The Introduction to the Conjugate of an Intuitionistic Multi L–Fuzzy Subgroup and the Strongest Intuitionistic Multi L–Fuzzy Relation

C.GOWRISHANKAR[1], **M. AKILESH**[2]
[1] Research Scholar, Department of Mathematics, SRMV College of Arts and Science, Coimbatore-641 020, India. Email: cgowrishankar88@gmail.com
[2] Department of Mathematics, SRMV College of Arts and Science, Coimbatore-641 020, Tamil Nadu, India.

Abstract

In this paper, the theory of intuitionistic multi L-fuzzy subgroups plays a significant role in extending classical group theory under uncertainty. In this work, we introduce the notion of the conjugate of an intuitionistic multi L–fuzzy subgroup and establish its fundamental properties in relation to group operations. This concept provides a deeper understanding of subgroup behavior under conjugation while accounting for multiple levels of membership, non-membership, and hesitation. Furthermore, we define the strongest intuitionistic multi L–fuzzy relation and explore its role in preserving the structural characteristics of fuzzy subgroups. These two notions together enrich the algebraic framework of intuitionistic multi L-fuzzy theory, laying a foundation for both theoretical development and practical applications in decision making, information processing, and related fields.

Keywords: Intuitionistic Multi L-Fuzzy Subset (IMLFS), Intuitionistic Multi L–Fuzzy Subgroup (IMLFSG), Conjugate of Intuitionistic Multi L – Fuzzy Subgroup (CIMLFSG), Strongest Intuitionistic Multi L–Fuzzy Relation (SIMLFR).

1. Introduction

L. A. Zadeh [9] established fuzzy set theory in 1965, and since then, it has shown promise in a variety of academic domains. Numerous studies have also been done on the idea of a fuzzy set's generalization. Krassimir T. Atanassov [1] introduced the concept of the intuitionistic fuzzy set in 1983. It addresses the degree of membership, the degree of hesitation, and the degree of non-membership function. The generalization of the Cartesian product of fuzzy subgroups was developed by B.A. Ersoy [2].

A few years later, Sabu Sebastian and T. V. Ramakrishnan and others [3-8] presented the notion of multi-fuzzy sets using a multi-dimensional membership function. The conjugate of an intuitionistic multi L-fuzzy subgroup and the strongest intuitionistic multi L-fuzzy relation are studied in this research, along with some of their features that are mentioned.

2. Preliminaries

The basic definitions that are utilized in the sequel are listed in this section.

2.1 Definition

Let X be a non-empty set. A fuzzy set A of X is defined by $A: X \to [0,1]$.

2.2 Definition

Let $(G,.)$ be a group. A fuzzy subset A of G is said to be L-fuzzy subgroup (LFSG) of G, if the following conditions are satisfied:

(i) $\mu(xy) \geq min\{\mu(x), \mu(y)\}, \forall\, x, y \in G.$

(ii) $\mu(x^{-1}) = \mu(x), \forall\, x \in G.$

2.3 Definition

Let X be non-empty set. Let $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x\epsilon X\}$ in X is defined as a set of ordered sequences.

ie., $A = \{\langle x, (\mu_{A_1}(x), \mu_{A_2}(x), \ldots., \mu_{A_i}(x), \ldots.), (\gamma_{A_1}(x), \gamma_{A_2}(x), \ldots., \gamma_{A_i}(x), \ldots.)\rangle : x\epsilon X\}$.

Where $\mu_{A_i}: X \to [0,1]$, $\gamma_{A_i}: X \to [0,1]$ and $0 \leq \mu_{A_i}(x) + \gamma_{A_i}(x) \leq 1$ for all i.

Here, $\mu_{A_1}(x) \geq \mu_{A_2}(x) \geq. \ldots. \geq \mu_{A_i}(x) \geq. \ldots.$, for all $x\epsilon X$ are decreasingly ordered sequence. Then the set A is said to be an Intuitionistic Multi L-fuzzy subset (IMLFS) of X.

Remark

Since we arrange the membership sequence in decreasing order, the corresponding non-membership sequence may not be in decreasing or increasing order.

