**}(B_k)$.

This shows that block labels are distinct.

Hence, A Kayak paddle graphs *KP* (*4,3, n*) admits AUM block geometric mean labeling.

Example 3.7:



AUM block geometric mean labeling for kayak paddle $K(4,3,2)$

Theorem 3.8: A kayak paddle graph *KP* (*l,m,n*) admits AUM block geometric mean labeling when both the cycles sizes are equal *to 3.*

Proof: Consider the kayak paddle graph *KP*(*l,m,n*) as G where l is the length of C_l and m is the length of C_m and n is the length of path.

Denote $S = l + m + n$.

Let $V(G) = \{v_1, v_2 \ldots v_S\}$, $E(G) = \{e_1, e_2, \ldots e_{S+1}\}$, $B(G) = \{b_1, b_2, \ldots b_{n+3}\}$ in G, respectively.

AUM block geometric mean labeling is defined as follows,

Define $\Psi: V(G) \to Z^+$ by $\Psi(v_i) = 2\text{i} - 1,\ for\ 1 \le i \le S.$

Ψ^* is defined as $E(G) \to Z^+$,

$\Psi^*(v_1 v_{i+1}) = i, for\ i = 1.$

$\Psi^*(v_i v_{i+1}) = 2i - 1, for\ 2 \le i \le s - 1.$

$\Psi^*(v_{n+4} v_{n+6}) = 2(n + 4), for\ all\ n.$

Define $\Psi^{**}: B(G) \to Z^+$ by $\Psi^{**}(B_i) = 2i, for\ i = 1.$

Ψ^{**} $(B_{i+1}) = 2i + 3;\ 1 \le i \le n + 1.$

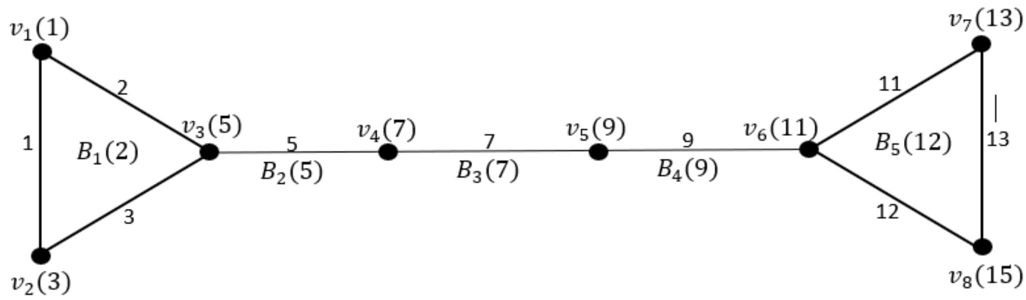
Ψ^{**} $(B_{n+3}) = 2(n + 4), for\ all\ n$.

For $i \neq k$, $\Psi^{**}(B_i) \neq \Psi^{**}(B_k)$.

This shows that block labels are distinct.

Hence, A Kayak paddle graphs KP (l, m, n) admits AUM block geometric mean labelingwhen both the cycle sizes are equal to 3.

Example 3.9:



AUM block geometric mean labeling for kayak paddle $K(3,3,2)$

Theorem 3.10: A kayak paddle graph KP (l,m,n) admits AUM block geometric mean labeling when both the cycles sizes are equal *to 4.*

Proof: Consider the kayak paddle graph KP(l,m,n) as G where l is the length of C_l and m is the length of C_m and n is the length of path.

Denote $S = l + m + n$.

Let $V(G) = \{ v_1, v_2\ \dots v_S\}$, $E(G) = \{ e_1, e_2, \dots e_{S+1}\}$, $B(G) = \{ b_1, b_2, \dots b_{n+3}\}$ in G, respectively.

