**UtilitasMathematica**

**ISSN 0315-3681 Volume 122 (2), 2025**

# LAPLACIAN SPECTRUM IN STEREOPHONIC SYSTEMS

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**Abstract.** Contemporary innovations are deeply intertwined with mathematics, which provides the foundational tools for modeling and problem-solving. Real-world challenges can be effectively represented through graph-theoretic frameworks, enabling structured analysis and solution design. In the context of emerging electric vehicle technologies, the customization of stereophonic systems for enhancing travel experiences can be formulated using fuzzy graphs. Furthermore, this paper emphasizes on the application of Laplacian spectrum of complete fuzzy graphs for optimizing system performance.

**Keywords:** Fuzzy graphs, Degree matrix, Adjacency matrix, Laplacian matrix, Laplacian spectrum, Complete fuzzy graph.

**Mathematical Subject Classification (2020):** 05C72, 05C85

## 1. Introduction

In 2024, Wafaa Fakieh, Amal Alsaluli and Hanaa Alashwali discussed the Laplacian spectrum of unit graphs associated with the ring of integers [5]. Laplacian spectrum for token graphs was deliberated during 2021 [3]. Al-Hawary, Talal and Al-Shalaldeh, Sumaya and Akram, Muhammad discussed Certain matrices and energies of fuzzy graphs in 2023 [1].

Yong Peng, Xin Zhu, Feiping Nie, Wanzeng Kong, Yuan Ge conferred Fuzzy graph clustering in 2021 [6]. Shi, Kosari, Talebi, investigated the Main Energies of Picture Fuzzy Graph and its Applications in 2022 [4].

A review of the existing literature reveals that studies on the Laplacian spectrum have been relatively limited, with only a small body of work dedicated to this area. Within this narrow scope, investigations concerning the Laplacian spectrum of fuzzy graphs are even scarcer. This paucity of research is largely attributed to the intricate mathematical challenges associated with the computation of Laplacian spectrum in fuzzy environments. While traditional graph theory and its spectral properties have been widely explored, the extension of these concepts to fuzzy graphs remains underdeveloped. Consequently, there exists a significant research gap in applying Laplacian spectral techniques to fuzzy graph models, particularly in domains where uncertainty and imprecision are inherent. Addressing this gap not only contributes to the theoretical advancement of fuzzy spectral graph theory but also opens pathways for practical applications, such as optimizing systems in emerging technologies like electric vehicles by using the short cut method developed in finding the Laplacian spectrum [2].

## 2. Preliminaries

In the course of investigating Laplacian spectrum of complete fuzzy graphs were examined, leading to the development of an effective algorithm that directly utilizes vertex membership values to compute the Laplacian spectrum [2]. This would decrease the calculation part of finding its corresponding degree matrix, adjacency matrix, Laplacian matrix and then finding its eigen values. Using this algorithm in any vehicle’s stereophonic systems, makes the travel more comfortable for the travelling people.

## 3. Laplacian spectrum in stereophonic systems

Laplacian spectrum has enormous applications in various fields. It also helps in finding the dominant components that produces noise so as to reconstruct the original signal to reduce the impact of noise. Assembling a stereo system in a car needs to be pleasurable to ears and vibrations caused by the frequency while playing the stereo system should be minimized so as to relish a happy travelling. There are many types of cars, such as, hatchback, SUV, Sedan, Coupe, Convertible, Minivan Station Wagon etc.,

There are three frequency ranges

- High (2.5kHz to 20kHz)
- Medium (200Hz to 2.5kHz)
- Low (20Hz to 200Hz)

Each type of car needs different assembling of stereo system. Considering the maximum sound produced by speakers/woofers/passengers as vertex membership values ($\sigma$) and the frequency level between these vertices as edge membership values ($\mu$), a fuzzy Graph is being depicted. By finding its Laplacian spectrum we could adjust the frequency level of the volume of that corresponding speaker to minimize the vibration caused inside the vehicle. Eigen values is a number that tells us how the frequency is spread out on the line. If the eigen value is zero then the frequency level is stable, if it is less than one the factor does not amplify the effect of each vertex and if the eigen values are more than one then the corresponding frequency level will amplify the effect at each vertex and that speaker or woofer should be tuned to minimize the quivering.

Fuzzifying the frequency levels, let universe of discourse be the interval,

u $\epsilon$ [20Hz, 20kHz]. Let A denote the frequency levels,

$$\mu_A(u) = \begin{cases} 0 & \text{for} \quad u \leq 20Hz \\ 0.2 & \text{for} \quad 20Hz \leq u \leq 200Hz \\ 0.5 & \text{for} \quad 200Hz \leq u \leq 2.5\,kHz \\ 0.8 & \text{for} \quad 2.5kHz \leq u \leq 20\,kHz \\ 1 & \text{for} \quad u \geq 20kHz \end{cases}$$