2.4 Definition

The Intuitionistic Multi L-fuzzy subset $A = \{\langle x, \mu_{A_i}(x), \gamma_{A_i}(x)\rangle : x\epsilon X\}$ of a group G is said to be Intuitionistic Multi L-fuzzy subgroup of G (IMLFSG) if it satisfies the following: For all $x, y\epsilon G$,

(i) $\mu_{A_i}(xy) \geq min\{\mu_{A_i}(x), \mu_{A_i}(y)\}$ and $\gamma_{A_i}(xy) \leq max\{\gamma_{A_i}(x), \gamma_{A_i}(y)\}$,

(ii) $\mu_{A_i}(x^{-1}) = \mu_{A_i}(x)$ and $\gamma_{A_i}(x^{-1}) = \gamma_{A_i}(x)$.

Or Equivalentlyμ is IMLFSG of G iff

$$\mu_{A_i}(xy^{-1}) \geq min\{\mu_{A_i}(x), \mu_{A_i}(y)\} \text{ and } \gamma_{A_i}(xy^{-1}) \leq max\{\gamma_{A_i}(x), \gamma_{A_i}(y)\}.$$

3. The Conjugate of the Intuitionistic Multi L-Fuzzy Subgroup and the Strongest Intuitionistic Multi L–Fuzzy Relation

3.1 Definition

Let k be a positive integer and, let A and B be two intuitionistic multi L-fuzzy subsets of X and Y.

$A = \{\langle x, (\mu_{A_1}(x_1), \mu_{A_2}(x_2), \ldots., \mu_{A_k}(x_k)), (\gamma_{A_1}(x_1), \gamma_{A_2}(x_2), \ldots., \gamma_{A_k}(x_k))\rangle : x\epsilon X\}$ and

$B = \{\langle y, (\mu_{B_1}(y_1), \mu_{B_2}(y_2), \ldots., \mu_{B_k}(y_k)), (\gamma_{B_1}(y_1), \gamma_{B_2}(y_2), \ldots., \gamma_{B_k}(y_k))\rangle : y\epsilon Y\}$.

Then the Cartesian product AxB of two intuitionistic multi L- fuzzy subsets A and B is given by

$A \text{ x B} = \{\langle (x,y), \mu_{A_k \text{x} B_k}(x_k, y_k), \gamma_{A_k \text{x} B_k}(x_k, y_k) \rangle\}$

Where, $\mu_{A_k \text{x} B_k}(x_k, y_k) = \min\{\mu_{A_k}(x_k), \mu_{B_k}(y_k)\}$ and

$\gamma_{A_k \text{x} B_k}(x_k, y_k) = \max\{\gamma_{A_k}(x_k), \gamma_{B_k}(y_k)\}$, $\forall\ x\epsilon X$ and $y\epsilon Y$.

3.2 Definition

Let A and B be an IMLFSG's of a group G. Then A is said to be Conjugate of Intuitionistic Multi L – Fuzzy Subgroup (CIMLFSG) B of G, if there exists $g_i \in G_i$ such that for all $x_i \in G_i$ satisfies the following:

(i) $\mu_{A_i}(x_i) = \mu_{B_i}({g_i}^{-1} x_i g_i)$ and

(ii) $\gamma_{A_i}(x_i) = \gamma_{B_i}({g_i}^{-1} x_i g_i)$, for every g_i,$x_i \in G_i$.

3.3 Theorem

If A is an IMLFSG of a group G and B be an IMLF Subset of a group G. If A and B are conjugate IMLF Subsets of the group G, then B is an IMLFSG of a group G.

Proof

Given A is an IMLFSG of a group G and B be an IMLF Subset of a group G.

To prove that B is an IMLFSG of a group G.

i.e., to prove μ_{B_i} and γ_{B_i} be an IMLFSG's of a group G_i.

Let A and B are conjugate IMLF Subsets of the group G, then there exists $g_i \in G_i$ such that for all $x_i \in G_i$,

$$\mu_{A_i}(x_i) = \mu_{B_i}({g_i}^{-1} x_i g_i) \text{ for } i = 1,2, \ldots\ldots, n \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

and $$\gamma_{A_i}(x_i) = \gamma_{B_i}({g_i}^{-1} x_i g_i) \text{ for } i = 1,2, \ldots\ldots, n \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Let $x_i, y_i \in G_i$, then $x_i {y_i}^{-1} \in G_i$.