AUM block geometric mean labeling is defined as follows,

Define Ψ: $V(G) \to Z^+$ by $\Psi\ (v_i) = 2\text{i} - 1,\ for\ 1 \le i \le S.$

Ψ^* is defined as $E(G) \to Z^+$,

Ψ^* $(v_1 v_{i+1}) = i, for\ i = 1.$

Ψ^* $(v_i v_{i+1}) = 2i - 1, for\ 1 \le i \le s - 1.$

Ψ^* $(v_{n+5} v_{n+8}) = 2(n + 6), for\ all\ n.$

Define Ψ^{**}: $B(G) \to Z^+$ by Ψ^{**} $(B_i) = 2i, for\ i = 1.$

Ψ^{**} $(B_{i+1}) = 2i + 5;\ 1 \le i \le n + 1.$

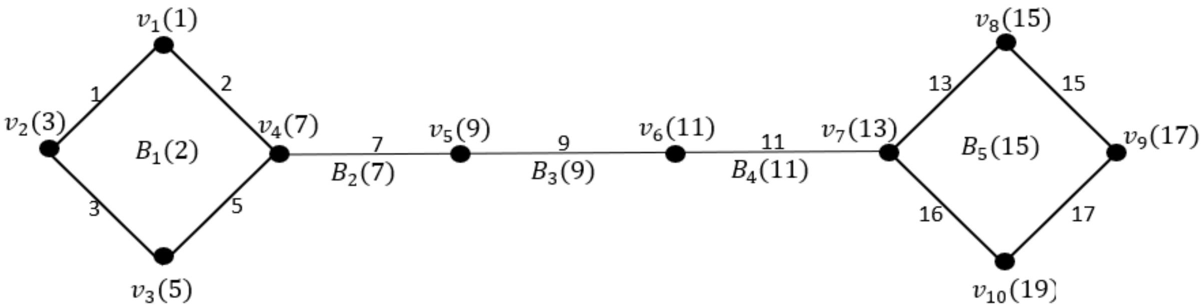
Ψ^{**} $(B_{n+3}) = 2n + 11, for\ all\ n$.

For $i \neq k$, $\Psi^{**}(B_i) \neq \Psi^{**}(B_k)$.

This shows that block labels are distinct.

Hence, A Kayak paddle graphs *KP* (*l, m, n*) admits AUM block geometric mean labeling when *l=m=4.*

Example: 3.11



AUM block geometric mean labeling for kayak paddle $K(4, 4, 2)$

Theorem 3.12: A Bull graph admits AUM block geometric mean labelling.

Proof: A bull graph admits AUM block geometric mean labeling.

Consider the bull graph as G Let $V(G) = \{ v_1, v_2 \ \dots v_n\}$, $E(G) = \{ e_1, e_2, \dots e_n\}$,

$B(G) = \{ b_1, b_2, \dots b_{n-2}\}$ in G, respectively.

AUM block geometric mean labeling is defined as follows,

Define Ψ: $V(G) \to Z^+$ by $\Psi\ (v_i) = 4\text{i} - 1,\ for\ 1 \le i \le n.$

Ψ^* is defined as $E(G) \to Z^+$

Ψ^* $(v_i v_{i+1}) = 4i, for\ i = 1 \le i \le n.$

Ψ^* $(v_{2i} v_{2(i+1)}) = 2(i + 4), for\ i = 1.$

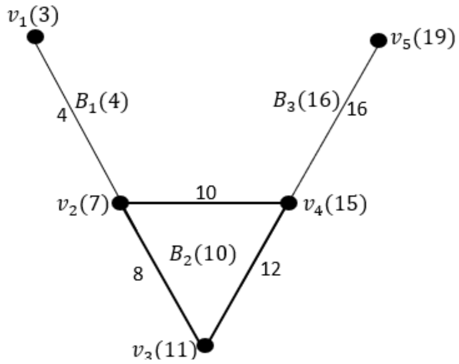
Define Ψ^{**}: $B(G) \to Z^+$ by Ψ^{**} $(B_i) = 2(3i - 1), for\ 1 \le i \le n - 2.$

For $i \neq k$, $\Psi^{**}(B_i) \neq \Psi^{**}(B_k)$.

This shows that block labels are distinct.

Hence, Bull graph admits AUM block geometric mean labelling.

Example: 3.13



AUM block geometric mean labeling for bull graph $n = 5$

Conclusion:

The AUM block geometric mean labeling is a recently introduced method. This paper demonstrates AUM block geometric mean labeling for tadpole, kayak paddle, and bull graphs. There is significant potential for further research on AUM block geometric mean labeling, particularly in exploring its application to various fields such as medicine, cryptography, and more.

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