# RADIO ANTIPODAL CONTRA HARMONIC MEAN GRAPHS

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**Abstract:** In graph theory, graph labeling is a technique that is evolving quickly. A novel graph labeling parameter, radio antipodal contra harmonic mean labeling, has been defined in this study. Radio antipodal contra harmonic mean labeling is also investigated for existence and nonexistence of graphs. Additionally, the procedure for determining the radio antipodal contra harmonic mean number of mean graphs is also displayed.

**Keywords:** distance, diameter, radio antipodal contra harmonic mean number.

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## 1. Introduction

Graph labeling was introduced by Rosa A in 1969. The radio contra harmonic mean number of graphs was introduced by Ashika T S and Asha S[3]. Antony Xavier D, Thivyarathi R C defined radio antipodal mean number of certain graphs[2]. Motivated by the above concepts here we are introducing the concept radio antipodal contra harmonic mean graphs. For basic terminologies we referred Frank Harary and J Gallian[4, 5]. In this paper helm[5], cycle[4] and sparkler graph[1] are examined with respect to radio antipodal contra harmonic mean labeling.

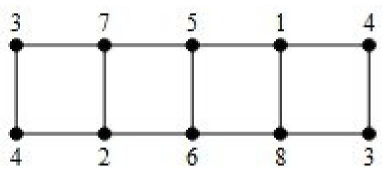
## 2. Main Results

**Definition 2.1.** The radio antipodal contra harmonic mean labeling of $G$ is a function $f$ that assigns to each vertex a non-negative integer such that $f(u) \neq f(v)$ if $d(u,v) < diam(G)$, $f(u) = f(v)$ if $d(u,v) = diam(G)$ then

$$d(u,v) + \left\lceil \frac{(f(u))^2 + (f(v))^2}{f(u)+f(v)} \right\rceil \geq diam(G) \qquad (2.1)$$

for any two distinct vertices $u, v \in V(G)$. The radio antipodal contra harmonic mean number of $f$denoted by $rachmn(f)$, is the maximum number assigned to any vertex of $G$. The radio antipodal contra harmonic mean number of $G$, denoted by $rachmn(G)$ is the minimum value of $rachmn(f)$ taken over all radio antipodal contra harmonic mean labelings $f$ of $G$. A graph which admit radio antipodal contra harmonic mean labeling is called a radio antipodal contra harmonic mean graph.

**Example 2.2.** The below graph $G$ depicts the radio antipodal contra harmonic mean labeling of $G$ and $rachmn(G) = 8$.



**Theorem 2.3.** *Let $G$ be a connected graph. If $G$ has exactly only one $u$-$v$ path with $d(u,v) = diam(G)$ then $G$ is a radio antipodal contra harmonic mean graph.*

**Proof:** Let $G$ be a connected graph with exactly only one $u$-$v$ path $u = v_0, v_1, v_2, \ldots, v_{n-1}, v_n = v$ with $d(u,v) = diam(G)$.

To prove $G$ is a radio antipodal contra harmonic mean graph.

Define a mapping $f: V(G) \to \mathbb{Z}^+$. Since there is only one such $u$-$v$ path we can assign $f(u) = f(v) = diam(G) - 1$ and $f(v_i) = diam(G) + i - 1$ for $1 \le i \le n-1$.

If there is any other vertices other than the vertices in this path will receive consecutive labels next to $diam(G) + n - 3$. Clearly $G$ admits radio antipodal contra harmonic mean labeling and hence $G$ is a radio antipodal contra harmonic mean graph.