(i) Now, $\mu_{B_i}(x_i {y_i}^{-1}) = \mu_{A_i}({g_i}^{-1} x_i {y_i}^{-1} g_i)$

$= \mu_{A_i}({g_i}^{-1} x_i g_i {g_i}^{-1} {y_i}^{-1} g_i)$

$= \mu_{A_i}\big(({g_i}^{-1} x_i g_i)({g_i}^{-1} y_i g_i)^{-1}\big)$

$\geq min\{\mu_{A_i}({g_i}^{-1} x_i g_i), \mu_{A_i}({g_i}^{-1} y_i g_i)^{-1}\}$

$\geq min\{\mu_{A_i}({g_i}^{-1} x_i g_i), \mu_{A_i}({g_i}^{-1} y_i g_i)\}$ [∵ A is an IMLFSG of G]

$$= min\{\mu_{B_i}(x_i), \mu_{B_i}(y_i)\}$$ [∵ by using (1)]

i.e., $\mu_{B_i}(x_i {y_i}^{-1}) \geq min\{\mu_{B_i}(x_i), \mu_{B_i}(y_i)\}$

Hence, μ_{B_i} is an IMLFSG of the group G_i.

(ii) Also, $\gamma_{B_i}(x_i {y_i}^{-1}) = \gamma_{A_i}({g_i}^{-1} x_i {y_i}^{-1} g_i)$

$$= \gamma_{A_i}({g_i}^{-1} x_i g_i {g_i}^{-1} {y_i}^{-1} g_i)$$

$$= \gamma_{A_i}\big(({g_i}^{-1} x_i g_i)({g_i}^{-1} y_i g_i)^{-1}\big)$$

$$\leq max\{\gamma_{A_i}({g_i}^{-1} x_i g_i), \gamma_{A_i}({g_i}^{-1} y_i g_i)^{-1}\}$$

$$\leq max\{\gamma_{A_i}({g_i}^{-1} x_i g_i), \gamma_{A_i}({g_i}^{-1} y_i g_i)\}$$ [∵ A is an IMLFSG of G]

$$= max\{\gamma_{B_i}(x_i), \gamma_{B_i}(y_i)\}$$ [∵ by using (2)]

i.e., $\gamma_{B_i}(x_i {y_i}^{-1}) \leq max\{\gamma_{B_i}(x_i), \gamma_{B_i}(y_i)\}$

Hence, γ_{B_i} is an IMLFSG of the group G_i.

Hence by cases (i) & (ii), we have B is an IMLFSG of a group G.

3.4 Theorem

If an IMLFSG's $A_1, A_2, \ldots\ldots.., A_n$ are conjugate to the IMLFSG's $B_1, B_2, \ldots\ldots.., B_n$ be of groups $G_1, G_2, \ldots\ldots, G_n$ respectively. Then the product of an IMLFSG's $A_1 \times A_2 \times \ldots\ldots.. \times A_n$ is conjugate to the IMLFSG's $B_1 \times B_2 \times \ldots\ldots.. \times B_n$ are of $G_1 \times G_2 \times \ldots\ldots \times G_n$.

Proof

Given IMLFSG's $A_1, A_2, \ldots\ldots.., A_n$ are conjugate to the IMLFSG's $B_1, B_2, \ldots\ldots.., B_n$ be of groups $G_1, G_2, \ldots\ldots, G_n$ respectively.

Then prove that the product of an IMLFSG's $A_1 \times A_2 \times \ldots\ldots.. \times A_n$ is conjugate to the IMLFSG's $B_1 \times B_2 \times \ldots\ldots.. \times B_n$ are of $G_1 \times G_2 \times \ldots\ldots \times G_n$.

i.e., to prove $\mu_{A_1 \times A_2 \times \ldots\ldots\ldots \times A_n}$ and $\gamma_{A_1 \times A_2 \times \ldots\ldots\ldots \times A_n}$ are conjugate to the IMLFSG's $\mu_{B_1 \times B_2 \times \ldots\ldots\ldots \times B_n}$ and $\gamma_{B_1 \times B_2 \times \ldots\ldots\ldots \times B_n}$ be of groups $G_1 \times G_2 \times \ldots\ldots \times G_n$.