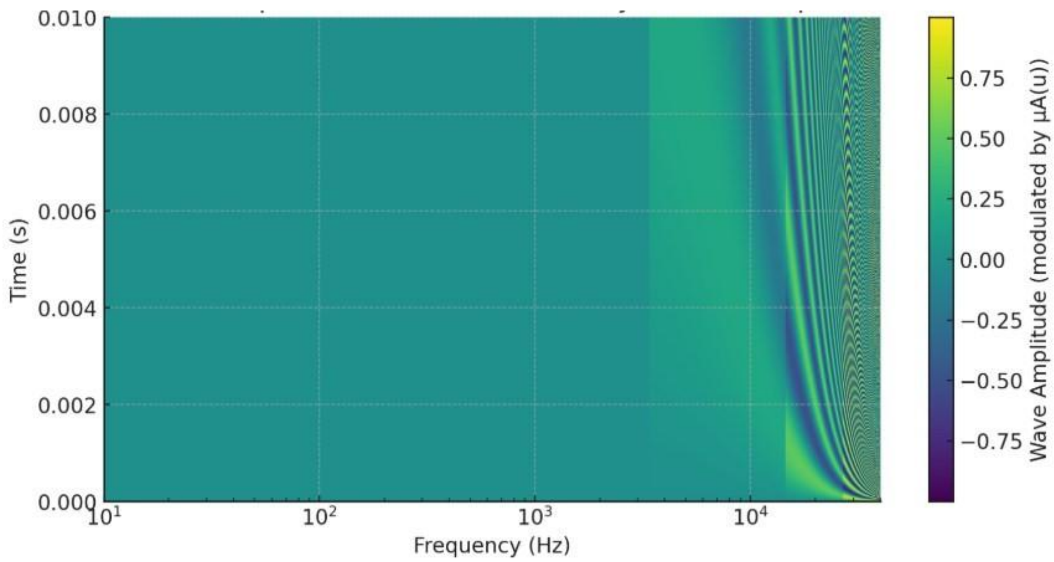


Fig.1 Sound wave representation modulated by membership function

In Fig 1. the sound wave-style visualization of your membership function

- The x-axis shows frequency (Hz, log scale)
- The y-axis shows time (s)
- The colour intensity represents the wave amplitude, modulated according to the membership values $\mu_A(u)$

So, low frequencies (≤ 20 Hz) are suppressed, mid frequencies (200 Hz – 2.5 kHz) are moderately strong, and high frequencies (≥ 20 kHz) are fully amplified.

Consider a sedan car with 5 speakers and 4 passengers as in Fig. 1. Depicting a fuzzy graph by considering the speakers and passengers as vertices and sound frequency between them as edges by considering it as a weighted edge. The vertex membership function denotes the maximum frequency from the respective speaker/woofer and the maximum frequency level that is acceptable for each passenger.

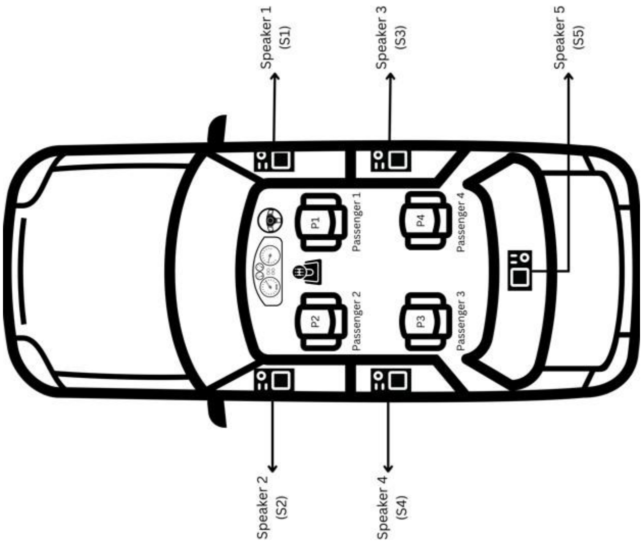


Fig. 1 Considering 5 speakers S1, S2, S3, S4, S5 and 4 passengers P1, P2, P3, P4

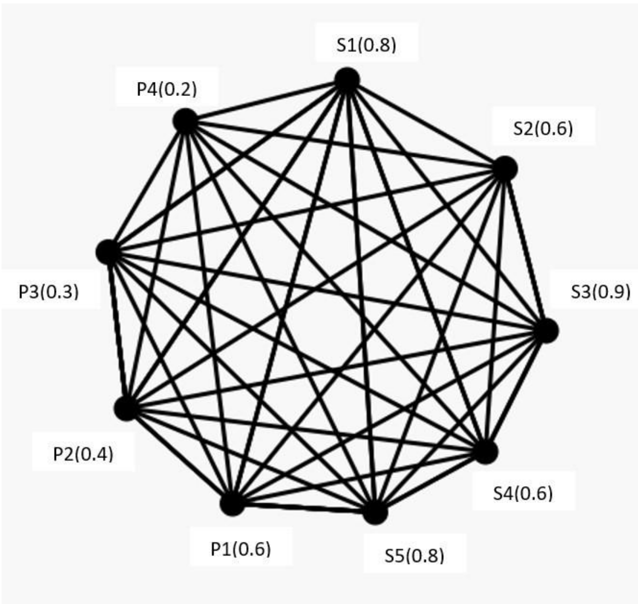


Fig. 2 Constructed fuzzified complete graph.