**Corollary 2.4.** *Converse of the theorem need not be true.*

**Theorem 2.5.** *$C_n$ will not admit radio antipodal contra harmonic mean labelling for odd $n \ge 5$.*

**Proof:** If $n$ is odd then $diam(C_n) = \lfloor \frac{n}{2} \rfloor$. Then from $u_1$ the vertices $u_{\lfloor \frac{n}{2} \rfloor}$ and $u_{\lfloor \frac{n}{2} \rfloor + 1}$ are at distance $\lfloor \frac{n}{2} \rfloor$ that is, $d(u_1, u_{\lfloor \frac{n}{2} \rfloor}) = d(u_1, u_{\lfloor \frac{n}{2} \rfloor + 1}) = diam(C_n)$. Then $f(u_1) = f(u_{\lfloor \frac{n}{2} \rfloor}) = f(u_{\lfloor \frac{n}{2} \rfloor + 1})$. But $d(u_{\lfloor \frac{n}{2} \rfloor}, u_{\lfloor \frac{n}{2} \rfloor + 1}) = 1 \ne diam(C_n)$, therefore $f(u_{\lfloor \frac{n}{2} \rfloor}) \ne f(u_{\lfloor \frac{n}{2} \rfloor + 1})$, which is not possible. Therefore $C_n$ will not admit radio antipodal contra harmonic mean labeling for odd $n \ge 5$.

**Theorem 2.6.** *For $n \le 8$, $rachmn(C_n) = diam(C_n)$ if and only if $n$ is even or $n = 3$.*

**Proof:** Let $V(C_n) = \{v_i : 1 \le i \le n\}$ and $E(C_n) = \{v_i v_{i+1}, v_1 v_n : 1 \le i \le n-1\}$.

Assume that $n$ is even with $n \le 8$ or $n = 3$. To prove $rachmn(C_n) = diam(C_n)$. Since $n$ is even $diam(C_n) = \lfloor \frac{n}{2} \rfloor = d(v_i, v_{\lfloor \frac{n}{2} \rfloor + 1})$. Then inequality (2.1) reduces to

$$d(u,v) + \left\lceil \frac{(f(u))^2 + (f(v))^2}{f(u) + f(v)} \right\rceil \ge \lfloor \frac{n}{2} \rfloor \qquad (2.2)$$

Now define a function $f: V(C_n) \to \mathbb{Z}^+$ as follows:

If $n = 3$ assign $f(v_i) = 1$ for $1 \le i \le 3$.

If $n = 4$ or 6 assign $f(v_i) = f(v_{\frac{n}{2}+i}) = i$ for $1 \le i \le \frac{n}{2}$.

If $n = 8$ assign $f(v_i) = f(v_{\frac{n}{2}+i}) = i$ for $i = 1, \frac{n}{2}$ and $f(v_i) = f(v_{\frac{n}{2}+i}) = 5 - i$ for $i = 2,3$. It is clear that every distinct pair of vertices will hold the inequality (2.2) and the largest integer assigned is $\frac{n}{2} = diam(C_n)$.

Conversely, assume $rachmn(C_n) = diam(C_n)$ for $n \le 8$. To prove $n$ is even or $n = 3$. Suppose $n$ is odd and $n > 3$. Then by theorem 2.5, $C_n$ will not admit radio antipodal contra harmonic mean labeling, which a contradiction to the existence of $rachmn(C_n)$. Hence $n$ is even or $n = 3$.

**Theorem 2.7.** If $n$ is even and $n \ge 10$, then $diam(C_n) < rachmn(C_n) < |V(C_n)|$.

**Proof:** Let $V(C_n) = \{v_i : 1 \le i \le n\}$ and $E(C_n) = \{v_i v_{i+1}, v_1 v_n : 1 \le i \le n-1\}$.

Assume that $n$ is even with $n \ge 10$. To prove $diam(C_n) < rachmn(C_n) < |V(C_n)|$.

Since $n$ is even, $diam(C_n) = \frac{n}{2} = d(v_i, v_{\frac{n}{2}+i})$. Then inequality (2.1) reduces to

$$d(u,v)+\left\lceil\frac{(f(u))^2+(f(v))^2}{f(u)+f(v)}\right\rceil\geq\frac{n}{2} \tag{2.3}$$

Now define a function $f:V(C_n)\to\mathbb{Z}^+$ such that $f(v_i)=f(v_{\frac{n}{2}+i})=\frac{n}{2}-1+i$ for $1\leq i\leq\frac{n}{2}$.