(i) By the definition of conjugate, if A be an IMLFSG of G conjugates to an IMLFSG B of G, then there exists $g_i \in G_i$ such that for all $x_i \in G_i$,

$$\mu_{A_i}(x_i) = \mu_{B_i}(g_i{}^{-1}x_i g_i) \quad \text{for} \quad i = 1,2, \dots \dots, n.$$

Thus we have

$$\mu_{A_1 \times A_2 \times \dots \dots \times A_n}(x_1, x_2, \dots \dots, x_n) \geq min\{\mu_{A_1}(x_1), \mu_{A_2}(x_2), \dots \dots \dots, \mu_{A_n}(x_n)\}$$
$$= min\{\mu_{B_1}(g_1{}^{-1}x_1 g_1), \mu_{B_2}(g_2{}^{-1}x_2 g_2), \dots \dots \dots, \mu_{B_n}(g_n{}^{-1}x_n g_n)\}$$
$$= \mu_{B_1 \times B_2 \times \dots \dots \times B_n}(g_1{}^{-1}x_1 g_1, g_2{}^{-1}x_2 g_2, \dots \dots \dots, g_n{}^{-1}x_n g_n)$$

$\therefore$ $\mu_{A_1 \times A_2 \times \dots \dots \times A_n}$ is conjugate to $\mu_{B_1 \times B_2 \times \dots \dots \times B_n}$ of $G_1 \times G_2 \times \dots \dots \times G_n$.

(ii) By the definition of conjugate, if A be an IMLFSG of G conjugates to an IMLFSG B of G, then there exists $g_i \in G_i$ such that for all $x_i \in G_i$,

$$\gamma_{A_i}(x_i) = \gamma_{B_i}(g_i{}^{-1}x_i g_i) \quad \text{for} \quad i = 1,2, \dots \dots, n.$$

Thus we have

$$\gamma_{A_1 \times A_2 \times \dots \dots \times A_n}(x_1, x_2, \dots \dots, x_n) \leq max\{\gamma_{A_1}(x_1), \gamma_{A_2}(x_2), \dots \dots \dots, \gamma_{A_n}(x_n)\}$$
$$= max\{\gamma_{B_1}(g_1{}^{-1}x_1 g_1), \gamma_{B_2}(g_2{}^{-1}x_2 g_2), \dots \dots \dots, \gamma_{B_n}(g_n{}^{-1}x_n g_n)\}$$
$$= \gamma_{B_1 \times B_2 \times \dots \dots \times B_n}(g_1{}^{-1}x_1 g_1, g_2{}^{-1}x_2 g_2, \dots \dots \dots, g_n{}^{-1}x_n g_n)$$

$\therefore$ $\gamma_{A_1 \times A_2 \times \dots \dots \times A_n}$ is conjugate to $\gamma_{B_1 \times B_2 \times \dots \dots \times B_n}$ of $G_1 \times G_2 \times \dots \dots \times G_n$.

Hence, the product IMLFSG's $A_1 \times A_2 \times \dots \dots \times A_n$ is conjugate to the IMLFSG's $B_1 \times B_2 \times \dots \dots \times B_n$ of the groups $G_1 \times G_2 \times \dots \dots \times G_n$.

3.5 Theorem

Let A and B be any two IMLFSG's of the group G. Then A and B are conjugate IMLFSG's of G iff $A = B$.

Proof

(i) Let A and B are conjugate IMLFSG's of the group G. To show that $A = B$.

i.e., to prove $\mu_{A_i} = \mu_{B_i}$ and $\gamma_{A_i} = \gamma_{B_i}$.