In addition, the analysis also considers the contribution of passenger-induced noise, which overlaps with the intrinsic noise of the system. As illustrated in Fig. 2, the resulting structure can be represented as a fuzzified complete graph. The fuzzification process is carried out by assigning frequency levels to the edges in such a way that each edge is labeled with minimum of its corresponding sigma values, that is,

edge (S1, S2) = $\mu_1$= min (0.8, 0.6) = 0.6

edge (S2, S3) = $\mu_2$= min (0.6, 0.9) = 0.6

edge (S3, S4) = $\mu_3$= min (0.9, 0.6) = 0.6

edge (S4, S5) = $\mu_4$= min (0.8, 0.6) = 0.6

edge (S5, P1) = $\mu_5$= min (0.8, 0.6) = 0.6

edge (P1, P2) = $\mu_6$= min (0.4, 0.6) = 0.4

edge (P2, P3) = $\mu_7$= min (0.4, 0.3) = 0.3

edge (P3, P4) = $\mu_8$= min (0.3, 0.2) = 0.2

edge (P4, S1) = $\mu_9$= min (0.8, 0.2) = 0.2

This approach ensures that the underlying structure forms a complete fuzzy graph, effectively capturing the interaction between system noise and passenger-induced disturbances.

Using the algorithm [10] the Laplacian spectrum of the complete fuzzy graph is calculated as follows. Since there are nine vertices, there will be nine eigen values are 4.1, 4.1, 4.1, 4.1, 3.1, 2.5, 1.8, 1.8, 0.

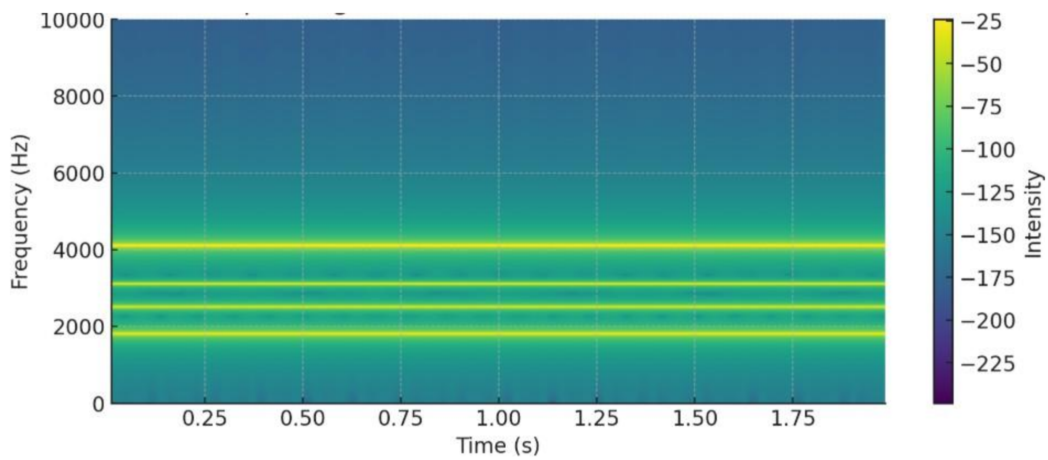


Fig 3. Spectrogram of combined sound wave

Here are the sound-wave representations for the Laplacian spectrum 4.1, 4.1, 4.1, 4.1, 3.1, 2.5, 1.8, 1.8 considered as combined frequencies in kHz

- Time-domain waveform (first 10 ms) — shows the actual oscillations of the mixed signal
- Spectrogram — shows constant lines near 1.8 kHz, 2.5 kHz, 3.1 kHz, and 4.1 kHz, which are the frequencies present in the mix

The spectrogram shows several prominent horizontal yellow lines at distinct frequency levels (e.g., around 2000 Hz, 2500 Hz, 3000 Hz, 4000 Hz). These bright, continuous lines indicate the presence of strong, consistent frequency components (or pure tones/harmonics) throughout the duration of the sound. The surrounding green and blue areas represent lower intensity frequencies, which could be background noise or other less prominent sound components.

Hence, by tuning the frequency level according to the Laplacian spectrum, the vibration can be minimized in a vehicle.

In this way, consumers can minimize the vibration caused and make the travelling comfortable by customizing their car stereo system by analyzing its Laplacian spectrum then tuning the frequency level and then assembling the speakers accordingly with the help of an engineer.

## 4 Conclusion

The investigation of complete fuzzy graphs and the formulation of an effective algorithm for deriving the Laplacian spectrum directly from vertex membership values establish a significant foundation for spectral analysis in fuzzy environments. Beyond its theoretical contribution, this approach opens avenues for extending spectral techniques to more complex fuzzy graph structures and real-world applications. In particular, it provides a promising framework for optimizing systems that inherently involve uncertainty and imprecision, such as stereophonic customization in electric vehicles, thereby linking mathematical innovation with practical technological advancement.

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