Clearly every distinct pair of vertices will hold the inequality and largest label assigned is $n-1$. Thus $rachmn(C_n)=n-1<|V(C_n)|$. Therefore $rachmn(C_n)<|V(C_n)|$.

Claim: $diam(C_n)<rachmn(C_n)$

Suppose $diam(C_n)\geq rachmn(C_n)$. If $diam(C_n)=rachmn(C_n)$ then by theorem 2.6, $n\leq 8$ which is not possible.

If $diam(C_n)>rachmn(C_n)$ then there are $\frac{n}{2}$ pair of vertices with geodesic distance $\frac{n}{2}$ between them and hence $\frac{n}{2}$ labels must be needed.

Suppose 1 and 2 are the two labels in that $\frac{n}{2}$ labels then the inequality 2.3 will not hold. So we have to remove any one label from this and choose the label $\frac{n}{2}+1$ and so on until the inequality hold.

Thus $rachmn(C_n)\geq\frac{n}{2}+1$ implies $rachmn(C_n)>\frac{n}{2}$. Therefore $rachmn(C_n)>diam(G)$. Clearly $diam(C_n)>rachmn(C_n)$ is not possible.

Thus $diam(C_n)<rachmn(C_n)$.

Hence $diam(C_n)<rachmn(C_n)<|V(C_n)|$.

**Theorem 2.8.** *If* $m=2,n\geq 2, rachmn(P_m^{+n})=m$ *and for* $m>2,n\geq 2$ *sparkler graph* $P_m^{+n}$ *does not admit radio antipodal contra harmonic mean labeling*.

**Proof:** Let $V(P_m^{+n})=\{u_i,v_j:1\leq i\leq m,1\leq j\leq n\}$ and $E(P_m^{+n})=\{u_iu_{i+1}+,u_mv_j:1\leq i\leq m-1,1\leq j\leq n\}$. The diameter of $P_m^{+n}$, $diam(P_m^{+n})=m$, then the inequality (2.1) reduces to $d(u,v)+\left\lceil\frac{(f(u))^2+(f(v))^2}{f(u)+f(v)}\right\rceil\geq m$

Now define a function $f:V(P_m^{+n})\to\mathbb{Z}^+$.

Now for $m>2,n\geq 2$, $d(u_1,v_j)=diam(P_m^{+n})=m$ for $1\leq j\leq n$ then $f(u_1)=f(v_j)$ for $j=1,2,3,\ldots n$. But $d(v_i,v_j)=2$ for $i\neq j$ and $f(v_i)\neq f(v_j)$ which is not possible. Therefore $P_m^{+n}$ will not admit radio antipodal contra harmonic mean labelling for $m>2,n\geq 2$. Hence $P_m^{+n}$ is not a radio antipodal contra harmonic mean graph for $m>2,n\geq 2$.

**Theorem 2.9.** *If $G$ is a radio antipodal contra harmonic mean graph then $G$ is connected.*

**Proof:** Assume that $G$ is a radio antipodal contra harmonic mean graph.

To prove $G$ is connected.

Suppose $G$ is not connected. Then $G$ must have atleast two components. Clearly for some pair of vertices $u$ and $v$ there is no path between them. Thus $d(u,v)$ is not defined, which is not possible since $G$ is a radio antipodal contra harmonic mean graph. Therefore our assumption is wrong.

Hence $G$ is connected.

**Corollary 2.10.** *Converse of the above theorem is not true.*

**Proof**: Proof is obvious from theorems 2.8.

## 3. Conclusion

We introduced radio antipodal contra harmonic mean graphs and looked at some of their behavior in this study. The characteristics of radio antipodal contra harmonic mean graphs will be examined in subsequent research.

## References

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