Since A and B are conjugate IMLFSG's of G, then there exists $g_i \in G_i$ such that for all $x_i \in G_i$, we have

$$\mu_{A_i}(x_i) = \mu_{B_i}(g_i{}^{-1}x_i g_i) \quad \text{for} \quad i = 1,2, \dots \dots, n \qquad \dots\dots\dots\dots (1)$$

and $$\gamma_{A_i}(x_i) = \gamma_{B_i}(g_i{}^{-1}x_i g_i) \quad \text{for} \quad i = 1,2, \dots \dots, n \qquad \dots\dots\dots\dots (2)$$

Let, $x_i = g_i x_i \in G_i$, then from (1) and (2) gives

$$\mu_{A_i}(g_i x_i) = \mu_{B_i}(g_i^{-1} g_i x_i g_i) \quad \text{and} \quad \gamma_{A_i}(g_i x_i) = \gamma_{B_i}(g_i^{-1} g_i x_i g_i)$$

$$= \mu_{B_i}(x_i g_i) \quad \text{and} \quad = \gamma_{B_i}(x_i g_i)$$

For some $g_i = e_i \in G_i$, we have

$$\mu_{A_i}(e_i x_i) = \mu_{B_i}(x_i e_i) \quad \text{and} \quad \gamma_{A_i}(e_i x_i) = \gamma_{B_i}(x_i e_i)$$

$$\mu_{A_i}(x_i) = \mu_{B_i}(x_i) \quad \text{and} \quad \gamma_{A_i}(x_i) = \gamma_{B_i}(x_i)$$

Therefore, $A = B$ is true.

(ii) Conversely, assume that $A = B$. To show that A and B are conjugate IMLFSG's of G.

Let, $\mu_{A_i} = \mu_{B_i}$ and $\gamma_{A_i} = \gamma_{B_i}$

$$\mu_{A_i}(x_i) = \mu_{B_i}(x_i) \quad \text{and} \quad \gamma_{A_i}(x_i) = \gamma_{B_i}(x_i)$$

$$\mu_{A_i}(x_i) = \mu_{B_i}(e_i^{-1} x_i e_i) \quad \text{and} \quad \gamma_{A_i}(x_i) = \gamma_{B_i}(e_i^{-1} x_i e_i), \quad \forall\ x_i, e_i \in G_i.$$

Hence, A and B are conjugate IMLFSG's of the group G.

3.6 Theorem

Let A_i, A_{i+1} be an IMLFSG's of the group G_i and B_j, B_{j+1} be IMLFSG's of the group G_j respectively. Suppose, A_i, A_{i+1} are conjugate of IMLFSG's of G_i and B_j, B_{j+1} are conjugate of IMLFSG's of G_j. Then the generalized Cartesian product of IMLFSG $A_i \times B_j$ of $G_i \times G_j$ is conjugate to the IMLFSG $A_{i+1} \times B_{j+1}$ of $G_i \times G_j$, for each $i = 1,2, \ldots., n$; $j = 1,2, \ldots\ldots, n$.

Proof

Given A_i, A_{i+1} be an IMLFSG's of the group G_i and B_j, B_{j+1} be IMLFSG's of the group G_j respectively.

Let A_i and A_{i+1} are conjugate of IMLFSG's of G_i, then there exists $g_i \in G_i$ such that

$$\mu_{A_i}(x_i) = \mu_{A_{i+1}}(g_i^{-1} x_i g_i) \quad \text{and} \quad \gamma_{A_i}(x_i) = \gamma_{A_{i+1}}(g_i^{-1} x_i g_i), \ \forall\, x_i \in G_i \quad \ldots\ldots\ldots\ldots\ldots.. (1)$$

Also B_j and B_{j+1} are conjugate of IMLFSG's of G_j, then there exists $g_j \in G_j$ such that

$$\mu_{B_j}(y_j) = \mu_{B_{j+1}}(g_j^{-1} y_j g_j) \quad \text{and} \quad \gamma_{B_j}(y_j) = \gamma_{B_{j+1}}(g_j^{-1} y_j g_j), \ \forall\, y_j \in G_j \quad \ldots\ldots\ldots\ldots\ldots. (2)$$

To prove that the generalized Cartesian product of IMLFSG $A_i \times B_j$ of $G_i \times G_j$ is conjugate to the IMLFSG $A_{i+1} \times B_{j+1}$ of $G_i \times G_j$.

(i) Now, $\mu_{A_i \times B_j}(x_i, y_j) = min\left\{\mu_{A_i}(x_i), \mu_{B_j}(y_j)\right\}$

$$= min\left\{\mu_{A_{i+1}}(g_i^{-1}x_ig_i), \mu_{B_{j+1}}(g_j^{-1}y_jg_j)\right\} \quad [\because \text{by using (1)}]$$

$$= \mu_{A_{i+1} \times B_{j+1}}\left((g_i^{-1}x_ig_i), (g_j^{-1}y_jg_j)\right)$$

$$= \mu_{A_{i+1} \times B_{j+1}}\left((g_i^{-1}, g_j^{-1})(x_i, y_j)(g_i, g_j)\right)$$

i.e., $\mu_{A_i \times B_j}(x_i, y_j) = \mu_{A_{i+1} \times B_{j+1}}\left((g_i^{-1}, g_j^{-1})(x_i, y_j)(g_i, g_j)\right)$

(ii) Also, $\gamma_{A_i \times B_j}(x_i, y_j) = max\left\{\gamma_{A_i}(x_i), \gamma_{B_j}(y_j)\right\}$

$$= max\left\{\gamma_{A_{i+1}}(g_i^{-1}x_ig_i), \gamma_{B_{j+1}}(g_j^{-1}y_jg_j)\right\} \quad [\because \text{ by using (2)}]$$

$$= \gamma_{A_{i+1} \times B_{j+1}}\left((g_i^{-1}x_ig_i), (g_j^{-1}y_jg_j)\right)$$

$$= \gamma_{A_{i+1} \times B_{j+1}}\left((g_i^{-1}, g_j^{-1})(x_i, y_j)(g_i, g_j)\right)$$

i.e., $\gamma_{A_i \times B_j}(x_i, y_j) = \gamma_{A_{i+1} \times B_{j+1}}\left((g_i^{-1}, g_2^{-1})(x_i, y_j)(g_i, g_j)\right)$

Hence, by cases (i) & (ii) gives the generalized Cartesian product of IMLFSG $A_i \times B_j$ is conjugate to the IMLFSG $A_{i+1} \times B_{j+1}$ of $G_i \times G_j$.

Hence the proof.

3.7 Definition

Let A be an Intuitionistic Multi L-Fuzzy subset of a group G, the Strongest Intuitionistic Multi L-Fuzzy relation on A is R given by

(i) $\mu_{R_i}(x, y) = min\{\mu_{A_i}(x), \mu_{A_i}(y)\}$ and

(ii) $\gamma_{R_i}(x, y) = max\{\gamma_{A_i}(x), \gamma_{A_i}(y)\}$, for every $x, y \in G$.

3.8 Theorem

Let A be an IMLF Subset of a group G and R be the Strongest Intuitionistic Multi L-Fuzzy relation on G. Then A is an IMLFSG of G if and only if R is an IMLFSG of $G \times G$.

Proof

Given, A be an IMLF Subset of a group G and R be the Strongest Intuitionistic Multi L - Fuzzy relation on G.

(i) First, we assume A be an IMLFSG of a group G.

To prove that R is an IMLFSG of $G \times G$.

Let, $x = (x_i, x_j)\ and\ y = (y_i, y_j)$ be any two elements of $G \times G$.

Now, $\mu_{R_i}(xy^{-1}) = \mu_{R_i}\left((x_i, x_j)(y_i, y_j)^{-1}\right)$

$= \mu_{R_i}\left((x_i, x_j)(y_i^{-1}, y_j^{-1})\right)$

$= \mu_{R_i}(x_i y_i^{-1}, x_j y_j^{-1})$

$= min\{\mu_{A_i}(x_i y_i^{-1}), \mu_{A_i}(x_j y_j^{-1})\}$ [∵ by definition 3.7]

$\geq min\{min\langle\mu_{A_i}(x_i), \mu_{A_i}(y_i)\rangle, min\langle\mu_{A_i}(x_j), \mu_{A_i}(y_j)\rangle\}$ [∵ A is an IMLFSG of G]

$= min\{min\langle\mu_{A_i}(x_i), \mu_{A_i}(x_j)\rangle, min\langle\mu_{A_i}(y_i), \mu_{A_i}(y_j)\rangle\}$

$= min\{\mu_{R_i}(x_i, x_j), \mu_{R_i}(y_i, y_j)\}$ [∵ by definition 3.7]

$= min\{\mu_{R_i}(x), \mu_{R_i}(y)\}$

i.e., $\mu_{R_i}(xy^{-1}) \geq min\{\mu_{R_i}(x), \mu_{R_i}(y)\}$

Also, $\gamma_{R_i}(xy^{-1}) = \gamma_{R_i}\left((x_i, x_j)(y_i, y_j)^{-1}\right)$

$= \gamma_{R_i}\left((x_i, x_j)(y_i^{-1}, y_j^{-1})\right)$

$= \gamma_{R_i}(x_i y_i^{-1}, x_j y_j^{-1})$

$= max\{\gamma_{A_i}(x_i y_i^{-1}), \gamma_{A_i}(x_j y_j^{-1})\}$ [∵ by definition 3.7]

$\leq max\{max\langle\gamma_{A_i}(x_i), \gamma_{A_i}(y_i)\rangle, max\langle\gamma_{A_i}(x_j), \gamma_{A_i}(y_j)\rangle\}$ [∵ A is an IMLFSG of G]

$= max\{max\langle\gamma_{A_i}(x_i), \gamma_{A_i}(x_j)\rangle, max\langle\gamma_{A_i}(y_i), \gamma_{A_i}(y_j)\rangle\}$

$= max\{\gamma_{R_i}(x_i, x_j), \gamma_{R_i}(y_i, y_j)\}$ [∵ by definition 3.7]

$= max\{\gamma_{R_i}(x), \gamma_{R_i}(y)\}$

i.e., $\gamma_{R_i}(xy^{-1}) \le max\{\gamma_{R_i}(x), \gamma_{R_i}(y)\}$

Hence from (i) & (ii), we conclude that R is an IMLFSG of $G \times G$.

(ii) Conversely, we assume that R be an IMLFSG of a group $G \times G$.

To prove A is an IMLFSG of G.

Let, for all $x, y \in G$.

We know, $\mu_{A_i}(xy^{-1}) = min\{\mu_{A_i}(xy^{-1}), \mu_{A_i}(ee^{-1})\}$

$= \mu_{R_i}(xy^{-1}, ee^{-1})$ [∵ by definition 3.7]

$= \mu_{R_i}\big((x, e)(y^{-1}, e^{-1})\big)$

$= \mu_{R_i}\big((x, e)(y, e)^{-1}\big)$

$\ge min\{\mu_{R_i}(x, e), \mu_{R_i}(y, e)\}$ [∵ R is an IMLFSG of $G \times G$]

$= min\{min\langle\mu_{A_i}(x), \mu_{A_i}(e)\rangle, min\langle\mu_{A_i}(y), \mu_{A_i}(e)\rangle\}$ [∵ by definition 3.7]

$= min\{\mu_{A_i}(x), \mu_{A_i}(y)\}$

i.e., $\mu_{A_i}(xy^{-1}) \ge min\{\mu_{A_i}(x), \mu_{A_i}(y)\}$

Also, $\gamma_{A_i}(xy^{-1}) = max\{\gamma_{A_i}(xy^{-1}), \gamma_{A_i}(ee^{-1})\}$

$= \gamma_{R_i}(xy^{-1}, ee^{-1})$ [∵ by definition 3.7]

$= \gamma_{R_i}\big((x, e)(y^{-1}, e^{-1})\big)$

$= \gamma_{R_i}\big((x, e)(y, e)^{-1}\big)$

$\le max\{\gamma_{R_i}(x, e), \gamma_{R_i}(y, e)\}$ [∵ R is an IMLFSG of $G \times G$]

$= max\{max\langle\gamma_{A_i}(x), \gamma_{A_i}(e)\rangle, max\langle\gamma_{A_i}(y), \gamma_{A_i}(e)\rangle\}$ [∵ by definition 3.7]

$= max\{\gamma_{A_i}(x), \gamma_{A_i}(y)\}$

i.e., $\gamma_{A_i}(xy^{-1}) \le max\{\gamma_{A_i}(x), \gamma_{A_i}(y)\}$

Thus A is an IMLFSG of G.

Conclusion

The concepts of Intuitionistic Multi L-Fuzzy Subgroups have developed significantly with the help of Intuitionistic Multi L-Fuzzy Sets. This research aims to investigate the Conjugate of Intuitionistic Multi L-Fuzzy Subgroups. It also defines and discusses the Strongest Characteristics associated with Intuitionistic Multi L- Fuzzy Relations.

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